95.144 Final Exam Fall 2014

Section instructor__________________________ Section number____

Last/First name ____________________________

Last 3 Digits of Student ID Number: ________

Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers. You may use an alphanumeric calculator during the exam as long as you do not program any formulas into memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK!

A Formula Sheet Is Attached To The Back Of This Examination
Be Prepared to Show your Student ID Card

Score on each problem:

1. (30) _____
2. (20) _____
3. (20) _____
4. (20) _____
5. (20) _____
6. (20) _____
7. (20) _____

Total Score (out of 150 pts) _____
1. **Conceptual Questions (30 point)**

1.1. (6pts) Figure shows the circular waves emitted by two in-phase sources. Are points a and b points of maximum constructive (C) interference or perfect destructive (D) interference? (write C or D in the table)

<table>
<thead>
<tr>
<th>Point A</th>
<th>Point B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

1.2. (6pts) Figure shows four Gaussian surfaces surrounding a distribution of charges. Which Gaussian surfaces have an electric flux of $+q/\varepsilon_0$ through them?

A) a
B) b
C) b and d
D) b and c
E) c

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$
1.3 (6pts) Two plane mirrors form a right angle. How many images of the ball can you see in the mirrors?

A) 1
B) 2
C) 3
D) 4
E) 0

1.4 (6pts) It can be concluded from Gauss's law for magnetism that

A) south magnetic poles do not exist
B) isolated magnetic poles sometimes exist
C) north magnetic poles do not exist
D) magnetic poles are pairs of electric charges
E) isolated magnetic poles do not exist

1.5 (6pts) A solid conductor carries a net positive charge \( Q \). There is a hollow cavity within the conductor, at whose center is a negative point charge \(-q\).

What is the charge on

(a) the inner surface of the conductor's cavity

\( Q_{\text{inner}} = +q \)

(b) the outer surface of the conductor?

\( Q_{\text{outer}} = 0 - (-q) = Q + q \)
Problem 2. (20 pts)

A 20-cm-tall object is placed 10 cm from a 20-cm-focal length converging lens.

a) Determine the image position using ray tracing (draw it).

b) Calculate the image position and height.

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}; \quad f > 0 \text{ (converging lens)}; \quad s = 10 \text{ cm}; \quad f = +15 \text{ cm}
\]

\[
s' = \frac{s \cdot f}{s - f} = \frac{10 \text{ cm} \cdot 20 \text{ cm}}{10 \text{ cm} - 20 \text{ cm}} = -20 \text{ cm} \quad \text{(virtual)}
\]

\[
m = -\frac{s'}{s} = -\frac{-20 \text{ cm}}{10 \text{ cm}} = +2 \quad \text{(image is upright)}
\]

\[
|m| = \frac{h'}{h} \quad \Rightarrow \quad h' = |m| \cdot h = 2 \cdot 2.0 \text{ cm} = 4.0 \text{ cm}
\]
Problem 3. (20 pts)

A light beam strikes a 2.0-cm-thick piece of plastic with a refractive index of 1.62 at a 45° angle. The plastic is on top of a 3.0-cm-thick piece of glass for which $n = 1.47$.

What is the distance $D$?

Given: $\theta_0 = 45^\circ$; $h_1 = 2 \text{ cm}$; $h_2 = 3 \text{ cm}$

$n_0 = 1$; $n_1 = 1.62$; $n_2 = 1.47$

- $n_0 \cdot \sin \theta_0 = n_1 \cdot \sin \theta_1$
  
  $\theta_1 = \sin^{-1} \left[ \frac{n_0}{n_1} \cdot \sin \theta_0 \right] = \sin^{-1} \left[ \frac{1}{1.62} \cdot \sin 45^\circ \right] = 25.88^\circ$

- $n_1 \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$
  
  $\theta_2 = \sin^{-1} \left[ \frac{n_1}{n_2} \cdot \sin \theta_1 \right] = \sin^{-1} \left[ \frac{1.62}{1.47} \cdot \sin 25.88^\circ \right] = 28.75^\circ$

- $D = BC + EF$

$\triangle ABC$: $\tan \theta_1 = \frac{BC}{AB} \Rightarrow BC = AB \cdot \tan \theta_1 = h_1 \cdot \tan \theta_1$

$BC = 2.0 \text{ cm} \cdot \tan 25.88^\circ = 0.970 \text{ cm}$

$\triangle CEF$: $EF = CE \cdot \tan \theta_2 = h_2 \cdot \tan \theta_2 = 3.0 \text{ cm} \cdot \tan 28.75^\circ = 1.6458 \text{ cm}$

$D = 0.970 \text{ cm} + 1.6458 \text{ cm} = 2.61 \approx 2.6 \text{ cm}$
Problem 4. (20 pts)

If 720-nm and 660-nm light passes through two slits 0.68 mm apart, how far apart are the second-order fringes for these two wavelengths on a screen 1.0 m away?

Given: \( \lambda_1 = 720 \text{ nm} \quad d = 0.68 \text{ mm} \quad \lambda_2 = 660 \text{ nm} \quad L = 1.0 \text{ m} \)

Find: \( \Delta y \) (see figure)

\[
\begin{align*}
\Delta y &= \frac{m \lambda L}{d} \\
&m = 2
\end{align*}
\]

\[
\begin{align*}
\Delta y &= \frac{2 \cdot \lambda_1 \cdot L}{d} - \frac{2 \cdot \lambda_2 \cdot L}{d} \\
&= \frac{2L}{d} (\lambda_1 - \lambda_2) \\
&= \frac{2 \cdot 1.0 \text{ m}}{0.68 \cdot 10^{-3} \text{ m}} (720 \text{ nm} - 660 \text{ nm}) \\
&= 1.46 \cdot 10^{-3} \text{ m} \\
&\approx 0.2 \text{ mm}
\end{align*}
\]
Problem 5. (20 pts)

A uniform electric field collapses to zero from an initial strength of $6.0 \times 10^5$ N/C in a time of $15 \mu s$ in a manner shown in the figure. Calculate the displacement current, through a $1.6 \text{ m}^2$ region perpendicular to the field, during each of the time intervals (a), (b), and (c) shown on the graph.

\[
I_D = E_0 \frac{d\Phi_E}{dt}
\]

\[
\Phi_E = E \cdot A = E \cdot 1.6 \text{ m}^2 = EA
\]

\[
I_D = E_0 \frac{d(EA)}{dt} = E_0 A \frac{dE}{dt} =
\]

\[
I_D = E_0 A \frac{\Delta E}{\Delta t} = E_0 A \cdot \frac{E_f - E_i}{\Delta t}
\]

\[
I_D = (8.85 \times 10^{-12} \frac{C^2}{Nm^2}) \cdot 1.6 \text{ m}^2 \cdot \frac{\Delta E}{\Delta t} = (14.16 \times 10^{-12} \frac{C^2}{N}) \frac{\Delta E}{\Delta t}
\]

\[\text{ⅰ) } I_D = (14.16 \times 10^{-12} \frac{C^2}{N}) \cdot \left[\frac{(4-6) \times 10^{-5} \frac{N}{C}}{4 \times 10^{-6} \text{ s}}\right] = -0.708 \text{ A}
\]

\[\text{ⅱ) } I_D = 0, \text{ since } \Delta E = 0
\]

\[\text{ⅲ) } I_D = (14.16 \times 10^{-12} \frac{C^2}{N}) \cdot \left[\frac{(0-4) \times 10^{-5} \frac{N}{C}}{5 \times 10^{-6} \text{ s}}\right] = -1.13 \text{ A}
\]
Problem 6. (20 pts)

The loop in the figure is being pushed into the 0.20 T magnetic field at 50 m/s. The resistance of the loop is 0.10 Ω.

a) What is the magnitude of the current in the loop?

\[ B = 0.20 \text{ T}; \quad v = 50 \text{ m/s}; \quad R = 0.10 \Omega \]

\[ L = 5.0 \text{ cm} \]

\[ \varepsilon = -\frac{d\Phi}{dt} = \frac{d}{dt} (B \cdot A) = \left\| \frac{\vec{E} \times \vec{A}}{\varepsilon} \right\| = B \frac{dA}{dt} = \left\| A = l \times \vec{v} \right\| = B \frac{d}{dt} (lx) = B l \frac{dx}{dt} = B l \cdot \dot{x} = \varepsilon \]

\[ \varepsilon = 0.20 \text{ T} \cdot 5.10^{-2} \text{ m} \cdot 50 \text{ m/s} = 0.5 \text{ V} \]

\[ I = \frac{\varepsilon}{R} = \frac{0.5 \text{V}}{0.10 \Omega} = 5 \text{A} \]

b) What is the direction of the induced current?

6) direction of an induced current:

\[ A \xrightarrow{\Phi} \vec{A} \xrightarrow{\text{Induced} \vec{B}} \vec{B}_{\text{ext}} \xrightarrow{\text{Ind} \vec{I} \text{ is \textit{CCW}} (\text{counterclockwise})} \]
Problem 7. (20 pts)

Two long wires are oriented so that they are perpendicular to each other. At their closest, they are 20.0 cm apart (see the figure). What is the magnitude of the magnetic field at a point midway between them if the top one carries a current of 20.0 A and the bottom one carries 12.0 A?

The net magnetic field is created by two currents.

\[ \mathbf{B} = \mathbf{B}_T + \mathbf{B}_B \]

View from above:

\[ l_T = 10.0 \text{ cm} \]

\[ l_B = 10.0 \text{ cm} \]

\[ I_T = 20.0 \text{ A} \]

\[ I_B = 12.0 \text{ A} \]

Bottom wire

The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. They are at right angles to each other.

\[ \mathbf{B} = \frac{\mu_0 I}{2\pi l} \]

so

\[ B_T = \frac{\mu_0 I_T}{2\pi l_T} \quad \text{and} \quad B_B = \frac{\mu_0 I_B}{2\pi l_B} \]

\[ B = \sqrt{B_T^2 + B_B^2} = \sqrt{\left(\frac{\mu_0 I_T}{2\pi l_T}\right)^2 + \left(\frac{\mu_0 I_B}{2\pi l_B}\right)^2} = \frac{\mu_0}{2\pi} \sqrt{\frac{I_T^2}{l_T^2} + \frac{I_B^2}{l_B^2}} = \sqrt{l_T^2 + l_B^2} \]

\[ B = \frac{\mu_0}{2\pi l} \sqrt{\frac{I_T^2}{l_T^2} + \frac{I_B^2}{l_B^2}} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi \times (0.100 \text{ m})} \sqrt{(20.0 \text{ A})^2 + (12.0 \text{ A})^2} \quad \text{= 4.66 \times 10^{-5} T} \]
**Formula Sheet:**

**Electricity and Magnetism**

**Coulomb's law**

\[ F = k \frac{qQ}{r^2} \]

**Electric Field**

\[ \vec{E} = -\frac{\vec{F}}{q} \]

Field of a point charge

\[ E = k \frac{Q}{r^2} \]

Electric field inside a capacitor

\[ E = \frac{\eta}{\varepsilon_0} \]

Principle of superposition

\[ \vec{E}_{net} = \sum_{i=1}^{N} \vec{E}_i \]

Electric flux

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

**Gauss's law**

\[ \Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \]

**Electric potential**

\[ V = \frac{U}{q} \]

\[ V(r) = -\int_{\infty}^{b} \vec{E} \cdot d\vec{l} \]

\[ V_{ba} = V_b - V_a = -\int_{a}^{b} \vec{E} \cdot d\vec{l} \]

For a point charge \( V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r} \)

For a paralle-plate capacitor

\[ V = Es \]

**Potential Energy**

\[ U = qV \]

**Capacitors**

\[ C = \frac{Q}{\Delta V} \]

Parallel-plate \( C = \varepsilon_0 \frac{A}{d} \)

Capacitors connected in parallel

\[ C_{eq} = C_1 + C_2 + \ldots \]

Capacitors connected in series

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \]

Energy stored in a capacitor \( U = \frac{Q^2}{2C} \)

**Ohm's law**

\[ V = IR \]

\[ R = \rho \frac{l}{A} \]

**Power**

\[ P = IV \]

**Resistors connected in series**

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

**Resistors connected in parallel**

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

**The potential difference across a charging capacitor in RC circuit**

\[ V(t) = \varepsilon(1 - e^{-t/\tau c}) \]
A magnetic field exerts a force
\[ \vec{d}F = I \vec{d}l \times \vec{B} \]
\[ \vec{F} = I \vec{l} \times \vec{B} \]
\[ \vec{F} = q \vec{v} \times \vec{B} \]

The Biot-Savart Law
\[ \vec{B} = \frac{\mu_0 q \vec{v} \times \vec{r}}{4\pi r^2} \]
\[ d\vec{B} = \frac{\mu_0 I ds \times \vec{r}}{4\pi r^2} \]

The magnetic field of:
A straight line wire
\[ \vec{B} = \frac{\mu_0 I}{2\pi r} \]

A solenoid
\[ \vec{B} = \mu_0 nI \]

Magnetic flux
\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]

Self-inductance
\[ L = N \frac{\Phi_B}{I} ; \quad \varepsilon = -L \frac{dI}{dt} \]

Energy stored in an inductor
\[ U = L \frac{I^2}{2} \]

"Discharged" LR circuit
\[ I = I_0 e^{-t/\tau} ; \quad \tau = L/R \]

Maxwell's equations
\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \]
\[ \oint \vec{B} \cdot d\vec{A} = 0 \]
\[ \varepsilon = \oint \vec{E} \cdot ds = -\frac{d\Phi_B}{dt} \]
\[ \oint \vec{B} \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

\[ \hat{F} = q(\hat{E} + \hat{v} \times \hat{B}) \]

The Poynting vector
\[ \hat{S} = \frac{1}{\mu_0} (\hat{E} \times \hat{B}) \]

Malus's Law
\[ I = I\cos^2 \theta \]

Traveling Wave
\[ y(x, t) = A\sin(kx - \omega t + \varphi_0) \]
\[ k = \frac{2\pi}{\lambda} ; \quad \omega = \frac{2\pi}{T} ; \quad v = \lambda f \]

Interference
\[ \Delta \varphi = 2\pi \frac{\Delta r}{\lambda} + \Delta \varphi_0 = m2\pi \quad (\text{constr}) \]
\[ \Delta \varphi = 2\pi \frac{\Delta r}{\lambda} + \Delta \varphi_0 = (m + \frac{1}{2})2\pi \quad (\text{destr}) \]
\[ A = 2a\cos\left(\frac{\Delta \varphi}{2}\right) \]

Standing Waves
\[ A(x) = 2a\sin(kx) \]
\[ \lambda_m = \frac{2L}{m} ; \quad f_m = m\frac{v}{2L} \]

Double Slit
\[ \gamma_m = \frac{m\lambda}{a} , \quad m=0,1,2 \]

Diffraction grating
\[ d \sin \theta_m = m\lambda \]
\[ \gamma_m = L \tan \theta_m \]
**Thin-lens equation:**
\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}
\]
\[
m = -\frac{s'}{s}; \quad |m| = \frac{h'}{h}
\]

**Snell’s Law:**
\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

TIR: \(\sin \theta_c = \frac{n_2}{n_1}\)

**Constants**
- Charge on electron
  \(e = 1.60 \cdot 10^{-19} \ C\)
- Electron mass \(m = 9.11 \cdot 10^{-31} \ kg\)
- Permittivity of free space
  \(\varepsilon_0 = 8.85 \cdot 10^{-12} \ C^2/Nm^2\)
- Permeability of free space
  \(\mu_0 = 4\pi \cdot 10^{-7} \ Tm/A\)
- \(k = \frac{1}{4\pi\varepsilon_0} = 8.99 \cdot 10^9 \ Nm^2/C^2\)
- \(c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3.0 \cdot 10^8 m/s\)

**Kinematic eq-ns with const. Acc.:**
\[
\begin{align*}
\nu(t) &= \nu_0 + at \\
x(t) &= x_0 + \nu_0 t + \frac{1}{2} at^2 \\
\nu^2 &= \nu_0^2 + 2a(x - x_0)
\end{align*}
\]

**Centripetal acceleration** \(a_R = \nu^2/r\)