1. Conceptual Questions

(20 point) Put a circle around the letter that you think is the best answer.

1.1. (4pts) A loop of wire is positioned in the vicinity of a very long straight wire carrying a increasing current. Find the direction of the induced current in the loop. (CW stands for a clockwise direction; CCW – counterclockwise)

   A) CW  
   B) CCW  
   C) No current

\[ I \Rightarrow \vec{B}_{\text{ext}} \Rightarrow \vec{F} \Rightarrow \vec{B}_{\text{ind}} \Rightarrow \vec{E}_{\text{ind}} \Rightarrow \Rightarrow I_{\text{ind}} \ (CW) \]

1.2. (4pts) Two particles of the same mass enter a magnetic field with the same speed and follow the circular paths shown in Fig. 2. Which particle has a positive/negative charge?

A) \( q_1 < 0 \) and \( q_2 < 0 \)
B) \( q_1 > 0 \) and \( q_2 < 0 \)
C) \( q_1 > 0 \) and \( q_2 > 0 \)
D) \( q_1 < 0 \) and \( q_2 > 0 \)
E) They are not charged

\[ \vec{F} = q \vec{v} \times \vec{B} \]

so \( q_1 > 0 \); \( q_2 < 0 \)
1.3. (4pts) Two particles of the same mass enter a magnetic field with the same speed and follow the circular paths shown in Fig 2. Which particle has the bigger absolute value of charge?

A) particle \( q_1 \)

B) particle \( q_2 \)

C) both charges are equal

D) impossible to tell from the picture

D) not enough information to tell

1.4. (4pts) The line integral of \( B \) around the loop shown in Fig.3 is \( \mu_0 \cdot 5.0 \, A. \) Current \( I_4 \) is

A) 0 A

B) 3.0 A into the page

C) 3.0 A out of the page

D) 1.0 A into the page

E) 1.0 A out of the page

Ampere's law \( \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{12} \)

\[ \mu_0 \cdot 5A = \mu_0 (-I_1 + I_3 + I_4) \]

\[ -2A + 6A + I_4 = 5A \Rightarrow I_4 = 1A \text{ (out of the page)} \]

1.5. (4pts) A negatively charged particle moves along a very long wire with a current as it is shown in the figure. What is the direction of the force acting on the particle?

A) there is no force

B) to the left

C) to the right

D) into the page

E) out of the page

\[ \mathbf{F} = q \mathbf{v} \times \mathbf{B} \]
Problem 2. (20 pts)

a) What is the current in the wire to the right of the junction? Does the current in this wire flow to the right or to the left?

A current flows from + to -. So, $I_1$ to the right.

$I_2$ - down

$I$ - assume (to the right)

- Ohm's law: $I_1 = \frac{4V}{2\Omega} = 2A$

$I_2 = \frac{15V}{5\Omega} = 3A$

- Conservation of current at the junction point: $\sum I_{in} = \sum I_{out}$

$I_1 = I + I_2 \Rightarrow 2A = I + 3A \Rightarrow I = -1A$

So, $I = -1A$ (to the left). Means that my assumption of $I$ direction was wrong. $I$ flows to the left (in).

b) What is $\Delta V$ across the unspecified circuit element?

Kirchhoff's loop rule:

$\sum \Delta V_i = 0$

$12V + \Delta V_x - 8V - 6V = 0$

$\Delta V_x = 2V$
Problem 3. (20 pts)

A long straight cylindrical wire conductor of radius $R$ carries a current $I$ of uniform current density in the conductor. Determine the magnetic field due to this current at:

a) Points outside the conductor ($r > R$),

b) Points inside the conductor ($r < R$),

(Show Amperean loops; at least for one of the cases (a or b) show how you handle a linear integral)

\[ A u m p e r e^ \text{'s law}: \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \]

a) $r > R$

\[ \oint \vec{B} \cdot d\vec{s} = \int B \, r \, dr = B \cdot 2\pi r = B \cdot 2\pi R = \mu_0 I_{in} \]

Amp. loop

\[ I_{in} = I \text{ (total current)}, \quad \text{so} \quad B \cdot 2\pi r = \mu_0 I \Rightarrow B(r) = \frac{\mu_0 I}{2\pi r}, \quad r > R \]

b) $r < R$

Similar $B \cdot 2\pi r = \mu_0 I_{in}$

\[ I_{in} = \frac{J \cdot \pi r^2}{\pi R^2} = \left( \frac{I}{\pi R^2} \right) \pi r^2 = I \cdot \frac{r^2}{R^2}, \quad \text{so} \]

\[ B \cdot 2\pi r = \mu_0 I \cdot \frac{r^2}{R^2} \]

\[ B(r) = \frac{\mu_0 I}{2\pi R^2} \cdot r, \quad r < R \]
Problem 2. (20 pts) A proton is accelerated by a voltage of 2700 V. After that, the proton enters a uniform magnetic 0.340-T field.

(a) What is the speed of the proton after acceleration?
(b) What will be the radius of curvature as it moves through the magnetic field?
(c) What is its period of revolution?

\[
\text{Initial: } V_i = 0 \quad \text{Final: } V_f \quad \text{Change: } \Delta V
\]

\[V_f = V_i + \Delta V \quad \Rightarrow \quad V_f = \frac{qV_i}{m} - qV_f = q \Delta V
\]

\[
\frac{1}{2} m v_f^2 = q \Delta V \quad \Rightarrow \quad v_f = \sqrt{\frac{2q \Delta V}{m}} = \sqrt{\frac{2 \cdot 1.6 \times 10^{-19} \cdot 2700}{1.67 \times 10^{-27}}} = 71.93 \times 10^3 \text{ m/s}
\]

(b) The uniform force forces the proton to move in a circular path

\[
F = q \vec{V} \times \vec{B} \quad \Rightarrow \quad \vec{V} \times \vec{B} = F = qVB
\]

\[
qVB = \frac{mV_f^2}{R_{\text{cy}}} \quad \Rightarrow \quad R_{\text{cy}} = \frac{mV_f^2}{qVB} = \frac{m}{qB} \sqrt{\frac{2q \Delta V}{m}}
\]

\[
= \frac{1.67 \times 10^{-27} \text{ kg} \cdot 71.93 \times 10^3 \text{ m/s}}{1.6 \times 10^{-19} \text{ C} \cdot 0.340 \text{ T}} = 2.21 \times 10^{-2} \text{ m}
\]

(c) Uniform circular motion

\[
T = \frac{2 \pi R_{\text{cy}}}{V_f} \quad \Rightarrow \quad T = \frac{2 \pi \cdot 2.21 \times 10^{-2} \text{ m}}{71.93 \times 10^3 \text{ m/s}} = 0.19 \times 10^{-6} \text{ s} = 0.19 \mu s
\]
Problem 5 (20 pts)

The loop is being pulled out of the 0.20 T magnetic field at 50 m/s. The resistance of the loop is 0.10 Ω. The width of the loop is 5 cm.

(a) What is the direction of the induced current in the loop?

(b) What is the magnitude of the induced current in the loop?

\[ B_{\text{ext}} \]

\[ l \]

\[ v \]

\[ \Rightarrow I_{\text{ind}} \text{ (cw)} \]

\[ \varepsilon = B \cdot l \cdot v = 0.20 \cdot 7.5 \cdot 50 \cdot 10^{-2} = 50 \text{ mV} \]

\[ I_{\text{ind}} = \frac{\varepsilon}{R} = \frac{50}{0.10} = 500 \text{ A} \]