Lecture 4

Chapter 27

Gauss’s Law

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Lecture Capture:
http://echo360.uml.edu/danylov201516/physicsII.html

Channel 61 (clicker)
Gauss’s Law

For any closed surface enclosing total charge $Q_{in}$, the net electric flux through the surface is:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

**Gaussian surface**

Properties:
1) It works for any closed surface called Gaussian (A Gaussian surface is an imaginary, mathematical surface)
2) $Q_{in}$ is the net charge enclosed by the Gaussian surface (charges outside must not be included) $Q_{in} = q_1 - q_2 + q_3$
3) Distribution of $Q_{in}$ doesn't matter

This result for the electric flux is known as Gauss’s Law.

Both, Gauss’s law and Coulomb’s law, help to find electric fields based on distribution of charges.
Gauss’s Law/Symmetry

Gauss’s law is always true, but it is not always useful

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \]

Gaussian surface

At first glance this doesn't seem to get us very far, because the quantity we want (E) is buried inside the surface integral.

Luckily, symmetry allows us to extract E from under the integral sign.

When the charge distribution has sufficient symmetry (spherical, cylindrical, planar), evaluation of the integral becomes simple.
Which spherical Gaussian surface has the larger electric flux?

- A) Surface A
- B) Surface B
- C) They have the same flux
- D) Not enough information to tell

Flux depends only on the enclosed charge, not the radius.

\[ \Phi_e = \int \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \]
Example  Electric Flux

Determine the electric flux through each surface

\[ \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]

Flux depends only on the enclosed charge, not the radius.

- S1: \( \Phi = (+Q - 3Q)/\varepsilon_0 \)
- S2: \( \Phi = (+Q + 2Q - 3Q)/\varepsilon_0 \)
- S3: \( \Phi = (+2Q - 3Q)/\varepsilon_0 \)
- S4: \( \Phi = 0 \) (no charge inside)
- S5: \( \Phi = (+2Q)/\varepsilon_0 \)
\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \]

**Examples**
A total charge $Q$ is spread uniformly throughout a dielectric sphere of radius $R$. What is the electric field outside the sphere ($r>R$)?

Notice a remarkable feature of this result: The field outside the sphere is exactly the same as it would have been if all the charge had been concentrated at the center.
A total charge $Q$ is spread uniformly throughout a dielectric sphere of radius $R$. What is the electric field inside the sphere ($r<R$)?

Let's plot it

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^3} \cdot r$$
Example 27.6: (The planar symmetry)

The electric field must be 1 to the plane because of the symmetry. Gaussian surface - a small cylinder that goes through the plane.

\[ \oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0} \]

Gaussian surface

\[ \oint E \cdot dA = \int_{\text{side}} E \cdot dA + \int_{\text{front}} E \cdot dA + \int_{\text{back}} E \cdot dA \]

Gaussian surface

\[ \oint E \cdot dA = \left\| \oint \overrightarrow{E} \cdot d\overrightarrow{A} \right\| = 0 \]

Side surface

\[ \oint E \cdot dA = \left\| \oint \overrightarrow{E} \cdot d\overrightarrow{A} \right\| = \int_{\text{front}} E \cdot dA = \int_{\text{back}} E \cdot dA = EA \]

Front

\[ \oint E \cdot dA = \left\| \text{Similar} \right\| = EA \quad \text{so} \]

\[ \oint E \cdot dA = 0 + EA + EA = 2EA = \frac{Q_{in}}{\varepsilon_0} \]

Now the charge inside the cylinder

\[ Q_{in} = \eta \cdot A \]

\[ 2EA = \frac{Q_{in}}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{1}{2 \varepsilon_0} \]

Find the electric field of an infinite nonconducting plane of charge with surface charge density \( \eta \) (C/m²).
What you should read

Chapter 27 (Knight)

Sections

- 27.2 Electric flux concept
- 27.3 Electric flux
- 27.4 Gauss’s law
- 27.5 Examples
Thank you

See you on Tuesday