4.12 Consider the second-order plant

\[
G(s) = \frac{1}{(s+1)(5s+1)}
\]

(a) Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P, PD, and PID controllers (as configured in Fig. 4.28). Let \( k_p = 19, k_i = 0.5, k_d = \frac{4}{19} \).

(b) Determine the system type and error constant of the system with respect to disturbance inputs for each of the three regulators in part (a) with respect to rejecting polynomial disturbances \( w(t) \) at the input to the plant.

(c) Is this system better at tracking references or rejecting disturbances? Explain your response briefly.

(d) Verify your results for parts (a) and (b) using MATLAB by plotting unit step and ramp responses for both tracking and disturbance rejection.

**Solution:**

(a) Given \( k_p = 19, k_i = 0.5, k_d = \frac{4}{19} \),

- P control: \( D(s) = k_p = 19, \quad D(s)G(s) = \frac{19}{(s+1)(5s+1)}, \) No poles at \( s = 0, \) Type 0
  \[ K_p = \lim_{s \to 0} D(s)G(s) = 19 \]

- PD control: \( D(s) = k_ds + k_p = \frac{4}{19}s + 19, \quad D(s)G(s) = \frac{4}{19}s + \frac{19}{(s+1)(5s+1)} \), No poles at \( s = 0, \) Type 0
  \[ K_p = \lim_{s \to 0} D(s)G(s) = 19 \]

- PID control: \( D(s) = k_ds + k_p + k_i/s = \frac{4}{19}s + 19 + 0.5/s, \quad D(s)G(s) = \frac{4}{19}s^2 + 19s + 0.5}{(s+1)(5s+1)}, \) \( s = 0 \) is 1-order pole, Type 1, \( K_v = \lim_{s \to 0} sD(s)G(s) = 0.5 \)

(b) \( T_w(s) = \frac{E(s)}{W(s)} = \frac{-G(s)}{1 + D(s)G(s)} \)

- P control: \( D(s) = k_p = 19, \quad T_w(s) = \frac{-1}{(s+1)(5s+1)+19}, \) No zeros at \( s = 0, \) Type 0
  \[ K_{p,w} = \lim_{s \to 0} T_w(s) = -20 \]

- PD control: \( D(s) = k_ds + k_p = \frac{4}{19}s + 19, \quad T_w(s) = \frac{-1}{(s+1)(5s+1)+\frac{4}{19}s + 19}, \)
No zeros at $s = 0$, Type 0, $K_{p,w} = \frac{1}{\lim_{s \to 0} T_w(s)} = -20$

- PID control: $D(s) = k_ds + k_p + k_i / s = \frac{4}{19}s + 19 + 0.5 / s$,

$$T_w(s) = \frac{-s}{s(s+1)(5s+1) + \frac{4}{19}s^2 + 19s + 0.5},$$

$s = 0$ is 1-order zero, Type 1,

$$K_{v,w} = \frac{1}{\lim_{s \to 0} T_w(s) / s} = -0.5$$

(c) There is reduced oscillatory behavior brought on by addition of the derivative term, and increased oscillatory but lower error brought on by the integral term.

(d) Reference Tracking:

$$E(s) = \frac{1}{R(s)} = \frac{5s^2 + 6s + 1}{5s^2 + 6s + 20},$$

P Control

$$E(s) = \frac{1}{R(s)} = \frac{5s^2 + 6s + 1}{5s^2 + \frac{118}{19}s + 20}$$

PD control

$$E(s) = \frac{1}{R(s)} = \frac{5s^2 + \frac{118}{19}s + 20}{5s^3 + 6s^2 + s}$$

PID control

Disturbance Rejection:

$$E(s) = -\frac{G(s)}{W(s)} = \begin{cases} \frac{1}{5s^2 + 6s + 20}, & \text{P Control} \\ \frac{1}{5s^2 + \frac{118}{19}s + 20}, & \text{PD control} \\ \frac{s}{5s^3 + \frac{118}{19}s^2 + 20s + 0.5}, & \text{PID control} \end{cases}$$

P Control

$\text{PD control}$

$\text{PID control}$

For Ramp Response, Use $y = \text{lsim(sys}, t, t)$ in MATLAB.

The followings are the plots of responses of $e(t)$. Note the steady state errors,

<table>
<thead>
<tr>
<th>P Control</th>
<th>PD Control</th>
<th>PID Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(t)$</td>
<td>$P$</td>
<td>$PD$</td>
</tr>
<tr>
<td>$l(t)$</td>
<td>$0.05$</td>
<td>$0.05$</td>
</tr>
<tr>
<td>$t1(t)$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reference Tracking: $l(t)$</th>
<th>$0.05$</th>
<th>$0.05$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t1(t)$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$2$</td>
</tr>
</tbody>
</table>
Step response for disturbance rejection

-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
-0.0
0

Errors \( r(t) - y(t) \)

Time (sec)

P Control
PD Control
PID Control

Ramp response for disturbance rejection

-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0

Errors \( r(t) - y(t) \)

Time (sec)

P Control
PD Control
PID Control
4.22 The transfer function for the plant in a motor position control is given by

\[ G(s) = \frac{A}{s(s + a)} \]

If we were able to select values for both A and a, what would they be to result in a system with \( K_v = 20 \) and \( \zeta = 0.707 \)?

Solution:

\[
K_v = \lim_{s \to 0} sD(s)G(s) = \lim_{s \to 0} s \frac{A}{s(s + a)} = \frac{A}{a} = 20 \quad \Rightarrow \quad A = 20a
\]

\[
\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{A}{s^2 + as + A} = \frac{A}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad \Rightarrow \quad \omega_n^2 = A, \quad a = 2\zeta \omega_n
\]

\[
\zeta = \frac{a}{2\sqrt{A}} = 0.707
\]

Thus, \( a = 40, \quad A = 800 \).