Today:
- Introduction
  - Motivation
  - Course Overview
  - Course project
- Math. Descriptions of Systems ~ Review
  - Classification of Systems
  - Linear Systems
  - LTI Systems
1. INTRODUCTION

1.1 Motivation

- What is a "system"?
  - A physical process or a mathematical model of a physical process that relates one set of signals to another set of signals.
  - Examples: Air conditioner, cars, filters, bank accounts.

- Two general categories of signals/systems:
  - **Continuous-time (CT) signals/systems**
    - Examples: Speed/car, current/circuit, temperature/room.
    - Described by differential eqs., e.g., \( \frac{dy}{dt} = ay(t) + bu(t) \).
    - Signals themselves could be discontinuous. But defined for each time instant.
  - **Discrete-time (DT) signals/systems**
    - Examples: Money in a bank account, quarterly profit.
    - Sequence of numbers.
    - Input/output related by difference equations, e.g., \( y[k+1] = ay[k] + bu[k] \), (on a daily or monthly base).
  - DT and CT are quite similar, and will be treated in parallel.

- The goal of "System Theory"?
  - Establish input/output relationship through models,
  - Predict output from input, know how to produce desired output.
  - Alter input automatically (via controller) to produce desired output.
Example: A simple electric circuit

\[ i(t) \sim \text{Output} \]
\[ u(t) \sim \text{Input} \]

- Use physical laws to model/describe the behavior of the system:
  - What are the components? What properties do they have?
    \[
    v_R = R_i_R, \quad v_L = L \frac{di}{dt}, \quad i_C = C \frac{dv}{dt}
    \]
  - Relationship among the variables by physical law:
    - KCL: Current to a node = 0, \( i_R = i_C = i_L = i \).
    - KVL: Voltage across a loop = 0.

- An integral-differential or differential equation
- Input-output description or external description

How to analyze the input-output relationship?
- For example, find the output \( i(t) \) given \( u(t) \) and IC.

- We can use Laplace transform
  - Note: only effective for LTI systems
Laplace Transform, A Quick Review

\[ f(t) \Leftrightarrow F(s) = \int_{0^-}^{\infty} f(t)e^{-st} \, dt \]

- **Key Properties**
  - Linearity: \( a_1 f_1(t) + a_2 f_2(t) \Leftrightarrow a_1 F_1(s) + a_2 F_2(s) \)
  - Derivative theorem:
    \[ \dot{f}(t) \Leftrightarrow sF(s) - f(0^-), \quad \int f(\tau) \, d\tau \Leftrightarrow F(s)/s \]

- **Converting linear constant coefficient differential equations into algebraic equations**

- **Other properties**
  - Differentiation in the frequency domain: \( tf(t) \Leftrightarrow (-1)F'(s) \)
  - Convolution: \( h(t) * f(t) \Leftrightarrow H(s)F(s) \)
  - Time and frequency shifting:
    \[ f(t-t_0)U(t-t_0) \Leftrightarrow e^{-st_0}F(s); \quad e^{st_0}f(t) \Leftrightarrow F(s - s_0) \]

- Time and frequency scaling: \( f(at) \Leftrightarrow 1/a F(s/a) \) for \( a > 0 \)
- Initial Value Theorem: \( f(0^+) = \lim_{s \to \infty} sF(s) \)
- Final Value Theorem: \( f(\infty) = \lim_{s \to 0} sF(s) \) if all the poles of \( sF(s) \) have strictly negative real parts

**Example (Continued)**

\[ R \frac{di}{dt} + \frac{1}{C} \int_{0}^{t} i(\tau) \, d\tau + v_0 = u(t) \]
\[ Ri(s) + L\left[s\hat{i}(s) - i_0\right] + \frac{\hat{i}(s)}{Cs} + \frac{v_0}{s} = \hat{u}(s) \]
- An algebraic equation vs integral-differential equation. Solution:
  \[
  \left( Ls + R + \frac{1}{Cs} \right) \hat{i}(s) = \hat{u}(s) + \left[ L\hat{i}_0 - \frac{v_0}{s} \right]
  \]
  \[
  \hat{i}(s) = \frac{cs}{LCs^2 + RCs + 1} \hat{u}(s) + \frac{LC\hat{i}_0 - cv_0}{LCs^2 + RCs + 1}
  \]

- Is there any pattern with the equation?
  - It has two components, one caused by input, and the other by IC

- How about the voltage across the capacitor?
  \[
  \hat{v}(s) = \frac{\hat{i}(s)}{Cs} + \frac{v_0}{s} = \frac{1}{LCs^2 + RCs + 1} \hat{u}(s) + \frac{Li_0 + \left( LC + RC \right) v_0}{LCs^2 + RCs + 1}
  \]

- What is the system's transfer function?

- Assume that the ICs are zero, then
  \[
  \hat{u}(s) \xrightarrow{g(s)} \hat{g}(s) \xrightarrow{i(s)} \hat{i}(s) = g(s) \hat{u}(s)
  \]
  \[
  \hat{i}(s) = \frac{Cs}{LCs^2 + RCs + 1} \hat{u}(s) \quad \hat{g}(s) = \frac{Cs}{LCs^2 + RCs + 1}
  \]
  - Frequency domain analysis

- How to obtain the response in time domain?
  \[
  i(t) = L^{-1}\left\{ \hat{i}(s) \right\}
  \]
  - Suppose that \( L = C = 1, R = 2, v_0 = i_0 = 0, \) and \( u(t) = U(t) \) (unit step function). Then
\[ \hat{u}(s) = \frac{1}{s} \quad \hat{i}(s) = \frac{Cs \hat{u}(s)}{LCs^2 + RCs + 1} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s + 1)^2} \]

\[ i(t) = te^{-t} \]

- Does this make sense for the circuit?

- Limitation of Laplace transform: not effective for time varying/nonlinear systems such as

\[ \ddot{y}(t) + a_1(y, t)\dot{y}(t) + a_0(y, t)y(t) = b(y, t)u(t) \]

- The \textbf{state space description} to be studied in this course will be able to handle more general systems
  - How can we do it?
  - Properties can be characterized without solving for the exact output

- To get some general idea about state space description, we consider the same circuit system.
State-Space Description
- State variables: Voltage across C and current through L
- State equation:

\[
\begin{align*}
\frac{dv}{dt} &= (1/C)i \\
\frac{di}{dt} &= -(R/L)i - (1/L)v + (1/L)u
\end{align*}
\]

- A set of first-order differential equations

It describes the behaviors inside the system by using the state variables \(v(t)\) and \(i(t)\)

How to describe the output?

\[
y = i = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix}
\]

- The output equation

Combined with the state equation, we have the state-space description or internal description

\[
\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} u,
\]

\[
x = Ax + Bu
\]

\[
y = Cx
\]
A general form:
\[
\begin{align*}
\dot{x} &= f(x,u,t), \\
y &= h(x,u,t),
\end{align*}
\] 
\[
x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_q \end{bmatrix}
\]

Main features of the state-space approach
- It describes the behaviors inside the system
- Characterizes stability and performances without solving the differential equations
- Applicable to more general systems, nonlinear, time-varying, uncertain, hybrid
- Most recent advancements in control theory are developed via a state-space approach

1.2 Course Overview

Textbook:

Reference:
- Peter B. Luh, Introduction to Systems Theory, Lecture note, University of Connecticut, Fall 2004
- **Goals:** To achieve a thorough understanding about systems theory and multivariable system design
- **Tentative Outline** (13 lectures):
  - Introduction
  - Modeling: Use diff. equ. to describe a physical system
  - The fundamentals of linear algebra
  - Analysis:
    - Quantitative: How to derive response for a given input
    - Qualitative: How to analyze controllability, observability, stability and robustness without knowing the exact solution?
  - Design:
    - How to realize a system given its mathematical description
    - How to design a control law so that desired output response is produced
    - How to design an observer to estimate the state of the system
    - How to design optimal control laws
  - Continuous-time and discrete-time systems will be treated in parallel
Prerequisites:
- 16:413 Linear Feedback Systems
- Background on
  - **Linear algebra:** Matrices, vectors, determinant, eigenvalue, solving a system of equations
  - z-transform
  - Ordinary differential equations
  - Laplace transform, and
  - Modeling of electrical and mechanical systems

Grading:
Homework  20%
Mid Term   30%
Project    20%
Final Examination  30%

All exams are open book, open notes
• **General Rules:**
  – Homework should be clear, concise, and complete
  – Discussion is allowed but no copying. Make sure you understand what you write down.
  – Homework due next class. Strong reasons needed for late homework
  – Homework solutions will be provided a week after the due date at class website

• **Attendance:** will be taken occasionally. Positive attitude is a key to success.

• If you decide to do something, use your heart and do it well. Otherwise it will be a waste of time.

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**Course Project**

A cart with an inverted pendulum (page 22, Chen’s book)

![Diagram of a cart with an inverted pendulum]

- u: control input, external force (Newton)
- y: displacement of the cart (meter)
- θ: angle of the pendulum (radian)

The control problems are:
1. Stabilization: bring the pendulum to the inverted position and keep it there. Assume the angle is initially small enough.
2. Assume the pendulum is initially downward. Design a control algorithm to bring it upward and keep inverted.

Assume that there is no friction or damping. The nonlinear model is as follows.

\[
\begin{bmatrix}
M + m & ml \cos \theta \\
\cos \theta & l
\end{bmatrix}
\begin{bmatrix}
y \\ \dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
u - ml\dot{\theta}^2 \sin \theta \\
0 - g \sin \theta
\end{bmatrix}
\]

- \( M = 5 \text{ kg} \): mass of the cart,
- \( g = 9.82 \text{ m/s}^2 \)
- \( m = 1 \text{ kg} \): mass of the pendulum
- \( l = 0.2 \text{ m} \): length of the pendulum
**Problems:**
1. Derive a linear model for the system.
2. Design feedback laws using Matlab.
3. Validate your designs with Simulink and animation.

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**A magnetic suspension system**

*Purpose:* used as an assist device to increase the blood flow rate so as to maintain necessary activities

- **How it works:** the rotor rotates at certain speed to produce the required pressure increase and flow rate (e.g., 2L/min)
- **Feature:** the rotor is magnetically suspended. There is no contact with the housing.
- **Advantage:** Minimal damage to blood cells, free flow path, can be used for a longer time than mechanically suspended pump.
- **The tricky part:** the magnetic suspension. It has to be achieved with high precision feedback control

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*The clearance* < 0.2mm

---
Control design plays the most important role in suspension

Active magnetic bearing, desired force generated via controlling the coil currents
Passive magnetic bearings (permanent magnets) help to stabilize the rotor.

- **Control objective**: Keep the rotor from touching the housing in the presence of flow disturbances and gravity
- **Feedback control**: the gap at different locations are determined by sensors. The control algorithm use this detected information to figure out the active magnetic force and coil current. The power amplifier produce the required currents.
- What are in the loop? Position sensor, AD converter, DSP, DA converter, power amplifier.

What are in the loop? Position sensor, AD converter, DSP, DA converter, power amplifier.

Advanced design method is needed
A magnetic suspension test rig (Ball Hall 406)

This experiment is part of the NSF project: (Sept. 06 – Aug. 09). The control objective is to keep the free end of the beam suspended. The gap between the beam and the electromagnet follows any set value via a nonlinear controller, which is implemented by a microprocessor, or the Labview. An eddie-current sensor converts the gap into a voltage signal which is fed into the microprocessor; The controller in the microprocessor computes the desired currents and output it to a power amplifier.

The beam rests on the stator when the controller is turned off

The beam suspended when a nonlinear feedback control is applied.
The robust controller adjusts the current of the electromagnet so that the gap is maintained at the same set value under different load conditions.

2. Mathematical Descriptions of Systems
(Review)

- Classification of systems
- Linear systems
- Linear time invariant (LTI) systems
2.1 Classification of Systems

- Basic assumption: When an input signal is applied to the system, a unique output is obtained

Q. How do we classify systems?
- Number of inputs/outputs; with/without memory; causality; dimensionality; linearity; time invariance

- The number of inputs and outputs
  - When \( p = q = 1 \), it is called a single-input single-output (SISO) system
  - When \( p > 1 \) and \( q > 1 \), it is called a multi-input multi-output (MIMO) system
  - MISO, SIMO defined similarly

• Memoryless vs. with Memory
  - If \( y(t) \) depends on \( u(t) \) only, the system is said to be memoryless, otherwise, it has memory
  - An example of a memoryless system?

A purely resistive circuit
\[
y(t) = \frac{R_2}{R_1 + R_2} u(t) \quad \sim \text{Memoryless}
\]

- An example of a system with memory?

\[
R_i + L \frac{di}{dt} = u \quad \text{or} \quad \frac{di}{dt} + \frac{R}{L} i = \frac{1}{L} u
\]

\[
i(t) = e^{-\frac{R}{L} (t-t_0)} i(t_0) + \frac{1}{L} \int_{t_0}^{t} e^{-\frac{R}{L} (\tau-t_0)} u(\tau) d\tau
\]
– $i(t)$ depends on $i(t_0)$ and $u(\tau)$ for $t_0 \leq \tau \leq t$, not just $u(t)$
– A system with memory

• **Causality:** No output before an input is applied

![Causal System Diagram]

– A system is causal or non-anticipatory if $y(t_0)$ depends only on $u(t)$ for $t \leq t_0$ and is independent of $u(t)$ for $t > t_0$
– Is the circuit discussed last time causal?

![Circuit Diagram]

– An example of a non-causal system?
– $y(t) = u(t + 2)$
– Can you truly build a physical system like this?
– All physical systems are causal!
The Concept of State

- The state of a system at $t_0$ is the information at $t_0$ that, together with $u_{[t_0, \infty)}$, uniquely determines the behavior of the system for $t \geq t_0$
- The number of state variables = the number of ICs needed to solve the problem
- For an RLC circuit, the number of state variables = the number of $C$ + the number of $L$ (except for degenerated cases)
- A natural way to choose state variables as what we have done earlier: $\{v_c\}$ and $\{i_L\}$
- Is this the unique way to choose state variables?

Any invertible transformation of the above can serve as a state, e.g.,

$$
\begin{bmatrix}
  x_1(t) \\
  x_2(t)
\end{bmatrix}
= \begin{bmatrix}
  2 & 1 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  v(t) \\
  i(t)
\end{bmatrix}
= \begin{bmatrix}
  2v(t) + i(t) \\
  i(t)
\end{bmatrix}
$$

- Although the number of state variables = 2, there are infinite numbers of representations

Order of dimension of a system: The number of state variables

- If the dimension is a finite number ⇒ Finite dimensional (or lumped) system
- Otherwise, an infinite dimensional (or distributed) system
• An example of an infinite dimensional system

\[ u(t) \rightarrow \text{System} \rightarrow y(t) = u(t-1) \quad \text{A delay line} \]

– Given \( u(t) \) for \( t \geq 0 \), what information is needed to know \( y(t) \) for \( t \geq 0 \)?

– We need an infinite amount of information \( \Rightarrow \) An infinite dimensional system

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2.2 Linear Systems

Linearity

• Double the efforts double the outcome?

  – Suppose we have the following (state,input)-output pairs:
    \[
    \begin{align*}
    x_i(t_0) \quad &\rightarrow y_i(t), \ t \geq t_0 \\
    u_i(t), \ t \geq t_0
    \end{align*}
    \]
    \[
    \begin{align*}
    x_2(t_0) \quad &\rightarrow y_2(t), \ t \geq t_0 \\
    u_2(t), \ t \geq t_0
    \end{align*}
    \]

  – What would be the output of
    \[
    \begin{align*}
    x_i(t_0) + x_2(t_0) \quad &\rightarrow y_i(t) + y_2(t), \ t \geq t_0 \\
    u_i(t) + u_2(t), \ t \geq t_0
    \end{align*}
    \]

  – If this is true ~ Additivity
  – How about
    \[
    \begin{align*}
    \alpha x_i(t_0) \quad &\rightarrow \alpha y_i(t), \ t \geq t_0 \\
    \alpha u_i(t), \ t \geq t_0
    \end{align*}
    \]

  – If this is true ~ Homogeneity
  – Combined together to have:
    \[
    \begin{align*}
    \alpha_i x_i(t_0) + \alpha_2 x_2(t_0) \quad &\rightarrow \alpha_i y_1(t) + \alpha_2 y_2(t) \\
    \alpha_i u_i(t) + \alpha_2 u_2(t), \ t \geq t_0
    \end{align*}
    \]

  – If this is true ~ Superposition or linearity property
  – A system with such a property: a Linear System
• Are R, L, and C linear elements?
  \[ v_R = Ri_R, \quad v_L = L \frac{di_L}{dt}, \quad i_C = C \frac{dv_C}{dt} \]
  – Yes (differentiation is a linear operation)

• Also, KVL and KCL are linear constraints. When put together, we have a linear system

\[ v = Ri \quad \text{Affine} \]
\[ v \quad \text{Linear} \]
\[ -v \quad \text{Nonlinear} \]

• The additivity property implies that

\[
y(t) \text{ due to } \begin{cases} x_i(t_0) \\ u_i(t), t \geq t_0 \end{cases} = y(t) \text{ due to } \begin{cases} x_i(t_0) \\ u_i(t) = 0 \end{cases} + y(t) \text{ due to } \begin{cases} x_i(t_0) = 0 \\ u_i(t), t \geq t_0 \end{cases}
\]

– Response = zero-input response + zero-state response

**Response of a Linear System**

- How can we determine the output \( y(t) \)?
- Can be derived from \( u(t) + \) the unit impulse response based on linearity
Let $\delta_{\Delta}(t-\tau)$ be a square pulse at time $\tau$ with width $\Delta$ and height $1/\Delta$

\[
\delta_{\Delta}(t-\tau)
\]

As $\Delta \to 0$, we obtain a shifted unit impulse

\[
\delta(t-\tau)
\]

Let the unit impulse response be $g(t, \tau)$. Based on linearity,

\[
y(t) = \int_{-\infty}^{\infty} g(t, \tau) u(\tau) d\tau
\]

If the system is causal,

\[
g(t, \tau) = 0 \text{ for } t < \tau \quad y(t) = \int_{-\infty}^{t} g(t, \tau) u(\tau) d\tau
\]

A system is said to be relaxed at $t_0$ if the initial state at $t_0$ is 0

- In this case, $y(t)$ for $t \geq t_0$ is caused exclusively by $u(t)$ for $t \geq t_0$

\[
y(t) = \int_{t_0}^{t} g(t, \tau) u(\tau) d\tau
\]
• How about a system with p inputs and q outputs?
  – Have to analyze the relationship for input/output pairs

\[
y(t) = \int_0^t G(t, \tau)u(\tau)d\tau
\]

\[
G(t, \tau) = \begin{bmatrix}
g_{11}(t, \tau) & g_{12}(t, \tau) & g_{1p}(t, \tau) \\
g_{21}(t, \tau) & g_{22}(t, \tau) & g_{2p}(t, \tau) \\
g_{p1}(t, \tau) & g_{p2}(t, \tau) & g_{pp}(t, \tau)
\end{bmatrix}
\]

\(g_{ij}(t, \tau)\): The impulse response between the \(j^{\text{th}}\) input and \(i^{\text{th}}\) output

**State-Space Description**

• A linear system can be described by

\[
\dot{x}(t) = A(t)x(t) + B(t)u(t)
\]

\[
y(t) = C(t)x(t) + D(t)u(t)
\]

---

**2.3 Linear Time-Invariant (LTI) Systems**

• Time Invariance: The characteristics of a system do not change over time
  – What are some of the LTI examples? Time-varying examples?
  – What happens for an LTI system if \(u(t)\) is delayed by \(T\)?

  – If the same IC is also shifted by \(T\), then
– This property can be stated as:

\[
x(0) = x_0 \quad \rightarrow \quad y(t), \ t \geq t_0
\]

\[
x(T) = x_0 \quad \rightarrow \quad y(t-T), \ t \geq T
\]

What happens to the unit impulse response when the system is LTI?

\[
g(t, \tau) = g(t + T, \tau + T) \quad \text{for any } T
\]

\[
g(t, \tau) = g(t - \tau, \tau - \tau) = g(t - \tau, 0) = g(t - \tau)
\]

– Only the difference between \( t \) and \( \tau \) matters

– What happens to \( y(t) \)?

\[
y(t) = \int_{\tau}^t g(t, \tau)u(\tau)d\tau
\]

\[
= \int_{\tau}^t g(t - \tau)u(\tau)d\tau
\]

\[
= g(\tau)u(t - \tau)d\tau
\]

\[
= g(t)*u(t) \quad \sim \text{Convolution integral}
\]

\[
\hat{y}(s) = \hat{g}(s)\hat{u}(s)
\]
Proof of \( \hat{y}(s) = \hat{g}(s) \hat{u}(s) \)

\[
\hat{y}(s) = \int_0^\infty y(t)e^{-st}dt
\]

\[
= \int_0^\infty \left( \int_{\tau=0}^\infty g(t-\tau)u(\tau)d\tau \right) e^{-st}dt
\]

\[
= \int_0^\infty \left( \int_{\tau=0}^\infty g(t-\tau)u(\tau)d\tau \right) e^{-s(t-\tau)} e^{-s\tau}dt
\]

\[
= \int_{\tau=0}^\infty \left( \int_{\tau=0}^\infty g(t-\tau)e^{-s(t-\tau)} dt \right) u(\tau)e^{-s\tau}d\tau, \quad \text{(Let } \nu = t - \tau) \]

\[
\hat{y}(s) = \int_{\tau=0}^\infty \left( \int_{\nu=-\tau}^\infty g(\nu)e^{-s\nu}d\nu \right) u(\tau)e^{-s\tau}d\tau, \quad \text{(Note } g(\nu) = 0 \text{ for } \nu < 0) \]

\[
= \int_{\tau=0}^\infty \left( \int_{\nu=0}^\infty g(\nu)e^{-s\nu}d\nu \right) u(\tau)e^{-s\tau}d\tau
\]

\[
= \left( \int_{\nu=0}^\infty g(\nu)e^{-s\nu}d\nu \right) \left( \int_{\tau=0}^\infty u(\tau)e^{-s\tau}d\tau \right)
\]

\[
\hat{y}(s) = \hat{g}(s) \cdot \hat{u}(s)
\]
Transfer-Function Matrix

- For SISO system, \( \hat{y}(s) = \hat{g}(s) \cdot \hat{u}(s) \)
- \( \hat{g}(s) \) is the transfer function, the Laplace transform of the unit impulse response

For MIMO system,

\[
\hat{y}(s) = \hat{G}(s) \cdot \hat{u}(s)
\]

\[
\hat{G}(s) = \begin{bmatrix}
\hat{g}_{11}(s) & \hat{g}_{12}(s) & \hat{g}_{1p}(s) \\
\hat{g}_{21}(s) & \hat{g}_{22}(s) & \hat{g}_{2p}(s) \\
\hat{g}_{q1}(s) & \hat{g}_{q2}(s) & \hat{g}_{qp}(s)
\end{bmatrix}
\]

~ Transfer-function matrix, or transfer matrix

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Next Time:
- Linearization; Examples; Discrete-time systems
- Linear Algebra
  - Basis, representation, and orthonormalization
Problem Set #1:

1. Give examples for nonlinear systems and infinite dimensional systems respectively. What are the inputs, outputs and states?

2. Suppose we have a linear time-invariant system. Its response to $u_1$ is $y_1(t) = t+1$, for $t \geq 0$, and its response to $u_2$ is $y_2(t) = 2t$, for $t \geq 0$. For $t < 0$, $y_1(t) = y_2(t) = 0$. Assume zero initial conditions. What is the response to $2u_1(t-1) + u_2(t+1)$? Roughly plot your response.

3. A LTI system is described by
   \[ y' + 5y' + 6y = u, \quad y(0) = 1, \quad y'(0) = -1, \]
   What is $y(t)$ for $u=0$ and $y(0)=1$, $y'(0)=-1$?
   What is $y(t)$ for a unit step $u = 1(t)$ and $y(0)=y'(0)=0$?
   What is $y(t)$ for $u=1(t)$ and $y(0)=2$, $y'(0)=-2$?
   What is the state of the system?