# Chapter 9: Alternating Current & Voltage

**Instructor**: Jean-François MILLITHALER

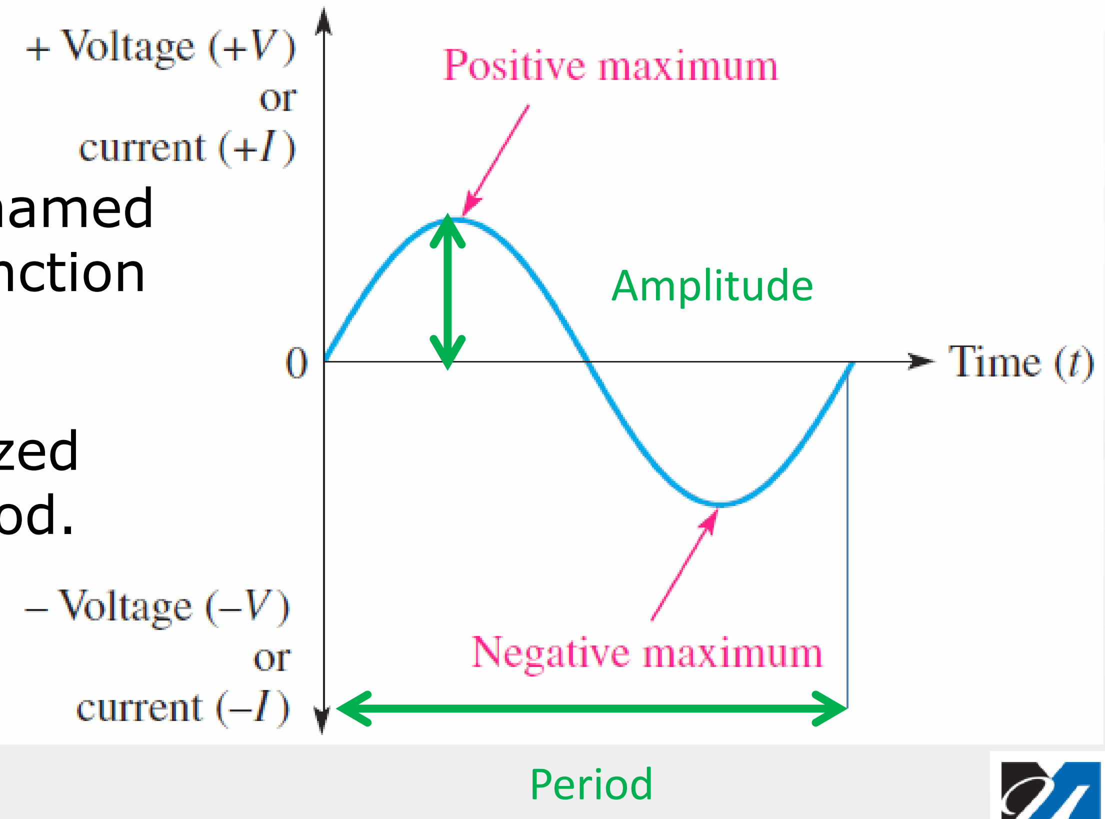
http://faculty.uml.edu/JeanFrancois_Millithaler/FunElec/Spring2017

# Sine Waves

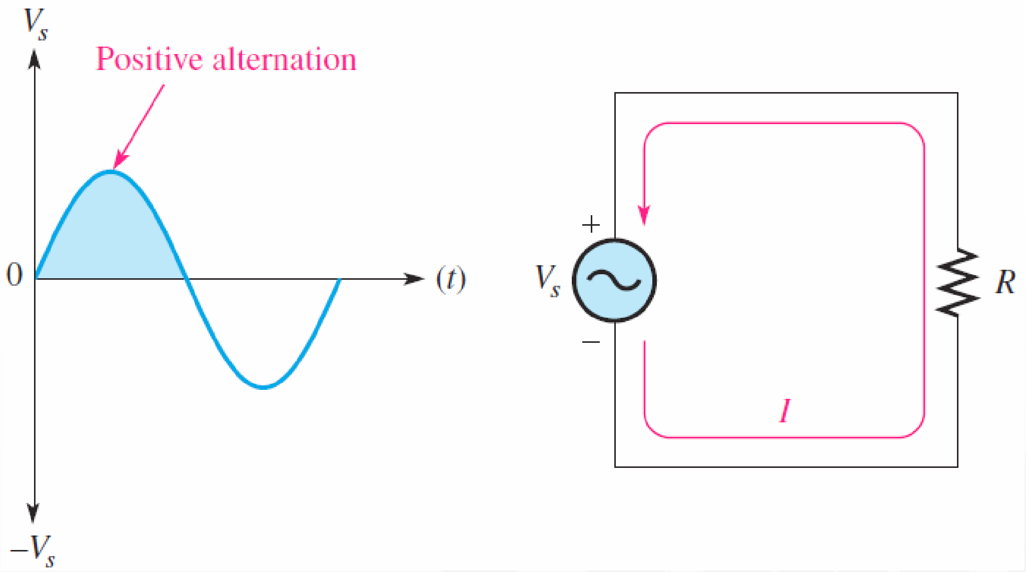
- The sinusoidal waveform (sine wave) is the fundamental alternating current (ac) and alternating voltage waveform.

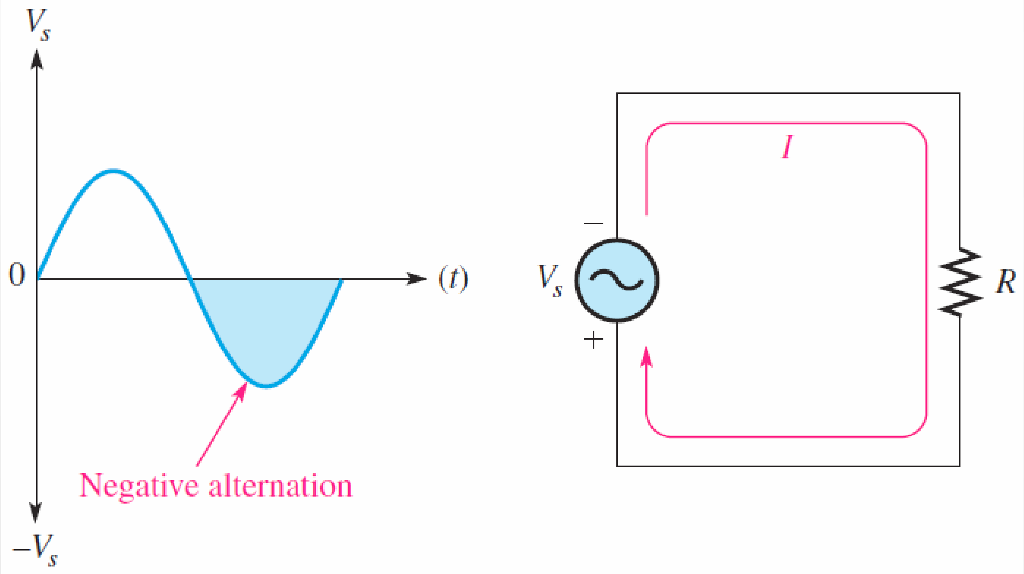
Electrical sine waves are named from the mathematical function with the same shape.

Sine waves are characterized by the amplitude and period.



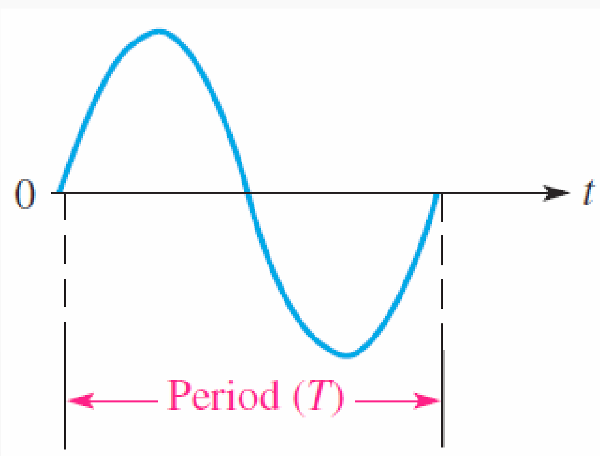
# Polarity of a Sine Wave

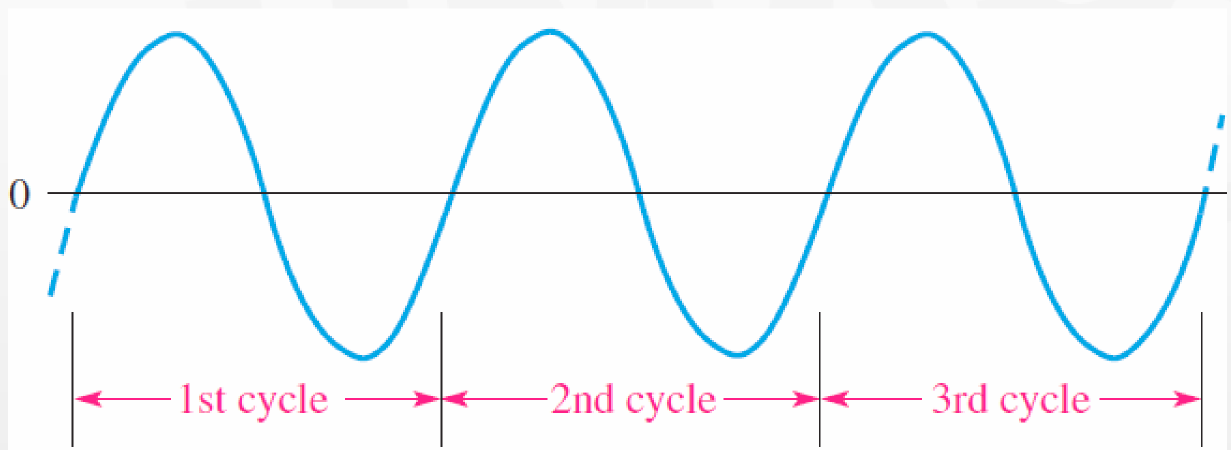


# Period of a Sine Wave

- The time required for a given sine wave to complete one full cycle is called the **period** (T).
- The Unit is the second (s)

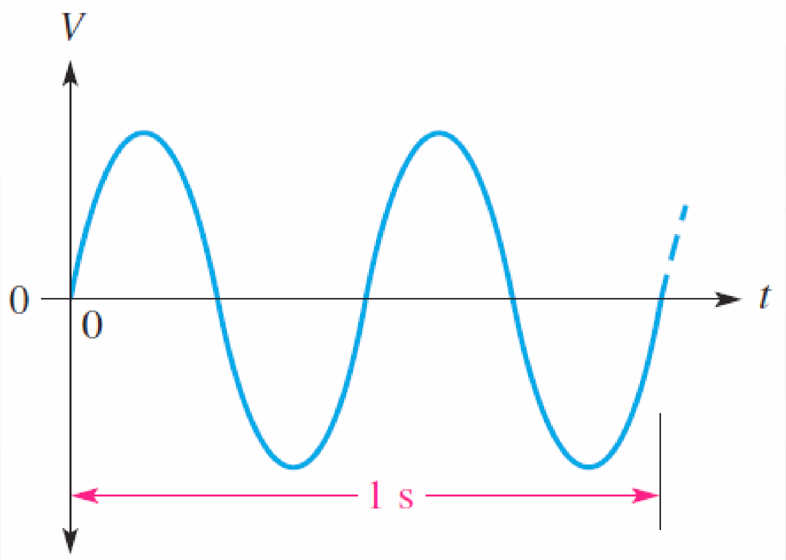


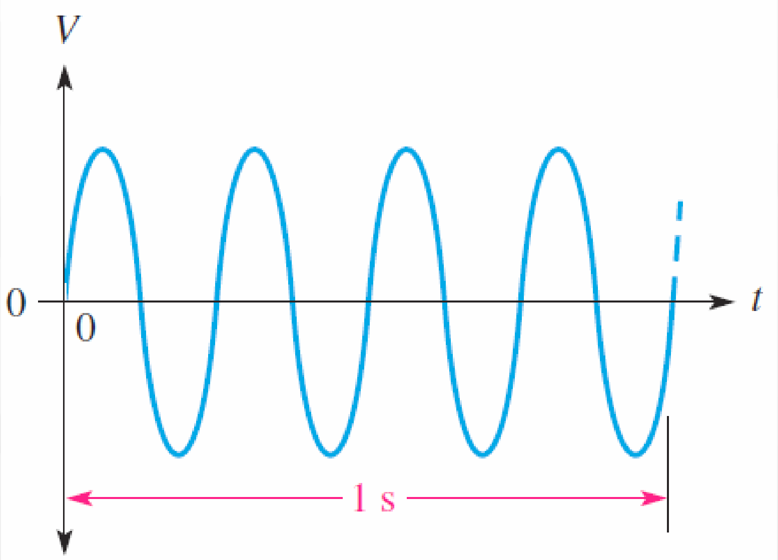



# Frequency of a Sine Wave

Heinrich Rudolf Hertz, German Physicist, 1857–1894

- **Frequency** is the number of cycles that a sine wave completes in one second.
- The Unit is the hertz (Hz)

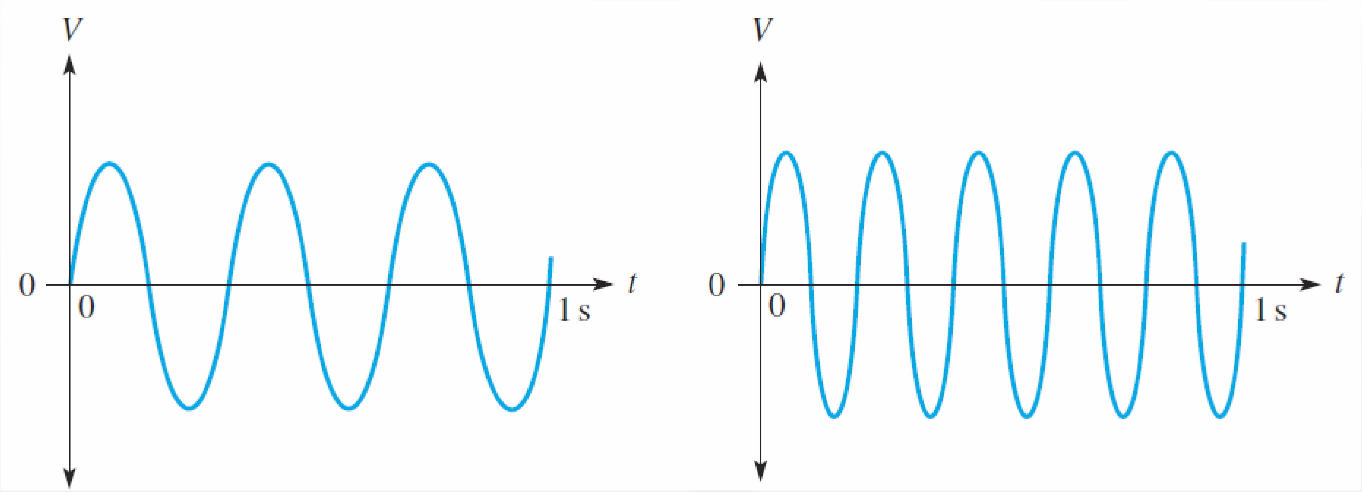


# Relationship of Frequency and Period

$$f = \frac{1}{T} \quad \Leftrightarrow \quad T = \frac{1}{f}$$

Question: Which sine wave has the higher frequency? Determine the frequency and the period of both waveforms.



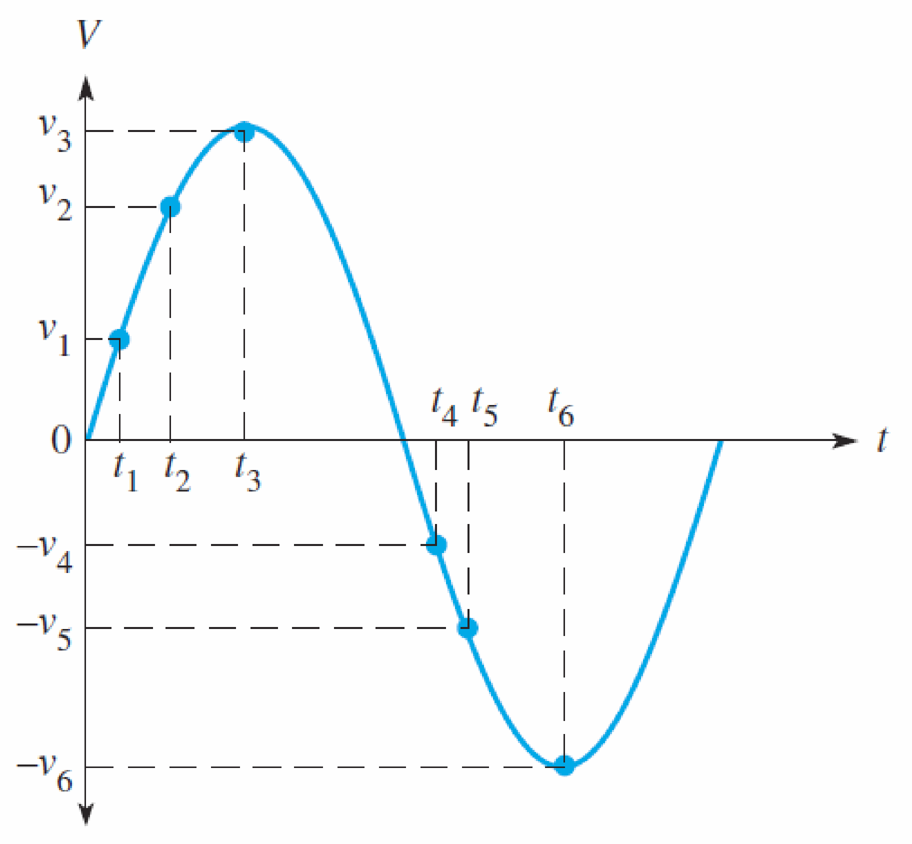

T=333 ms, f=3 Hz

T=200 ms, f=5 Hz

# VOLTAGE AND CURRENT VALUES OF SINE WAVES

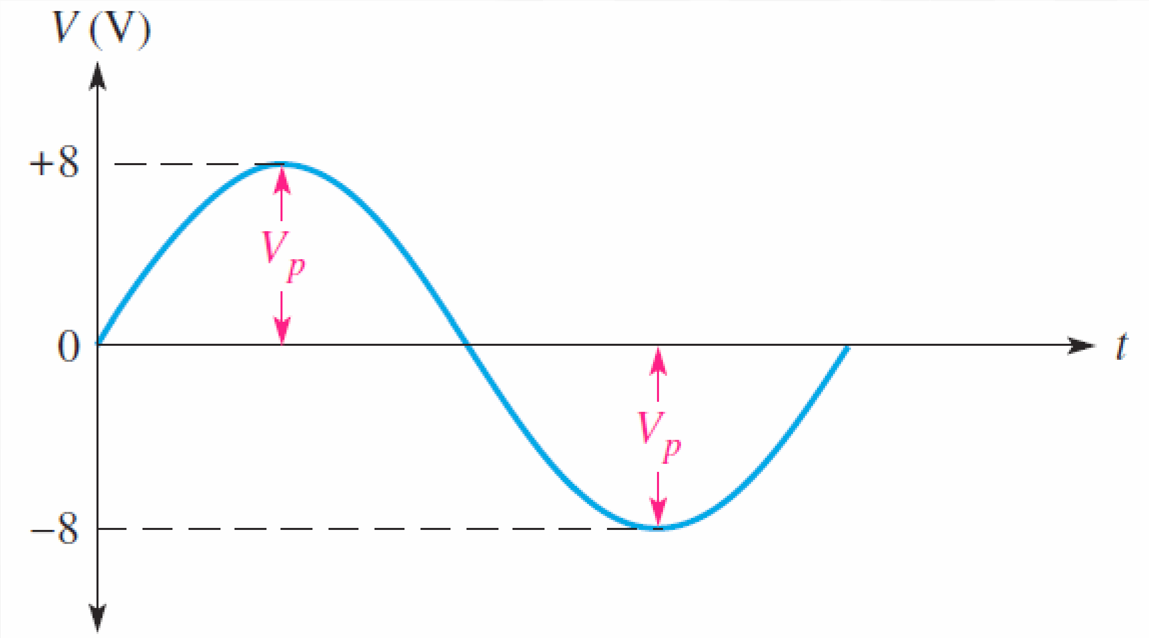
## Instantaneous Value

- The instantaneous value is different at different points along the curve.



## Peak Value

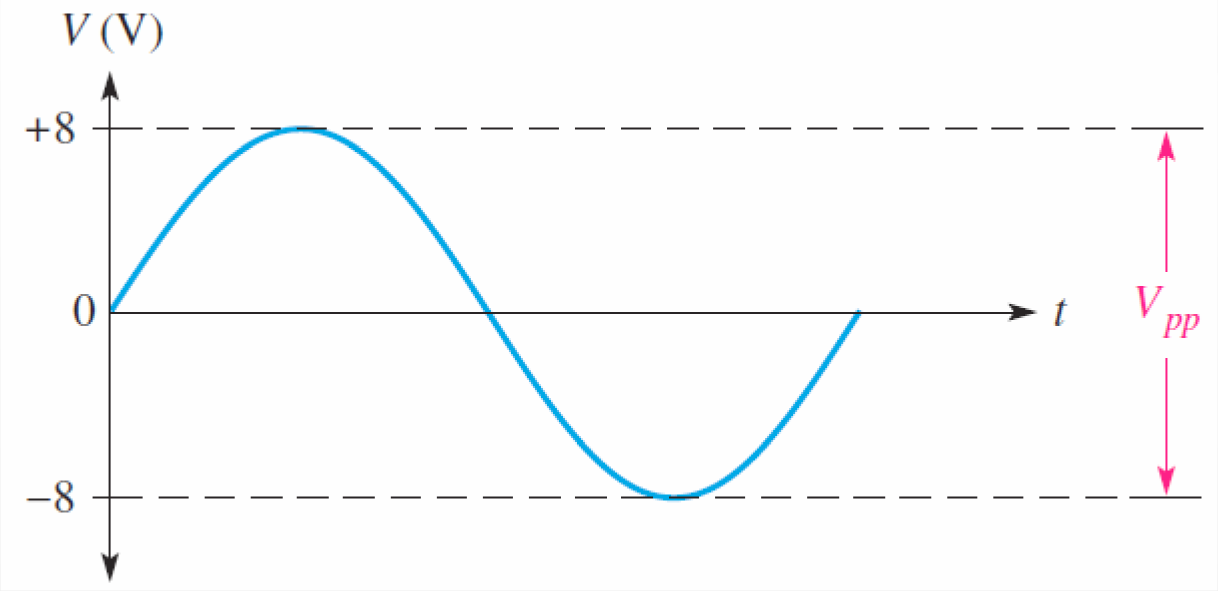
- The **peak value** of a sine wave is the value of voltage (or current) at the positive or the negative maximum (peaks) with respect to zero.
- Since positive and negative peak values are equal in magnitude, a sine wave is characterized by a single peak value



## Peak-to-Peak Value

- The **peak-to-peak value** of a sine wave is the voltage (or current) from the positive peak to the negative peak.

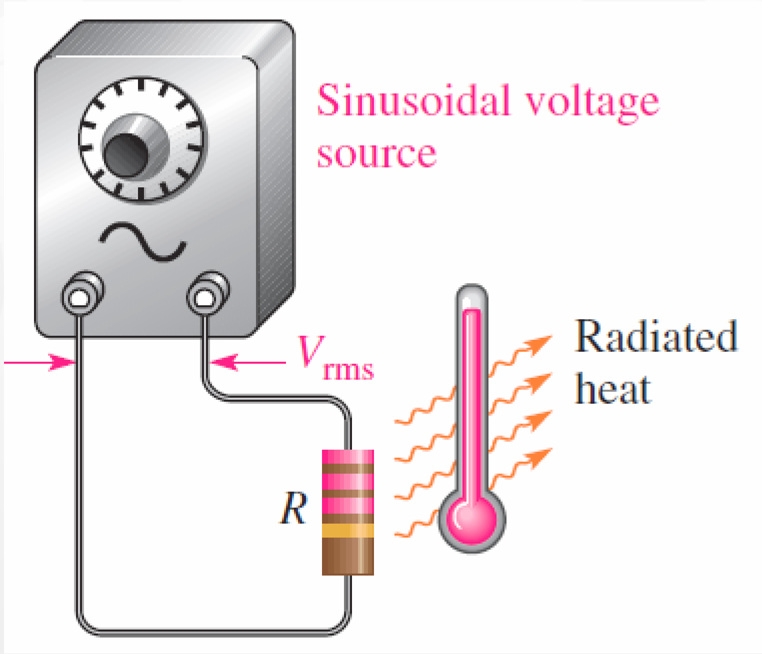
$$V_{pp} = 2\,V_p$$
$$I_{pp} = 2\,I_p$$



## rms Value

- The **rms value** (root mean square), also referred to as the **effective value**, of a sinusoidal voltage is actually a measure of the heating effect of the sine wave.

$$V_{rms} = 0.707\,V_p$$
$$I_{rms} = 0.707\,I_p$$



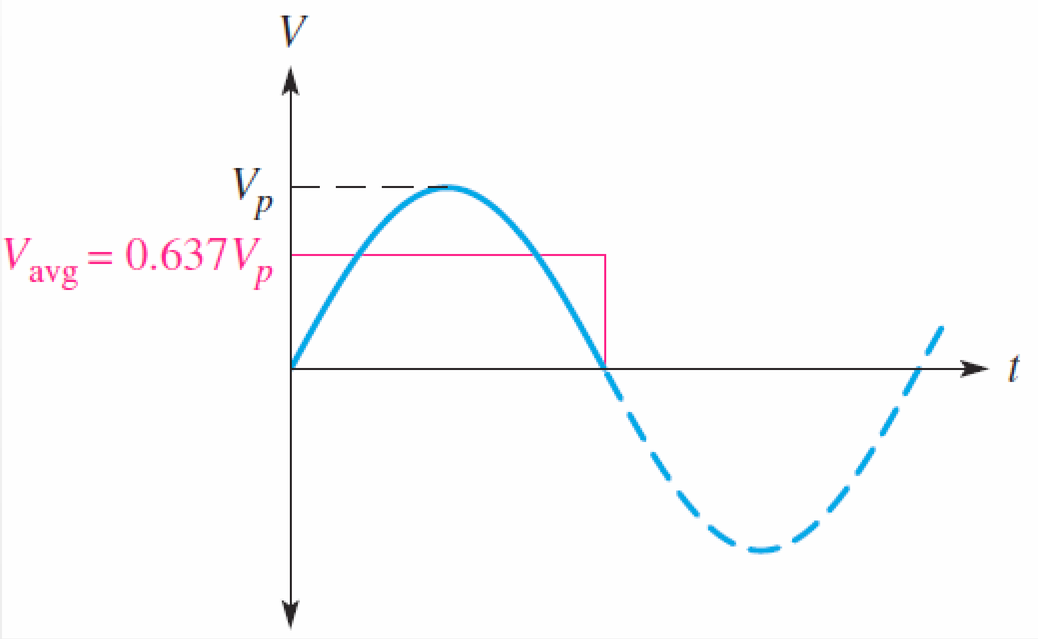
# VOLTAGE AND CURRENT VALUES OF SINE WAVES

## Average Value

- For some purposes, the **average value** (actually the halfwave average) is used to specify the voltage or current. By definition, the average value is as 0.637 times the peak value.

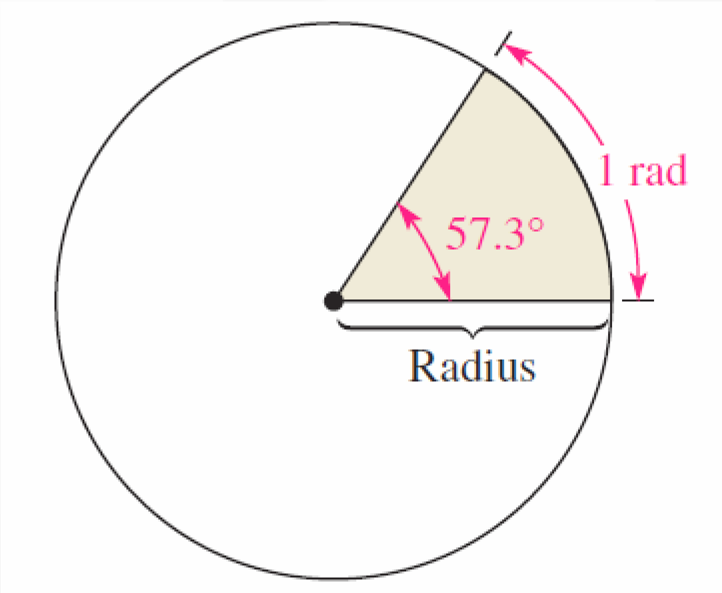
$$V_{avg} = 0.637\ V_p$$

$$I_{avg} = 0.637\ I_p$$



## Angular Measurement

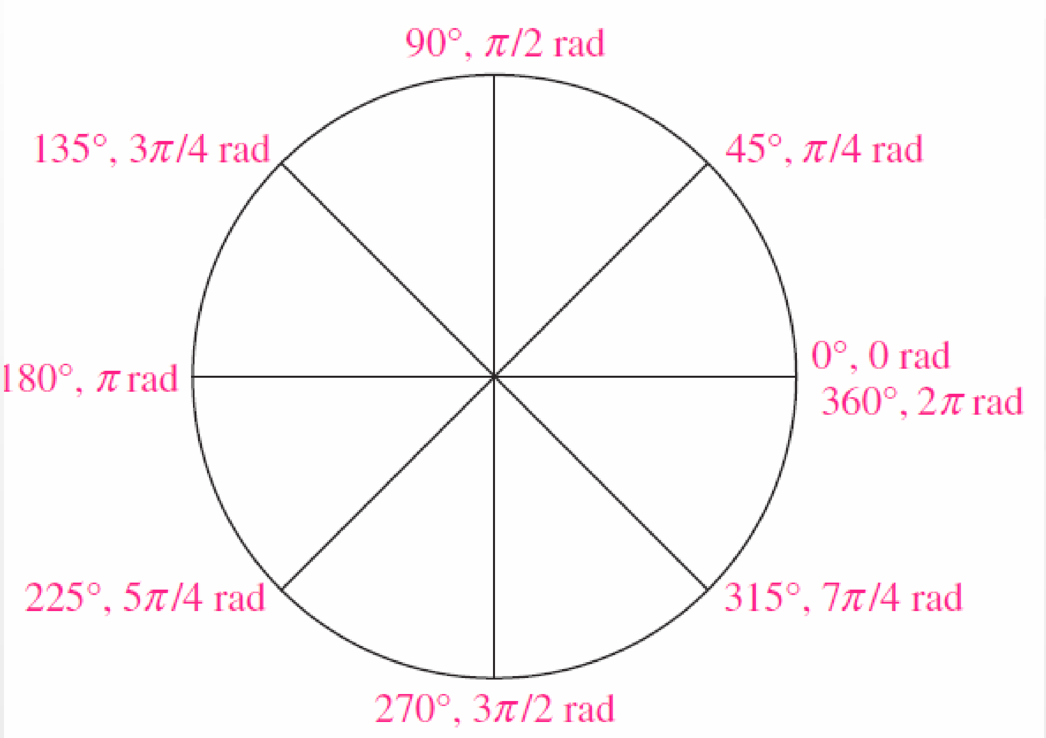
- Angular measurements can be made in degrees (°) or radians. The radian (rad) is the angle that is formed when the arc is equal to the radius of a circle. There are 360° or 2π radians in one complete revolution.



# Reminder

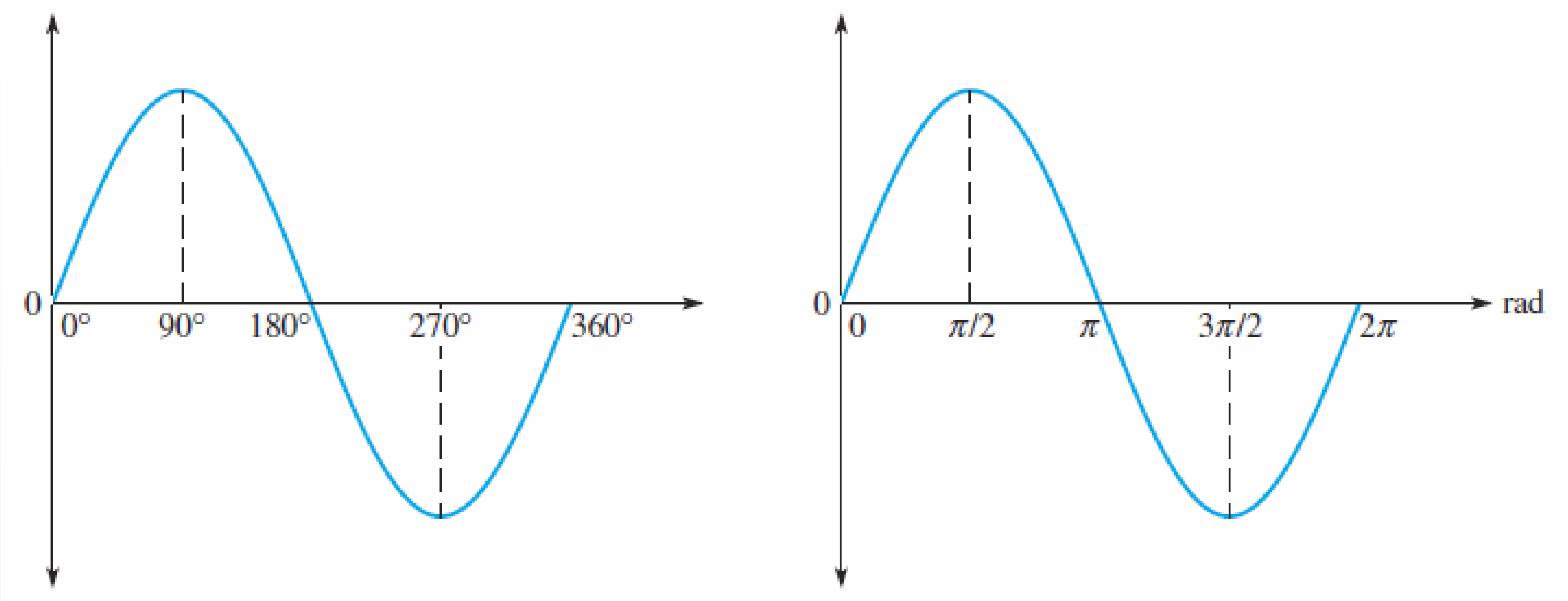
- Radian/Degree Conversion

$$rad = \left(\frac{\pi\, rad}{180°}\right) \times degrees \Leftrightarrow degrees = \left(\frac{180°}{\pi\, rad}\right) \times rad$$



## Sine Wave Angles

- The angular measurement of a sine wave is based on 360° or $2\pi$ rad for a complete cycle. A half-cycle is 180° or $\pi$ rad; a quarter-cycle is 90° or $\pi/2$ rad; and so on.



## Sin Wave Equation

- Instantaneous values of a wave are shown as v or i. The equation for the instantaneous voltage (v) of a sine wave is

$$v = V_p sin\theta$$

- where
- $V_p$ = Peak voltage
- $\theta$ = Angle in rad or degrees

## The Phase Shift

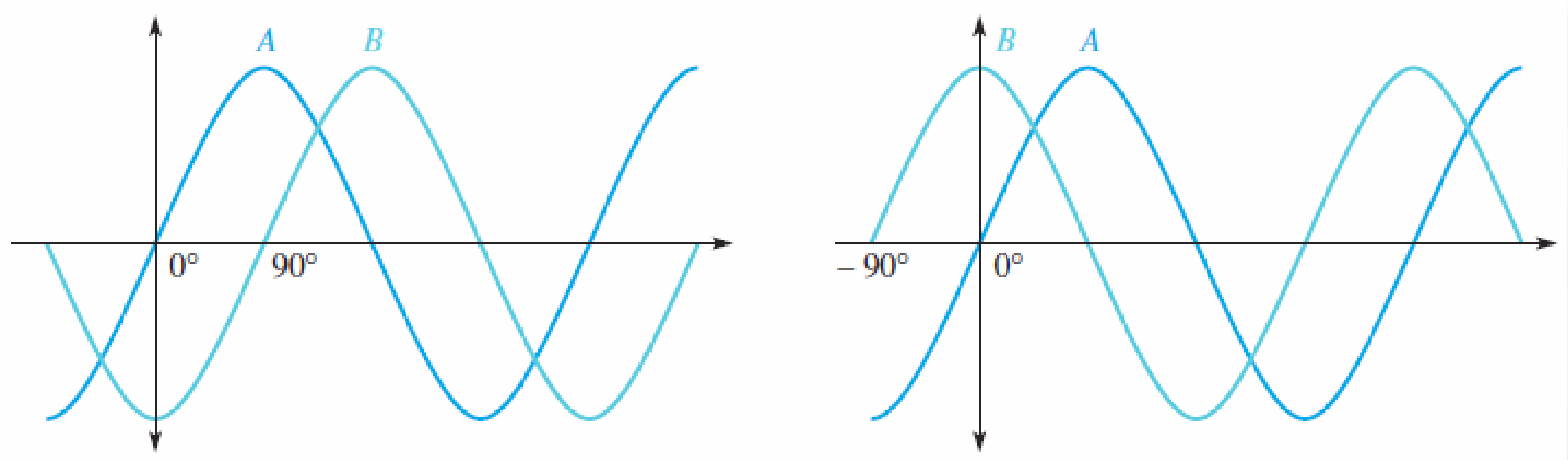
- The phase of a sine wave is an angular measurement that specifies the position of a sine wave relative to a reference. To show that a sine wave is shifted to the left or right of this reference, a term is added to the equation given previously.

$$v = V_p \sin(\theta \pm \emptyset)$$

- where
- $\emptyset$ = Phase shift

## Phase of Sine Wave

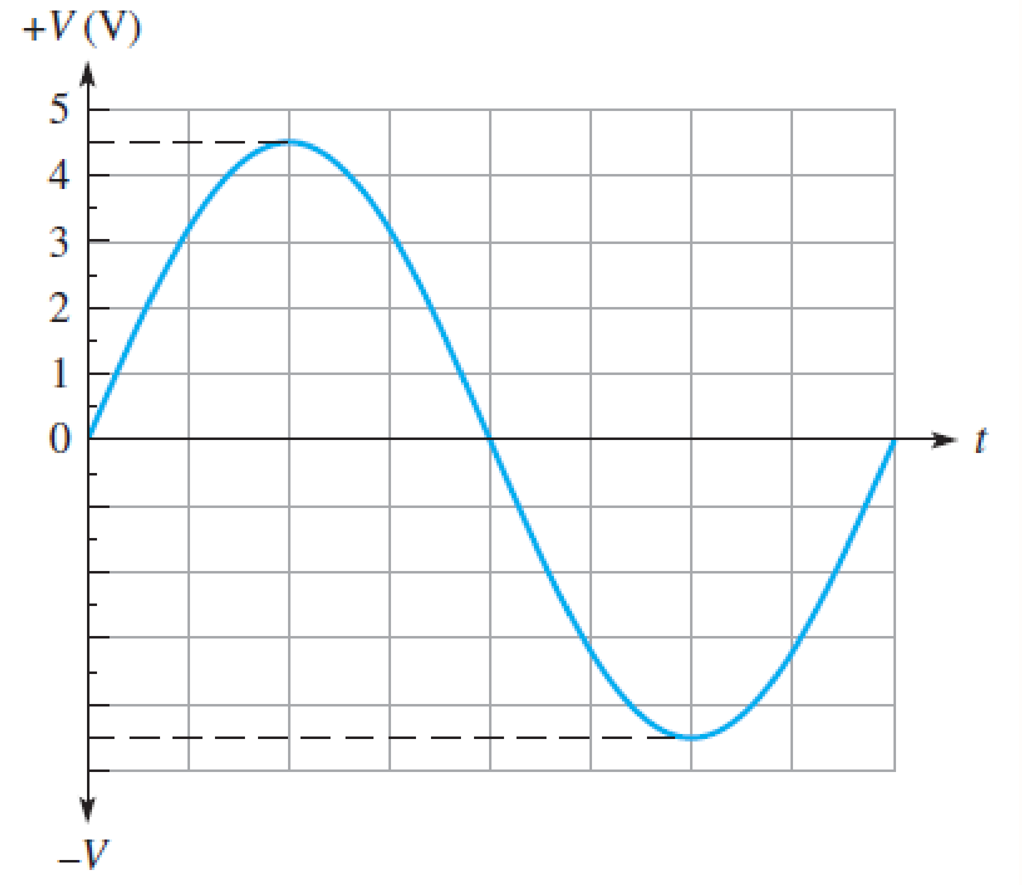
The **phase** of a sine wave is an angular measurement that specifies the position of that sine wave relative to a reference



# Exercise

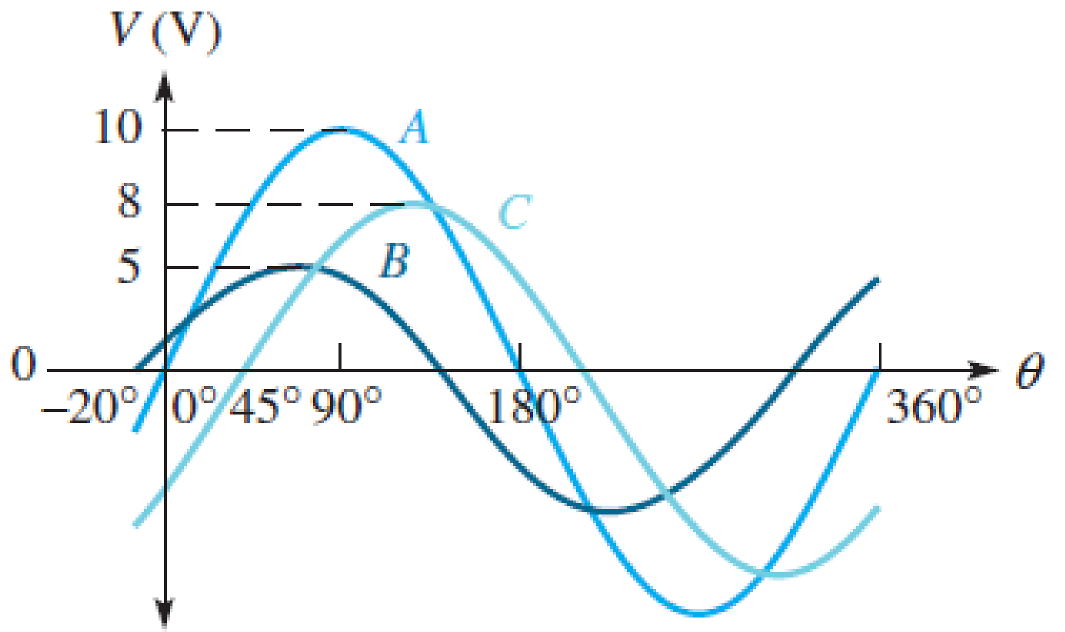
Determine $V_p$, $V_{pp}$, $V_{\text{rms}}$, and the half-cycle $V_{\text{avg}}$ for the sine wave

$V_p$=4.5 V, $V_{pp}$=9 V, $V_{rms}$=3.18 V, $V_{avg}$=2.87 V



# Exercise

Determine the instantaneous value at 90° on the horizontal axis for each voltage sine wave




Sine wave $A$ is the reference. Sine wave $B$ is shifted left 20° with respect to $A$, so $B$ leads. Sine wave $C$ is shifted right 45° with respect to $A$, so $C$ lags.

$$v_A = V_p \sin\theta = (10\text{ V})\sin 90^\circ = \mathbf{10\ V}$$

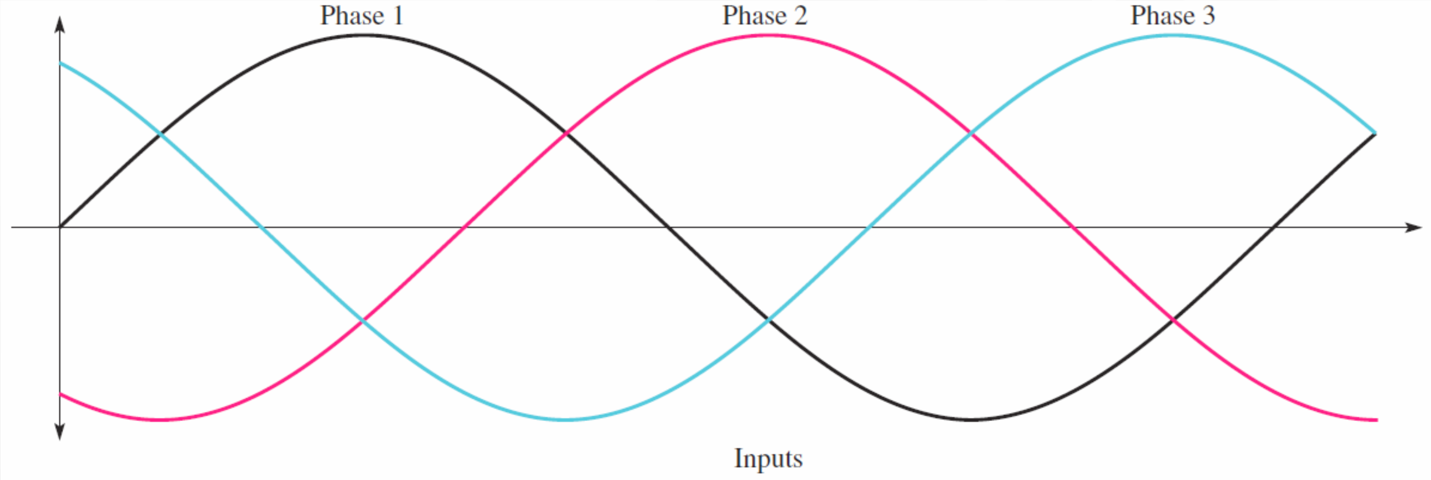
$$v_B = V_p \sin(\theta + \phi_B) = (5\text{ V})\sin(90^\circ + 20^\circ) = (5\text{ V})\sin 110^\circ = \mathbf{4.70\ V}$$

$$v_C = V_p \sin(\theta - \phi_C) = (8\text{ V})\sin(90^\circ - 45^\circ) = (8\text{ V})\sin 45^\circ = \mathbf{5.66\ V}$$

## The Phase Shift

- An important application of phase-shifted sine waves is in electrical power systems. Electrical utilities generate ac with three phases that are separated by 120° as illustrated.

- Normally, 3-phase power is delivered to the user with three hot lines plus neutral. The voltage of each phase, with respect to neutral is 120 V.

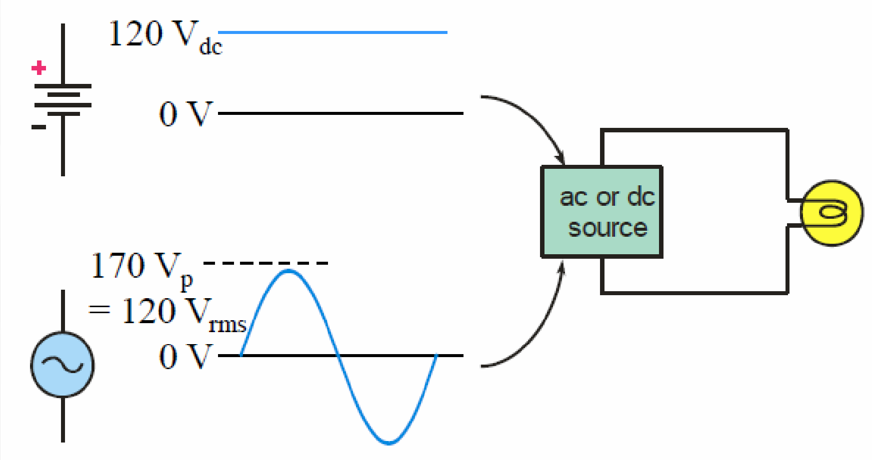


# Power in resistive AC circuits

- The power relationships developed for dc circuits apply to ac circuits except you must use rms values in ac circuits when calculating power.

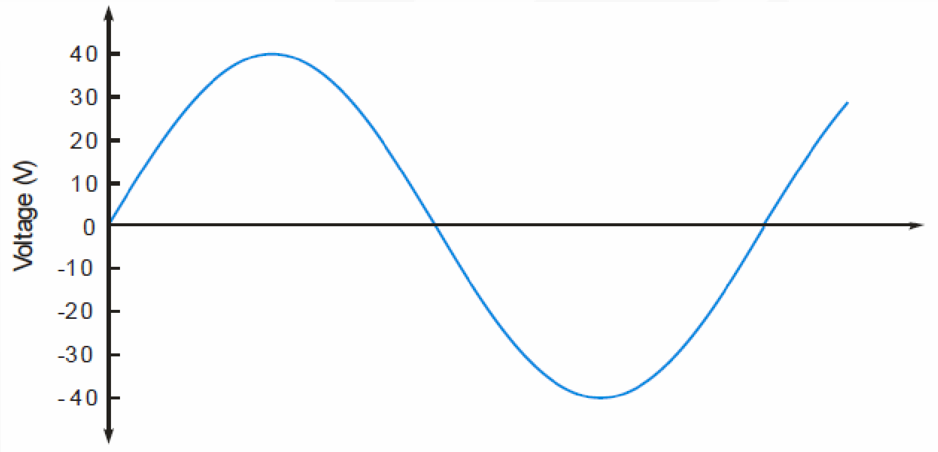
- Power formulas are:
- $P = V_{rms} I_{rms}$
- $P = \frac{{V_{rms}}^2}{R}$
- $P = {I_{rms}}^2 R$

- For example, the dc and the ac sources produce the same power to the bulb



# Power in resistive AC circuits

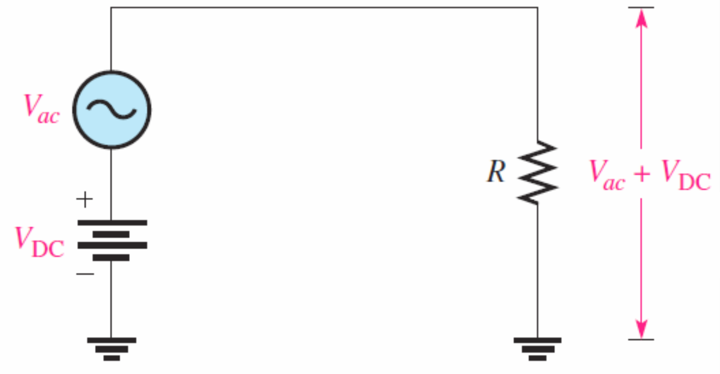
- <u>Example:</u>
- Assume a sine wave with a peak value of 40 V is applied to a 100 W resistive load. What power is dissipated?

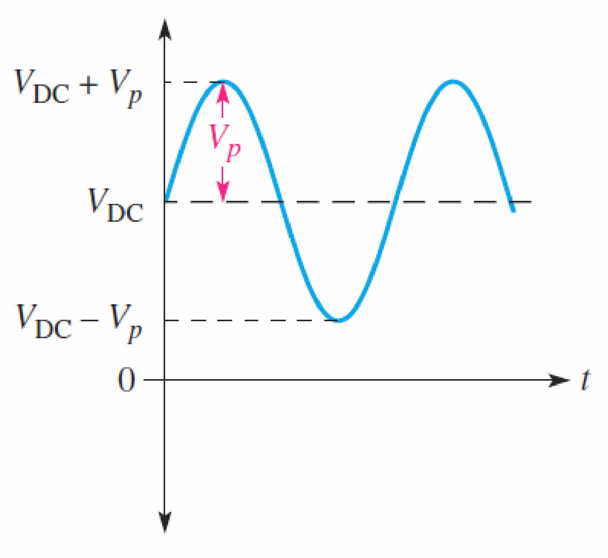



- <u>Solution:</u> $V_{rms} = 0.707 \times Vp = 0.707 \times 40\ V = 28.3\ V$
- $P = \frac{{V_{rms}}^2}{R} = \frac{28.3}{100} = 8\ W$

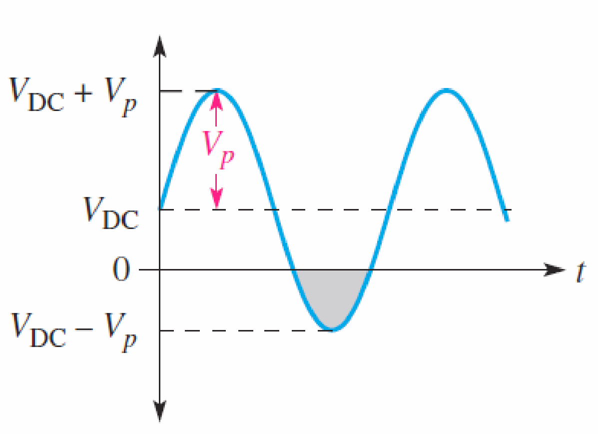
# Superimposed dc and ac voltages

Frequently dc and ac voltages are together in a waveform. They can be added algebraically, to produce a composite waveform of an ac voltage "riding" on a dc level.



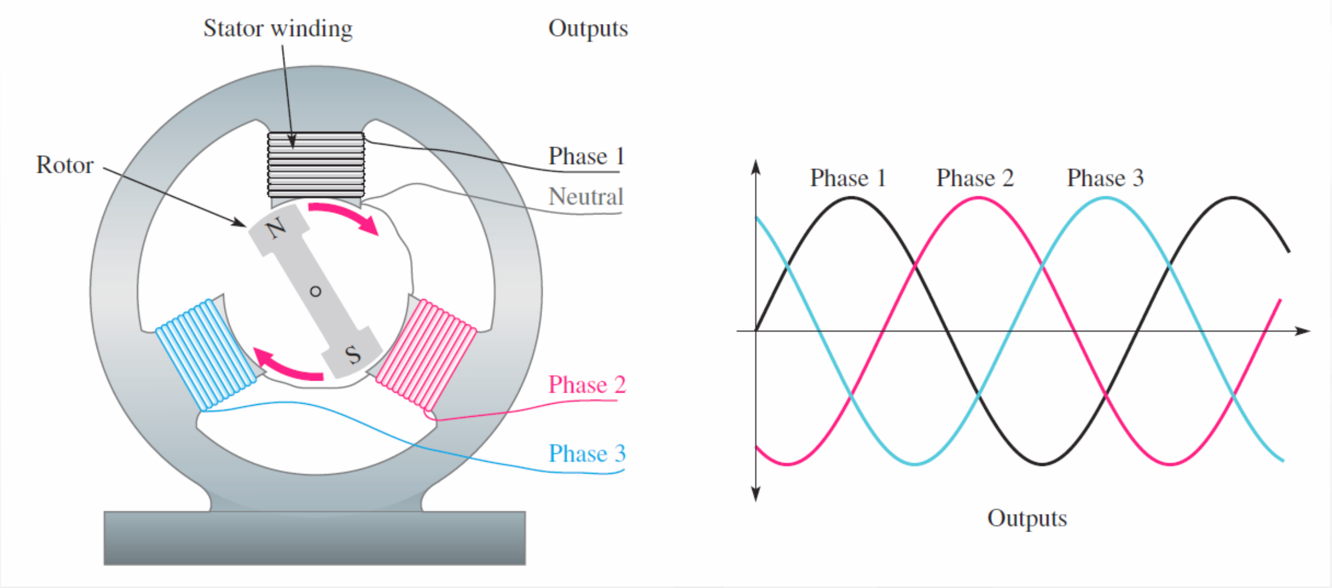

$V_{DC} > V_p$
Nonalternating




$V_{DC} < V_p$
Alternating

# Alternators

- Alternators are ac generators. Utility companies use 3-phase alternators and deliver all three phases to industrial customers.

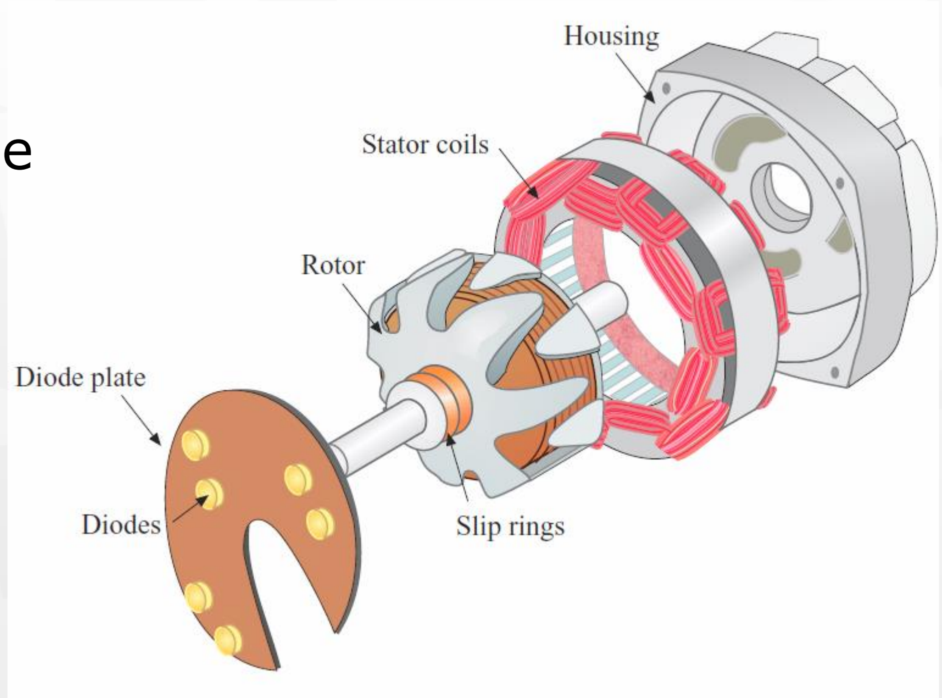



- The rotor shown is a permanent magnet that produces a strong magnetic field. As it sweeps by each stator winding, a sine wave is produced across that winding. The neutral is the reference.

# Alternators

- In vehicles, alternators generate ac, which is converted to dc for operating electrical devices and charging the battery. AC is more efficient to produce and can be easily regulated, hence it is generated and converted to dc by diodes.

- The output is taken from the rotor through the slip rings.

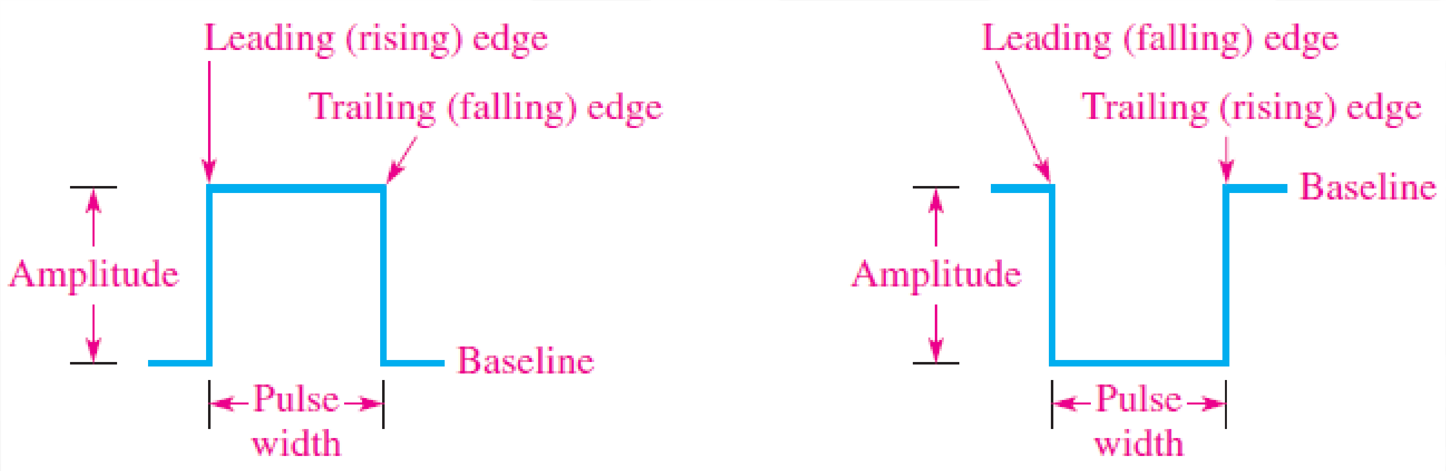



Basic vehicle alternator

# Nonsinusoidal Waveforms

## Pulse Waveform

Ideal pulses

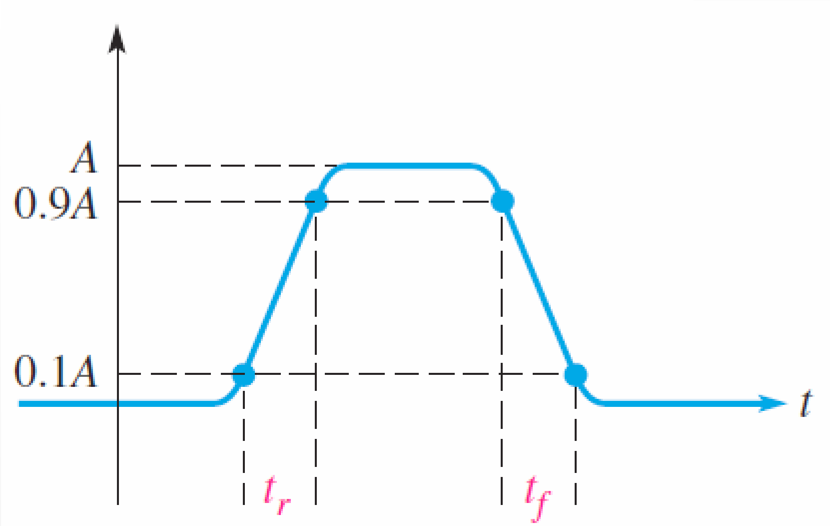



A **pulse** can be described as a very rapid transition (**leading edge**) from one voltage or current level (**baseline**) to another level; and then, after an interval of time, a very rapid transition (**trailing edge**) back to the original baseline level.

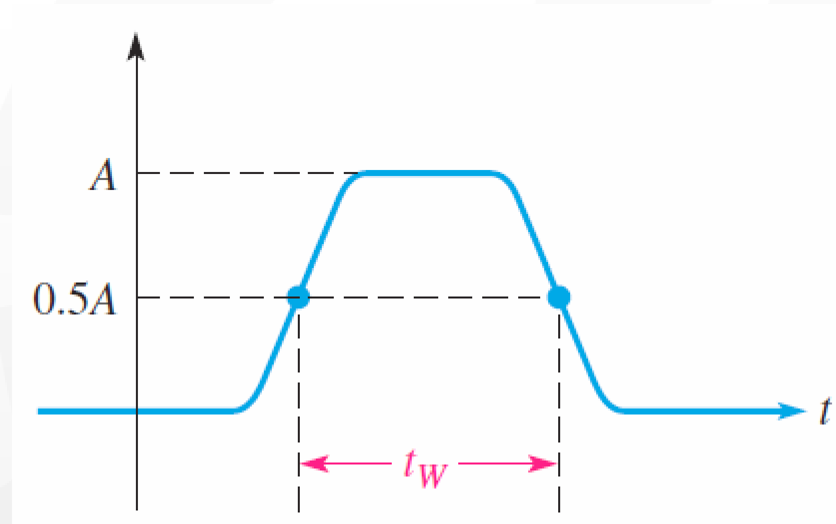
# Nonsinusoidal Waveforms

## Pulse Waveform

Actual pulses are never ideal




Rise and fall times




Pulse width

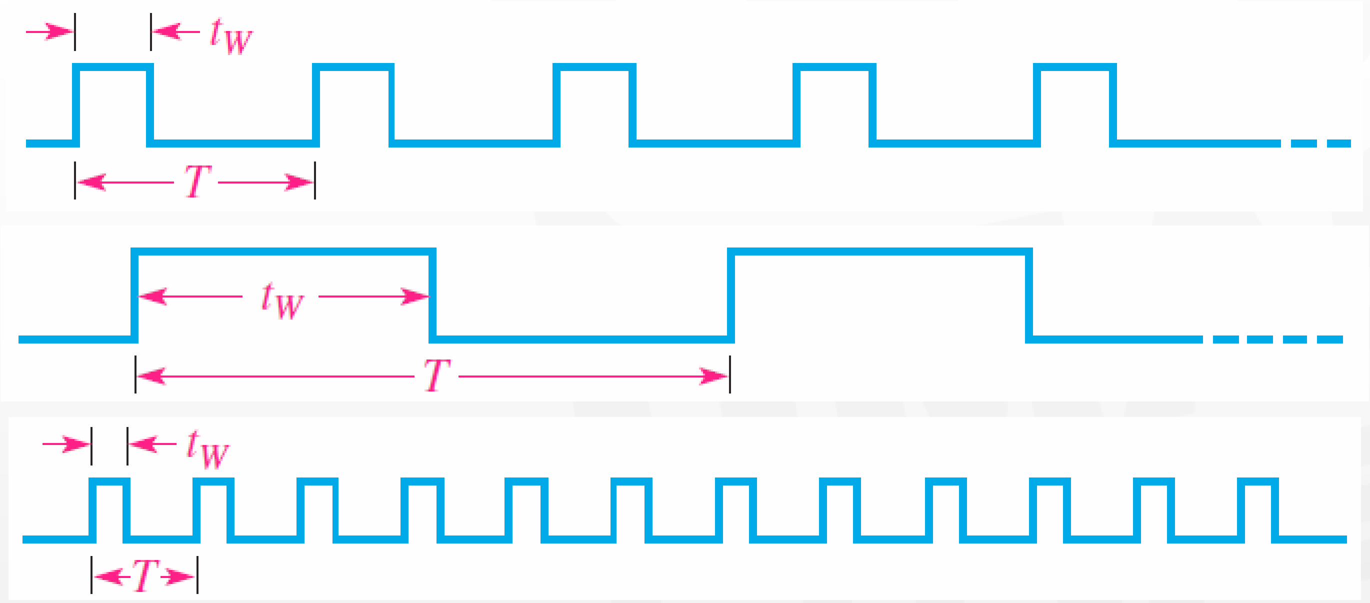
Rise and fall times are measured between the 10% and 90% levels.

Pulse width is measured at the 50% level.

# Nonsinusoidal Waveforms

## Pulse Waveform

**Repetitive Pulses**: Any waveform that repeats itself at fixed intervals is periodic.

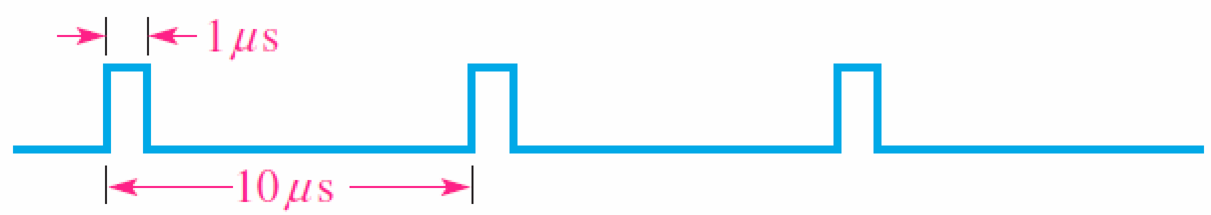



The **duty cycle** is the ratio of the pulse width ($t_W$) to the period (T) and is usually expressed as a percentage.

$$\text{Percent duty cycle} = \left(\frac{t_W}{T}\right)100\%$$

# Exercise

- Determine the period, frequency, and duty cycle for the pulse waveform.

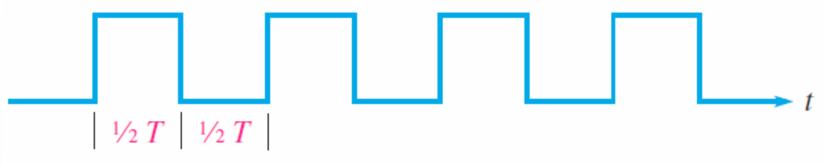



- $T = 10\ \mu s \quad f = \frac{1}{T} = \frac{1}{10} = 100\ kHz$
- Percent duty cycle= $\left(\frac{t_W}{T}\right) 100\% = \left(\frac{1 \mu s}{10 \mu s}\right) 100\% = 10\%$

# Nonsinusoidal Waveforms

## Pulse Waveform

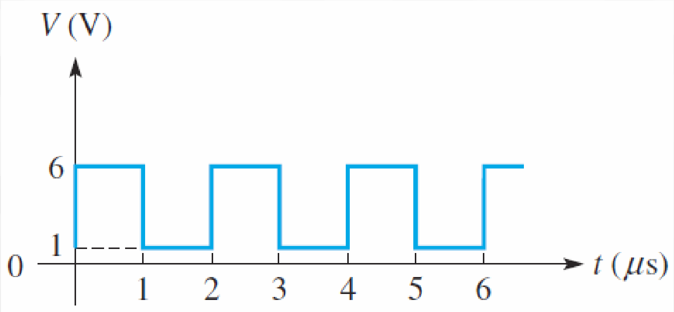
**A square wave** is a pulse waveform with a duty cycle of 50%.




**The average value of a pulse waveform** is equal to its baseline value plus the product of its duty cycle and its amplitude

$$V_{avg} = \text{baseline} + (\text{duty cycle})(\text{amplitude})$$

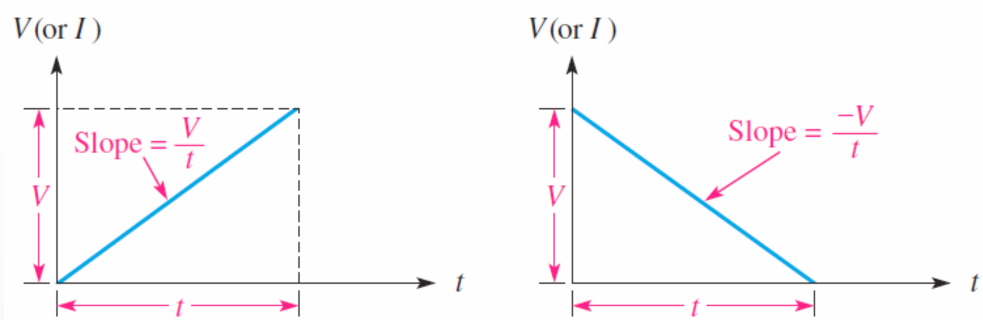
<u>Example:</u> Determine the average voltage of the positive-going waveforms

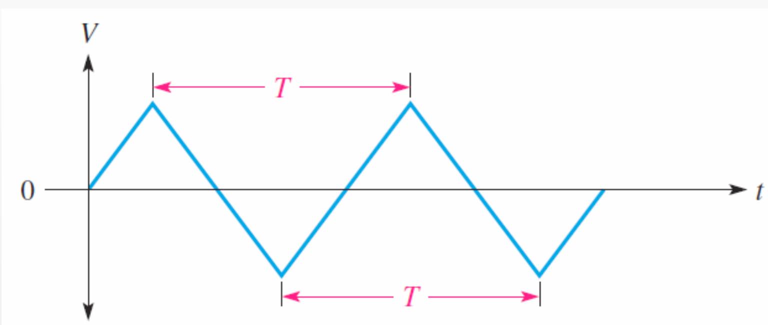


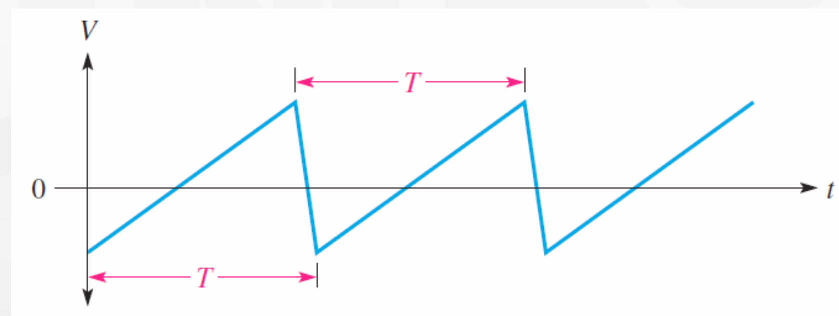

$$V_{avg} = 1\text{ V} + (50)(5\text{ V}) = 1 + 2.5\,V = 3.5\,V$$

# Triangular and Sawtooth waves

- Triangular and sawtooth waveforms are formed by voltage or current ramps (linear increase/decrease)



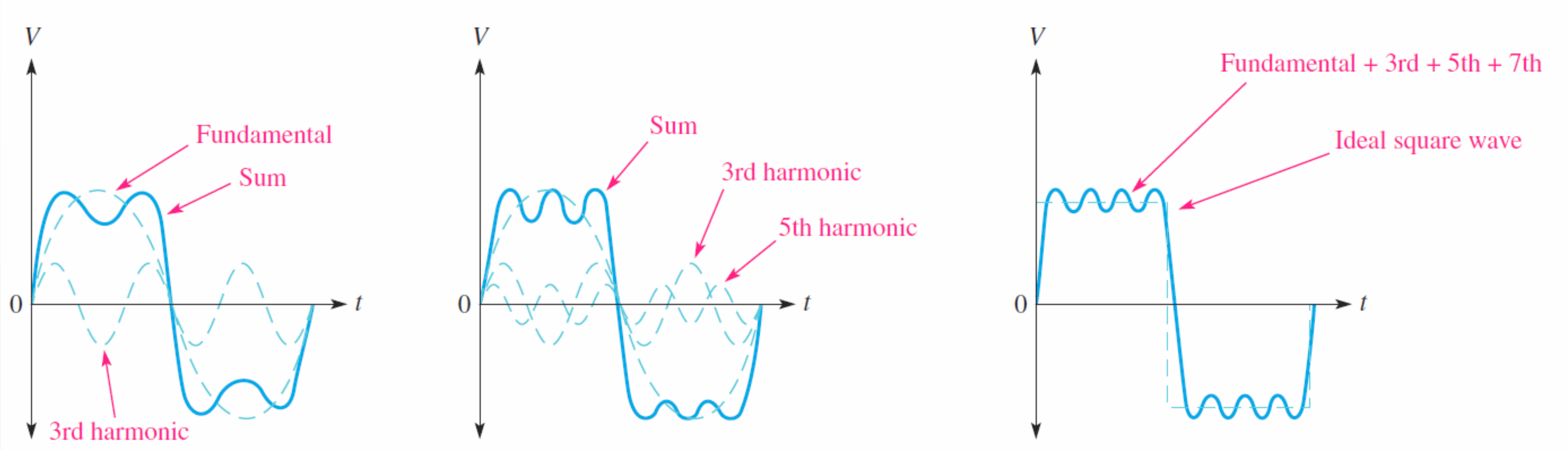




Triangular waveforms have positive-going and negativegoing ramps of equal duration.

The sawtooth waveform consists of two ramps, one of much longer duration than the other.

# Harmonics

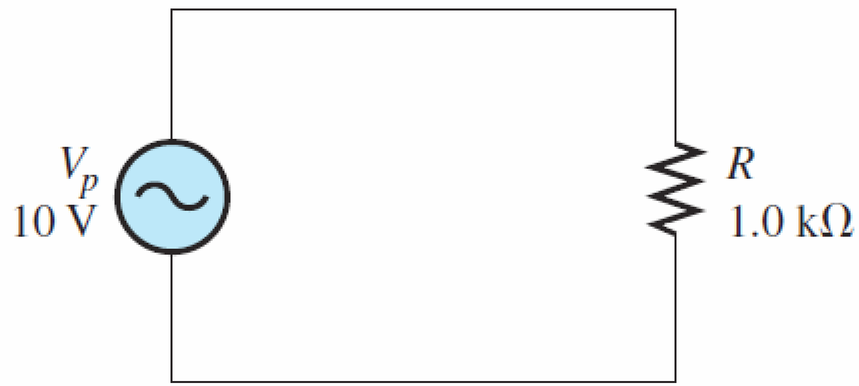
- All repetitive non-sinusoidal waveforms are composed of a **fundamental frequency** (repetition rate of the waveform) and **harmonic frequencies**.
- **Odd harmonics** are frequencies that are odd multiples of the fundamental frequency.
- **Even harmonics** are frequencies that are even multiples of the fundamental frequency.



# Oscilloscope


Typical oscilloscopes. Copyright © Tektronix

# Exercise




- A sinusoidal voltage is applied to the resistive circuit.
- Determine: $I_P$, $I_{PP}$, $I_{rms}$, $I_{avg}$ & i at the positive peak

- $I_P = V_P/R$ = 10 mA / $I_{PP}$ = 20 mA
- $I_{rms}$ = 7.07 mA / $I_{avg}$ = 0 A
- I at the positive peak = 10 mA