**Problem 2.17**  At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

**Solution:** Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$ 

Since the line is lossless, Eq. (2.79) is valid:

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = 50 \left[ \frac{(40 + j20) + j50 \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan (2\pi \text{ rad/m} \times 2.5 \text{ m})} \right]$$

$$= 50 \left[ (40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0$$

$$= (40 + j20) \Omega.$$
Problem 2.37  On a lossless transmission line terminated in a load $Z_L = 100 \, \Omega$, the standing-wave ratio was measured to be 2.5. Use the Smith chart to find the two possible values of $Z_0$.

Solution: Refer to Fig. P2.52. $S = 2.5$ is at point $L1$ and the constant SWR circle is shown. $z_L$ is real at only two places on the SWR circle, at $L1$, where $z_L = S = 2.5$, and $L2$, where $z_L = 1/S = 0.4$. so $Z_{01} = Z_L/z_{L1} = 100 \, \Omega / 2.5 = 40 \, \Omega$ and $Z_{02} = Z_L/z_{L2} = 100 \, \Omega / 0.4 = 250 \, \Omega$.

Figure P2.37: Solution of Problem 2.37.
Problem 2.38  A lossless 50-Ω transmission line is terminated in a load with $Z_L = (50 + j25) \, \Omega$. Use the Smith chart to find the following:

(a) The reflection coefficient $\Gamma$.

(b) The standing-wave ratio.

(c) The input impedance at $0.35\lambda$ from the load.

(d) The input admittance at $0.35\lambda$ from the load.

(e) The shortest line length for which the input impedance is purely resistive.

(f) The position of the first voltage maximum from the load.
Solution: Refer to Fig. P2.53. The normalized impedance

\[ z_L = \frac{(50 + j25) \, \Omega}{50 \, \Omega} = 1 + j0.5 \]

is at point Z-LOAD.

(a) \( \Gamma = 0.24e^{j76.0^\circ} \) The angle of the reflection coefficient is read off of that scale at the point \( \theta_r \).

(b) At the point SWR: \( S = 1.64 \).

(c) \( Z_{\text{in}} \) is 0.350\( \lambda \) from the load, which is at 0.144\( \lambda \) on the wavelengths to generator scale. So point Z-IN is at 0.144\( \lambda \) + 0.350\( \lambda \) = 0.494\( \lambda \) on the WTG scale. At point Z-IN:

\[ Z_{\text{in}} = z_{\text{in}}Z_0 = (0.61 - j0.022) \times 50 \, \Omega = (30.5 - j1.09) \, \Omega. \]
(d) At the point on the SWR circle opposite $Z-IN$,

$$
Y_{in} = \frac{y_{in}}{Z_0} = \frac{(1.64 + j0.06)}{50 \, \Omega} = (32.7 + j1.17) \text{ mS}.
$$

(e) Traveling from the point $Z-LOAD$ in the direction of the generator (clockwise), the SWR circle crosses the $x_L = 0$ line first at the point $SWR$. To travel from $Z-LOAD$ to $SWR$ one must travel $0.25\lambda - 0.144\lambda = 0.106\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be $0.106\lambda$.

(f) The voltage max occurs at point $SWR$. From the previous part, this occurs at $z = -0.106\lambda$. 