95.141 Exam 2 Fall 2013

Section number

Section instructor

Last/First name  DANYLOV

Last 3 Digits of Student ID Number: ___ ___ ___

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided in the cover sheet. You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!

A Formula Sheet Is Attached To The Back Of This Examination

For your convenience you may carefully remove it from the Exam. Please take it with you at the end of the exam or throw it in a waste basket.

Be Prepared to Show your Student ID Card

Score on each problem:

1. (10) ___
2. (25) ___
3. (29) ___
4. (18) ___
5. (18) ___

Total Score (out of 100 pts) ___
1. (10 points) Put a circle around the letter that you think is the best answer.

1.1. (2pts) You need to load a crate of mass m onto the bed of a truck. One possibility is to lift the crate straight up over a height h, equal to height of the truck’s bed. The work done in this case is \( W_1 \). The other possibility is to slide the crate up the frictionless ramp of length L as shown in figure. In this case you perform work \( W_2 \). What statement is true?

\[ A) \ W_1 < \ W_2 \]
\[ B) \ W_1 = \ W_2 \]
\[ C) \ W_1 > \ W_2 \]
\[ D) \ No \ simple \ relationship \ exists \ between \ \ W_1 \ and \ \ W_2. \]

1.2. (2pts) Does the centripetal force acting on an object do work on the object?

\[ A) \ Yes, \ since \ a \ force \ acts \ and \ the \ object \ moves, \ and \ work \ is \ force \ times \ distance \]
\[ B) \ Yes, \ since \ it \ takes \ energy \ to \ turn \ an \ object. \]
\[ C) \ No, \ because \ the \ object \ has \ constant \ speed. \]
\[ D) \ No, \ because \ the \ object's \ displacement \ is \ zero. \]
\[ E) \ No, \ because \ the \ force \ and \ the \ displacement \ of \ the \ object \ are \ perpendicular. \]

1.3. (2pts) If the net work done on an object is negative, then the object's kinetic energy

\[ A) \ decreases. \]
\[ B) \ remains \ the \ same. \]
\[ C) \ increases. \]
\[ D) \ is \ zero. \]
\[ E) \ cannot \ be \ determined \ without \ knowing \ the \ object's \ mass. \]
1.4. (4pts) An object is under the influence of a force as represented by the force vs. position graph in the figure. What is the work done as the object moves from 6 m to 12 m?

A) 20 J
B) 30 J
C) 0 J
D) 40 J
E) 70 J
2. (25 pts) It is the year 2030, and we have colonized the moon. In order to set up lunar GPS, a satellite must be launched to orbit the moon. The satellite is launched, to orbit at altitudes above the surface of \( h_a = 1 \times 10^6 \text{ m} \). For the moon, \( M_m = 7.35 \times 10^{22} \text{ kg} \) and \( R_m = 1.74 \times 10^6 \text{ m} \).

\[
\begin{align*}
\text{a) } \quad F_G &= G \frac{M \cdot m}{R_m^2} = (6 \frac{M}{R_m^2}) \frac{m}{R_m^2} = m g_M \Rightarrow g_M = \frac{GM}{R_m^2} \\
M &= \frac{7.35 \times 10^{22} \text{ kg}}{1.74 \times 10^6 \text{ m}} \\
g_M &= 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \cdot \frac{7.35 \times 10^{22} \text{ kg}}{(1.74 \times 10^6 \text{ m})^2} = 1.62 \text{ m/s}^2
\end{align*}
\]

b) \text{ N. 2nd law:} \quad \Sigma \vec{F} = m \vec{a} \Rightarrow m \frac{\Delta v}{\Delta t} = \frac{GMm}{R_m^2} \Rightarrow \Delta v = \sqrt{\frac{GM}{R_m}} \approx 1.34 \times 10^2 \text{ m/s}

\[
\begin{align*}
\text{c) } \quad & \quad r = R_m \\
& \quad K_t = 0 \\
& \quad U_t = 0 \\
& \quad K_i = \frac{1}{2} m V_{esc}^2 \\
& \quad U_i = -\frac{G M m}{R_m} \\
& \quad \text{Use conserv. of energy:} \\
& \quad E_i = E_f \\
& \quad \frac{1}{2} m V_{esc}^2 - \frac{G M m}{R_m} = 0 \\
& \quad V_{esc} = \sqrt{\frac{2 G M}{R_m}} \\
& \quad V_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2} \cdot \frac{7.35 \times 10^{22} \text{ kg}}{1.74 \times 10^6 \text{ m}}}{1.74 \times 10^6 \text{ m}}} \approx 2.37 \times 10^3 \text{ m/s} \\
& \quad = 2.37 \text{ km/s} \quad \text{(answer)}
\end{align*}
\]
3. (29 pts) A 1000-kg car is driven at a constant speed of 20 m/sec along a hilly road. The radii of curvature at the points A, B, and C are 100 m, 75 m, and \( \infty \) (infinite) m, respectively.

a) Draw a free body diagram (3 pts) and find the normal force of the road on the car at point A. (7 pts)

b) Draw a free body diagram (3 pts) and find the normal force of the road on the car at point B (7 pts).

c) Do the same for point C. (3 pts).

d) What will happen to the car at point A if it is driven at a speed of 40 m/s? (6 pts)

\[
\begin{align*}
\sum F = ma & \Rightarrow mg - F_{NA} = m \frac{v^2}{R_A} \\
F_{NA} &= m \left( g - \frac{v^2}{R_A} \right) \\
F_{NA} &= 1000 \text{ kg} \cdot \left( 9.80 \text{ m/s}^2 - \frac{(20 \text{ m/s})^2}{100 \text{ m}} \right) \\
&= 5800 \text{ N}
\end{align*}
\]

b) Point B

\[
\begin{align*}
\sum F = ma & \Rightarrow F_{NB} - mg = m \frac{v^2}{R_B} \\
F_{NB} &= m \left( g + \frac{v^2}{R_B} \right) \\
F_{NB} &= 1000 \text{ kg} \cdot \left( 9.80 \text{ m/s}^2 + \frac{(20 \text{ m/s})^2}{75 \text{ m}} \right) \\
&\approx 15133 \text{ N} \approx 15100 \text{ N}
\end{align*}
\]

c) Point C. \( R_c = \infty \). The surface is flat.

\[
F_{NC} = mg = 1000 \text{ kg} \cdot 9.80 \text{ m/s}^2 = 9800 \text{ N}
\]

d) If \( v = 40 \text{ m/s} \), Point A.

Use Eq. (1)

\[
F_{NA} = m \left( g - \frac{v^2}{R_A} \right) = m \cdot \left( 9.80 \text{ m/s}^2 - \frac{(40 \text{ m/s})^2}{100 \text{ m}} \right) = 1000 \text{ kg} \cdot (9.8 - 16) < 0
\]

When \( F_{NA} \leq 0 \), the car loses contact with a surface of the road. It flies.
4. (18 pts) A child of 20 kg slides down a 10-m long slide with a 30° incline (on which the coefficient of kinetic friction is $\mu = 0.2$).
   a) (4 pts) Draw a free-body diagram of all the forces on the child.
   b) (10 pts) Use Newton's second law to find the child's acceleration.
   c) (4 pts) Find the net work done on the child.

\[ \text{b) Newton's law:} \]

\[ \begin{align*}
\sum F_y &= ma_y \\
F_n - mg \sin \theta &= ma_y \\
F_n &= mg \sin \theta \\
F_k &= \mu F_n = \mu mg \sin \theta \\
\sum F_x &= ma_x = ma \\
ma &= mg \sin \theta - F_k \\
a &= g \left( \sin \theta - \mu \cdot \cos \theta \right) \\
a &= 9.8 \frac{m}{s^2} \left( \sin 30° - 0.2 \cdot \cos 30° \right) \\
a &= 3.2 \frac{m}{s^2}
\end{align*} \]

\[ \text{c) Net work:} \]

\[ \begin{align*}
W_{\text{net}} &= \mathbf{F}_{\text{net}} \cdot \mathbf{d} = (\sum \mathbf{F}) \cdot \mathbf{d} = \left\| \sum \mathbf{F} - ma \right\| = ma \cdot \mathbf{d} = 10 \text{ m} \cdot 3.2 \frac{m}{s^2} = 640 \text{ J}
\end{align*} \]
5. (18 pts) A 50-kg bungee jumper leaps from the bridge. He is tied to a bungee cord that is 10 m long when unstretched, and falls a total of 30 m. Calculate the spring constant \( k \) of a bungee cord assuming Hooke’s law applies.

\[ m = 50 \text{ kg} \]
\[ y_0 = 10 \text{ m} \]
\[ y_1 = 30 \text{ m} \]
\[ x = y_1 - y_0 = 20 \text{ m} \]

Use conservation of energy (mechanics):

\[ E_1 = E_2 \]

\[ k_1 + U_{g_1} + U_{sp_1} = k_2 + U_{g_2} + U_{sp_2} \]

\[ 0 = -mg \cdot y_1 + \frac{1}{2} k \cdot x^2 \]

\[ \frac{1}{2} k x^2 = mg \cdot y_1 \]

\[ k = \frac{2 \cdot mg \cdot y_1}{x^2} = \frac{2 \cdot 50 \text{ kg} \cdot 9.80 \text{ m/s}^2 \cdot 30 \text{ m}}{(20 \text{ m})^2} \approx 73.5 \text{ N/m} \approx 74 \text{ N/m} \]
Trig:
\[
\sin \theta = a/c \\
\cos \theta = b/c \\
\tan \theta = a/b \\
c^2 = a^2 + b^2
\]

Quadratic Formula:
\[
Ax^2 + Bx + C = 0 \text{ has solutions:} \\
x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

Misc Formulas:
- Circumference of a circle = \(2\pi R\)
- Area of a circle = \(\pi R^2\)
- Surface Area of a Sphere = \(4\pi R^2\)
- Volume of sphere = \((4/3)\pi R^3\)
- Volume of cylinder = \(\pi R^2 L\)

Differentiation:
\[
dx^n/dx = nx^{n-1} \quad (n \neq 0) \\
d\cos(x)/dx = -\sin(x) \quad (x \text{ in radians}) \\
d\sin(x)/dx = \cos(x) \quad (x \text{ in radians}) \\
d(f(x) + g(x))/dx = df(x)/dx + dg(x)/dx
\]

Integration:
\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + C
\]

1-D Motion:
displacement = \(\Delta x\)
\[
\text{average} = \frac{\Delta x}{\Delta t} = \frac{(x_2 - x_1)}{(t_2 - t_1)} \\
\text{acceleration} = \frac{\Delta v}{\Delta t} = \frac{(v_2 - v_1)}{(t_2 - t_1)}
\]

Given \(x(t)\)
\[
v(t) = dx/dt \quad \text{(instantaneous)} \\
a(t) = dv/dt = d^2x/dt^2 \quad \text{(instantaneous)}
\]

1-D Motion with Const. Acc.:
\[
x(t) = x_0 + v_{0t}t + (1/2)at^2 \\
v(t) = v_0 + at \\
v^2 = v_0^2 + 2ax_0
\]

Projectile Motion:
\[
x(t) = x_0 + v_{0x}t \\
v_x(t) = v_{0x} \\
a_x(t) = 0 \\
y(t) = y_0 + v_{0y}t + (1/2)at^2 \\
v_y(t) = v_{0y} + at \\
a_y(t) = a_y
\]

For motion over level ground
Range = \(v_0^2 \sin(2\theta_0)\)/\(g\)

Acceleration due to gravity:
\(g = 9.8 \text{ m/s}^2\) downward

Newton's Second Law:
\[
\vec{F}_{\text{net}} = \sum \vec{F}_{\text{ext}} = m\ddot{a}
\]

Circular Motion:
\[
a_c = \frac{v^2}{R} \\
T = \frac{1}{f} \\
v = 2\pi R/T
\]

Frictional Forces:
\[
f_k = \mu_k F_N \\
\mu_s > \mu_k
\]

Work and Kinetic Energy
\[
W = F:dx \\
W = \int_{0}^{\pi} \vec{F} \cdot d\vec{r}
\]

\(K = (1/2)mv^2\)
\(W_{\text{net}} = \Delta K\)
\(\Delta K = K_f - K_i\)

Potential Energy
\[
\Delta U = U(x) - U_i(x_i) = -\int_{x_i}^{x} Fdx \\
F(x) = -dU(x)/dx
\]

For gravity on earth's surface:
\[
F_g = mg \\
U(y) = U_0 + mgy
\]

For gravity in general:
\[
F_g = -GmM/R^2 \\
U(r) = -GmM/R
\]

For springs: \(F = -kx\)
\(U(x) = (1/2)kx^2\)

With conservative forces only:
\(E_{\text{tot}} = K + U\) \(\text{(a constant)}\)
\(\Delta E_{\text{ext}} = \Delta K + \Delta U = 0\)

With non-conservative forces:
\(\Delta E_{\text{ext}} = \Delta K + \Delta U = W_{\text{NC}}\)

Gravity
Kepler's third law:
\[
T^2/R^3 = 4\pi^2/GM_{\text{sun}} \\
G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2
\]

Power
\[
P_{\text{avg}} = W/t \\
P = dW/dt \\
P = \vec{F} \cdot \vec{v}
\]