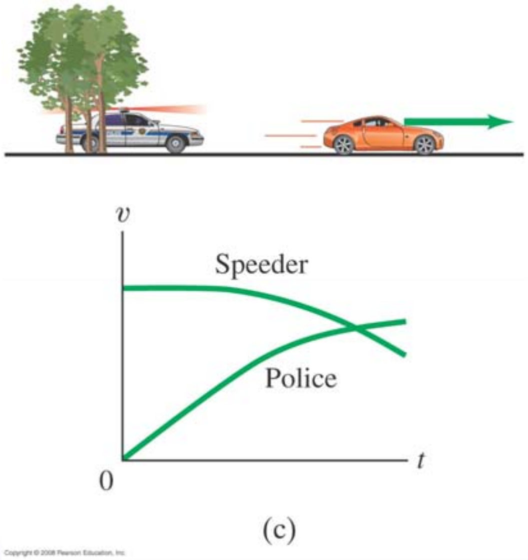
Lecture 3

Physics I

Chapter 2

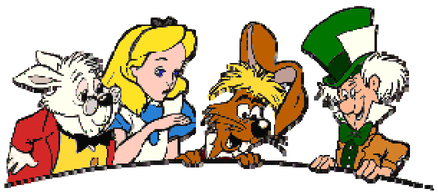
Equations of motion for constant acceleration



Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Today we are going to discuss:

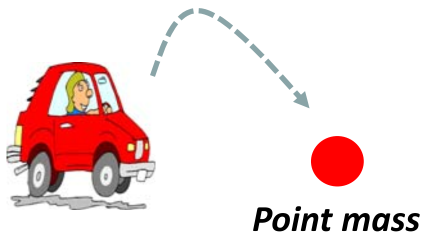
Chapter 2:



- ***Motion with constant acceleration:*** *Section 2.4*
- ***Free fall (gravity):*** *Section 2.5*

Simplifications

- ***Objects are point masses:*** ***have mass, no size***



- ***In a straight line:*** ***one dimension***

Consider a special, important type of motion:

- ***Acceleration is constant ($a = const$)***

$t = 0$ $\qquad\qquad\qquad\qquad$ $t > 0$

$a = \text{const.}$

we need to predict <u>future</u>, $x(t)$, $v(t)$

NEED: Equations ???

The Kinematic Equations of Constant Acceleration

Velocity equation. Equation 1.

(constant acceleration)

Since a=const, v is a straight line and it doesn't matter which acceleration to use, instantaneous or average.
Let's use average acceleration.

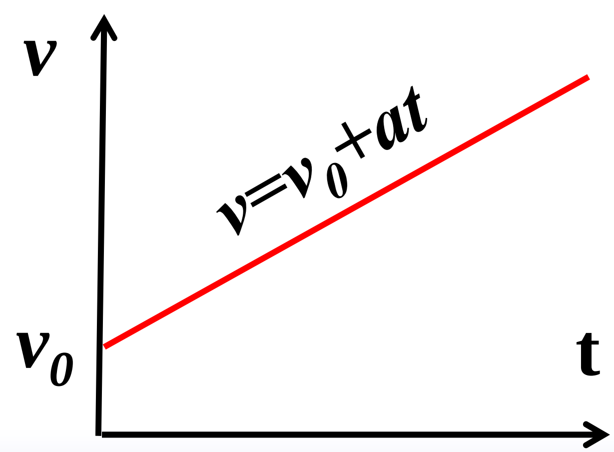
by definition, acceleration $$a = \frac{v(t) - v_o}{t - t_0} \text{ and } t_0 = 0$$

$$a = \frac{v(t) - v_o}{t} \quad \Rightarrow$$

Velocity equation

$$\boxed{v(t) = v_o + at} \quad (1)$$

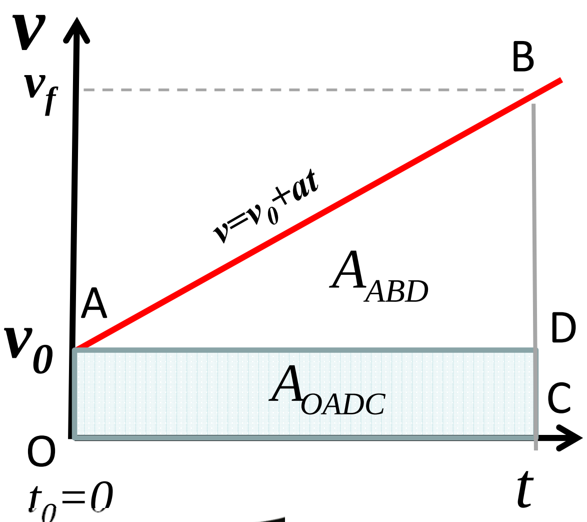
the velocity is increasing at a constant rate



Position equation. Equation 2

(constant acceleration)

Recall Eq (2.11) $x_f = x_0 + Area\ under\ v - vs - t\ between\ t_0\ and\ t_f$



$$\boxed{x_f = x_0 + A_{OADC} + A_{ABD}}$$

$$\boxed{x_f = x_0 + v_0 t + \tfrac{1}{2}(v_f - v_0)t} \qquad v_f(t) = v_o + at$$

Position equation

$$\boxed{x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2} \quad (2)$$

No time equation. Equation 3

(constant acceleration)

We can also combine these two equations so as to *eliminate t*:

Velocity equation

$$v(t) = v_o + at$$

Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

No time equation

$$v^2 = v_0{}^2 + 2a(x - x_0) \qquad (3)$$

It's useful when time information is not given.

Motion at Constant Acceleration (all equations)

We now have all the equations we need to solve constant-acceleration problems.

Velocity equation

$$v(t) = v_o + at \qquad (1)$$

Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (2)$$

No time equation

$$v^2 = v_0{}^2 + 2a(x - x_0) \qquad (3)$$

Problem Solving

How to solve:

- Divide problem into "knowns" and "unknowns"
- Determine best equation to solve the problem
- Input numbers

Example

A plane, taking off from rest, needs to achieve a speed of 28 m/s in order to take off. If the acceleration of the plane is constant at 2 m/s^2, what is the minimum length of the runway which can be used?

Example — A plane taking off

o which eq-n to use?

(1) ~~$v = v_0 + at$~~ (no time info)

(2) ~~$x = x_0 + v_0 t + at^2/2$~~ (no time info)

(3) $v^2 = v_0^2 + 2a(x - x_0)$ ✓ (our eq-n)

$$v^2 = \cancel{v_0^2}^{\,0} + 2a(x - \cancel{x_0}^{\,0})$$

known, known, ?

$$x = \frac{v^2}{2a} = \frac{(28\,\mathrm{m/s})^2}{2 \cdot 2\,\mathrm{m/s^2}} = \boxed{196\,\mathrm{m}}$$

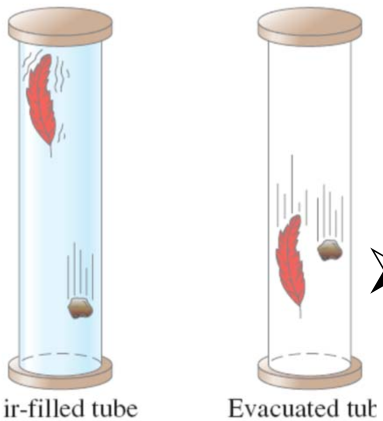
The runway <u>must</u> be 196 m long.


Free Fall


How?

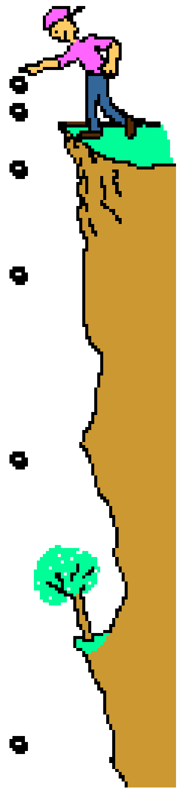
Freely Falling Objects

One of the most common examples of motion with constant acceleration is freely falling objects.



Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.

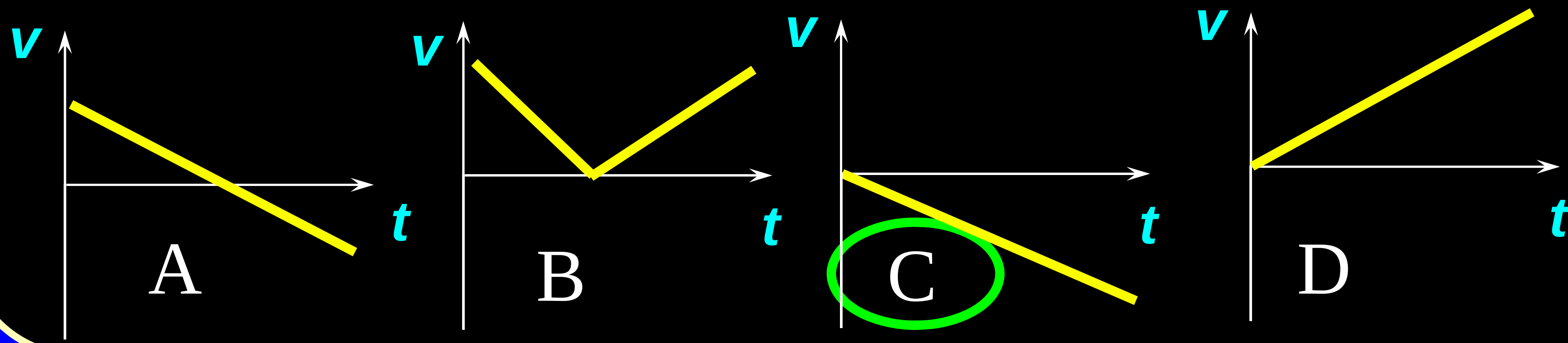
- ***All free-falling objects (on Earth) accelerate downwards at a rate of 9.8 m/s^2***
- ***Air resistance is neglected***



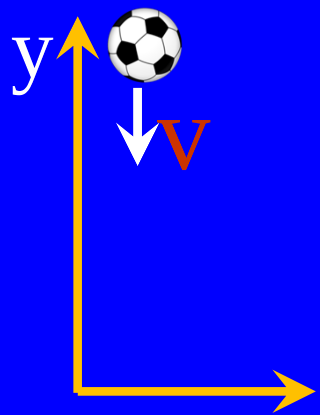
ConcepTest

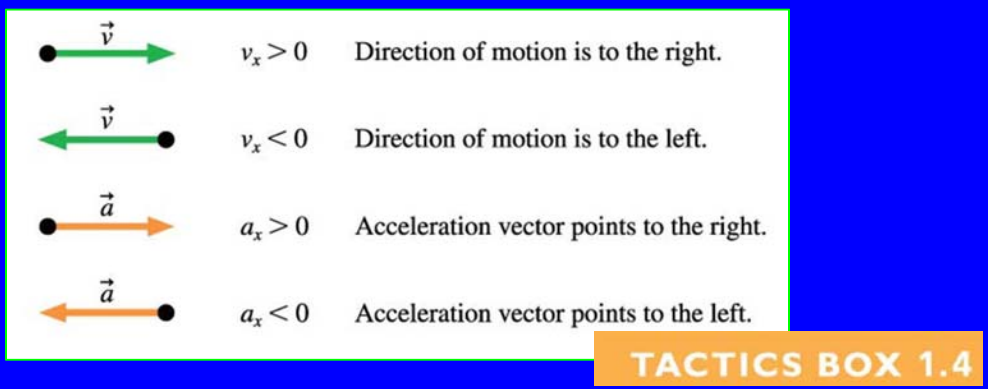
Free Fall

You drop a ball. Right after it leaves your hand and before it hits the floor, which of the above plots represents the v vs. t graph for this motion? (Assume your y-axis is pointing up).



The ball is dropped from rest, so its initial velocity is zero.
Because the y-axis is pointing upward and the ball is falling downward, its velocity is negative and becomes more and more negative as it accelerates downward.





Freely Falling Objects

Velocity equation

$$v(t) = v_o + at \qquad (1)$$

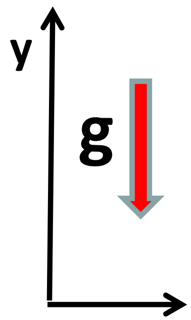
Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (2)$$

No time equation

$$v^2 = v_0{}^2 + 2a(x - x_0) \qquad (3)$$

if



then $a = -g$

$$\begin{cases} v = v_0 - gt \\ y = y_0 + v_0 t - gt^2/2 \\ v^2 = v_0{}^2 - 2g(y - y_0) \end{cases}$$

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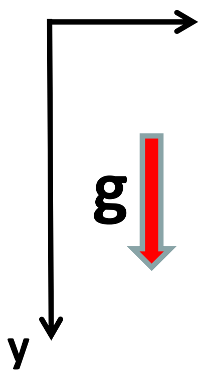
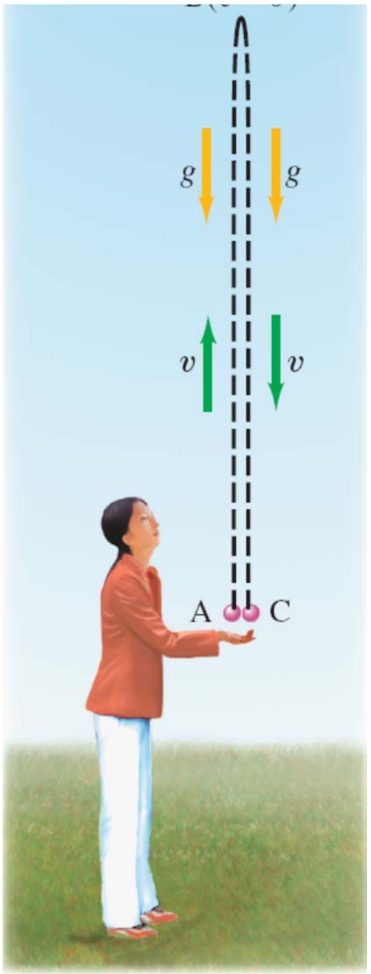
if



then $a = g$

$$\begin{cases} v = v_0 + gt \\ y = y_0 + v_0 t + gt^2/2 \\ v^2 = v_0{}^2 + 2g(y - y_0) \end{cases}$$

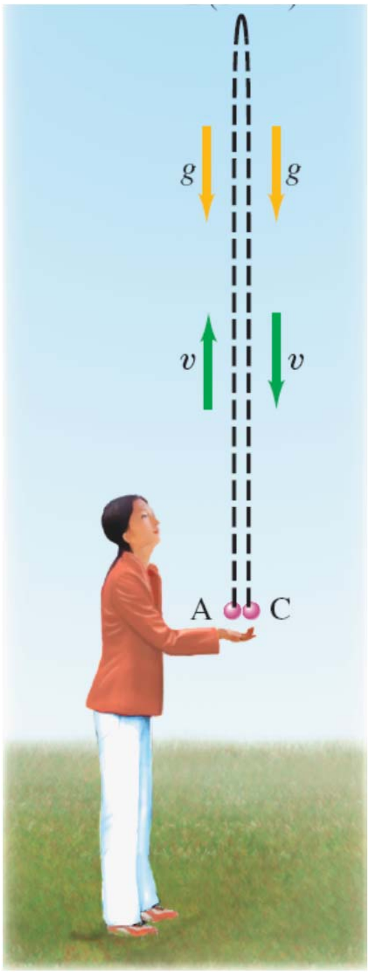
Example: Ball thrown upward.

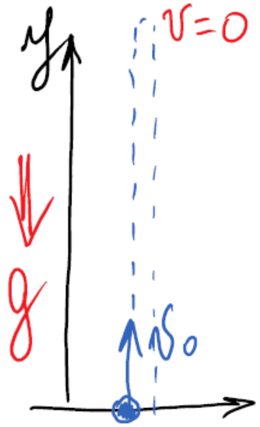


A person throws a ball upward into the air with an initial velocity of 10.0 m/s.
Calculate

(a) how high it goes, and
(b) how long the ball is in the air before it comes back to the hand.
(Ignore air resistance.)

Example





Given: $v_0 = 10\ m/s$; $y_0 = 0$

Calculate how high it goes: y - ?

1. Choose a coord. system:

y - upward,

g - downward (always)

so $a = -g$

$$\begin{cases} y = y_0 + v_0 t + \frac{a t^2}{2} \\ v = v_0 + a t \\ v^2 = v_0^2 + 2a(y - y_0) \end{cases}$$

$a = -g$

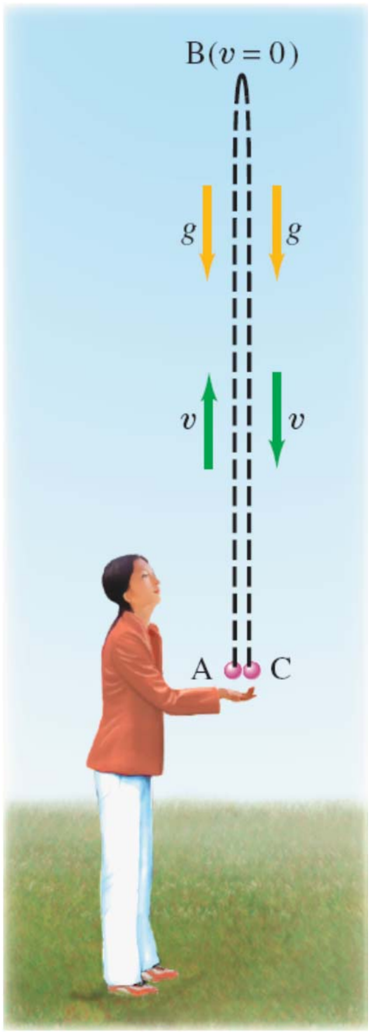
$$\begin{cases} y = y_0 + v_0 t - \frac{g t^2}{2} & \times \text{ (no } t \text{ info)} \\ v = v_0 - g t & \times \text{ (no } t \text{ info)} \\ \cancel{v^2}^{\,0} = v_0^2 - 2g(y - \cancel{y_0}^{\,0}) & \checkmark \end{cases}$$

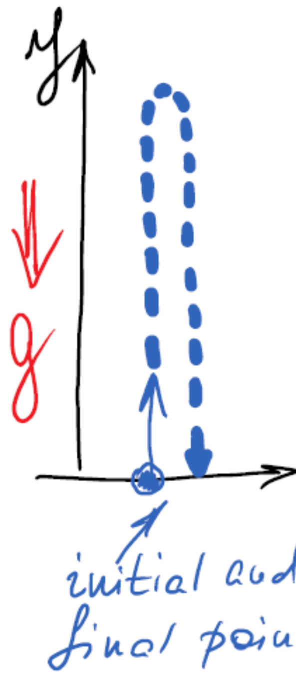
at max. hight, $v = 0$

$$0 = v_0^2 - 2 \cdot g \cdot y$$

$$y = \frac{v_0^2}{2 \cdot g} = \frac{(10\ m/s)^2}{2 \cdot 9.8\ m/s^2} \approx \frac{100}{20}\ m = 5m$$

Example





- How long the ball is in the air?

$$1.\ y = y_0 + v_0 t - \frac{gt^2}{2} \quad \checkmark$$
$$2.\ v = v_0 - gt \quad \checkmark$$
$$3.\ v^2 = v_0^2 - 2g(y - y_0) \quad \times$$

both OK!!

- At the final point: $y = 0$

let's use eq-n 1.

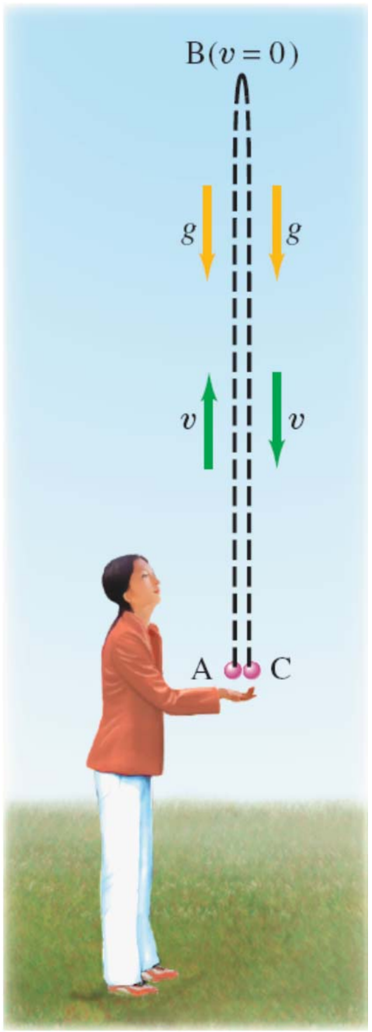
$$\cancel{y}^{0} = \cancel{y_0}^{0} + v_0 t - \frac{gt^2}{2}$$

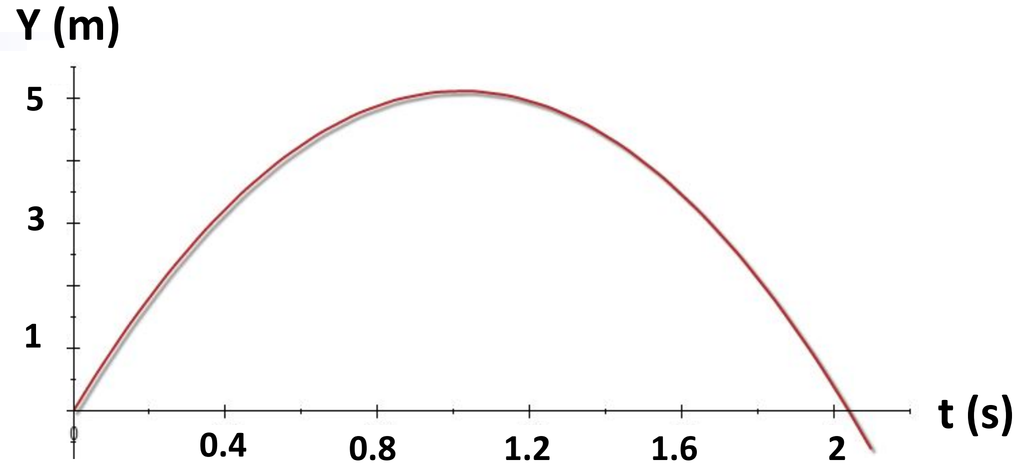
$$0 = t\left(v_0 - \frac{gt}{2}\right)$$ there are two solution.

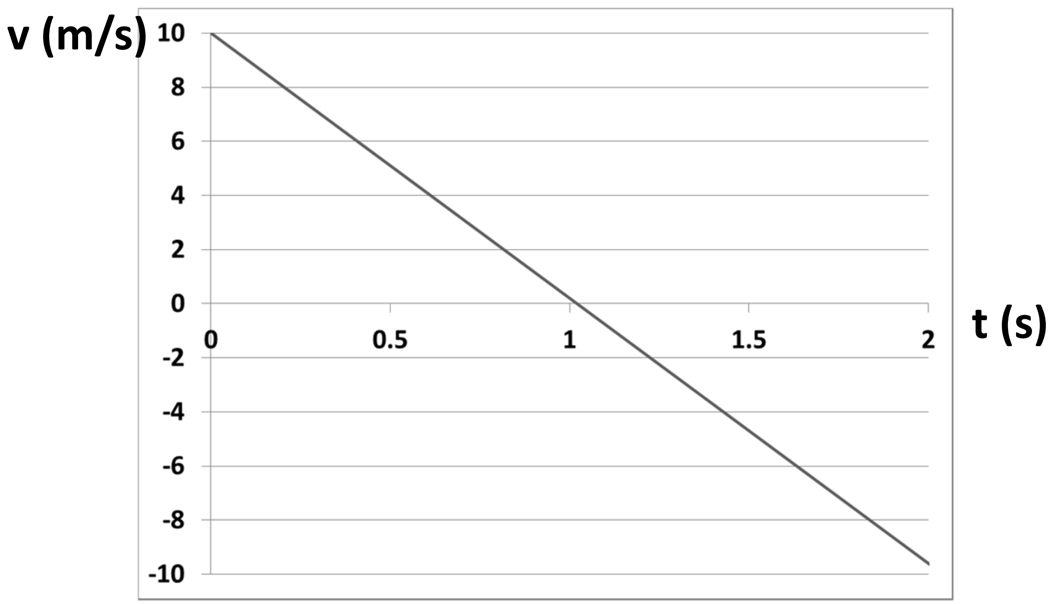
$$t_1 = 0; \quad v_0 - \frac{gt}{2} = 0$$

$$t_2 = \frac{2v_0}{g} = \frac{2 \cdot 10\,m/s}{9.8\,m/s^2} \simeq 2\,s$$

Example


B(v = 0)
g
g
v
v
A
C


Y (m)
5
3
1
0
0.4
0.8
1.2
1.6
2
t (s)


v (m/s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.5
1
1.5
2
t (s)


Thank you
Bye Bye For Now

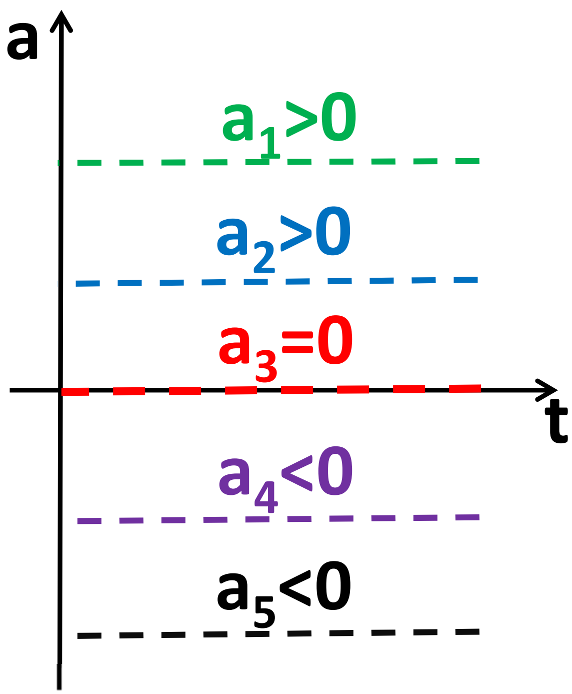
Determining the Sign of the Position, Velocity, and Acceleration

TACTICS BOX 1.4

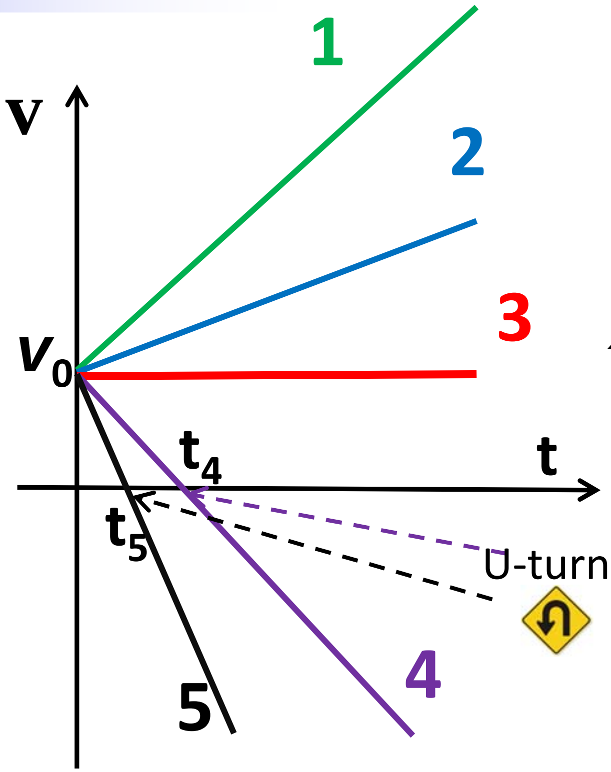
$x > 0$	Position to right of origin.
$x < 0$	Position to left of origin.
$v_x > 0$	Direction of motion is to the right.
$v_x < 0$	Direction of motion is to the left.
$a_x > 0$	Acceleration vector points to the right.
$a_x < 0$	Acceleration vector points to the left.

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.

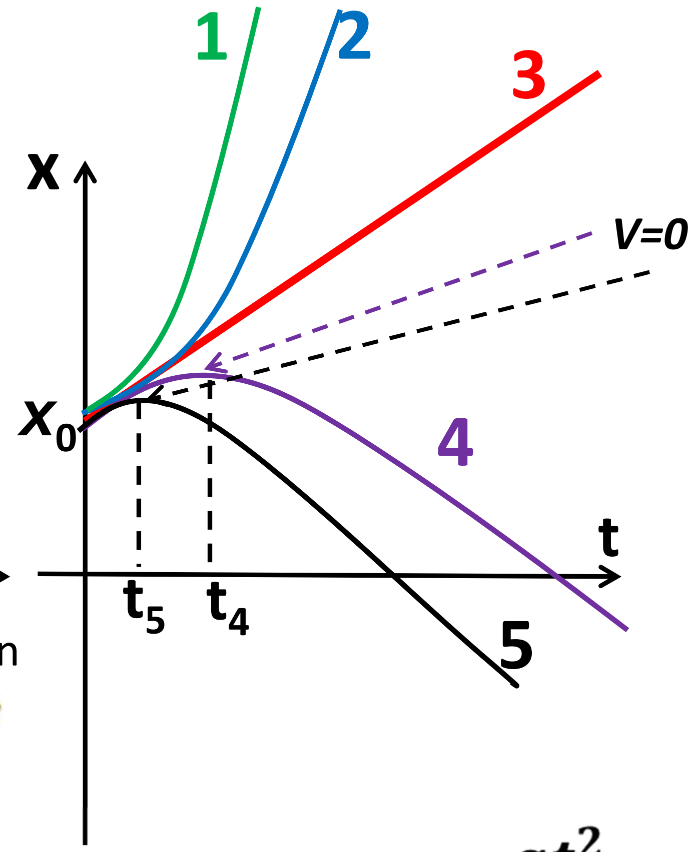
Velocity/Acceleration/Position



$$a = const$$



$$v = v_0 + at$$



$$x = x_0 + v_0 t + \frac{at^2}{2}$$

- 4,5 – negative acceleration,

  but from $0<t<t_4$ or t_5 – decceleration

  but for $t>t_4$ or t_5 – acceleration

ConcepTest

Position from velocity

Here is the velocity graph of an object that is at the origin (x = 0 m) at t = 0 s.

At t = 4.0 s, the object's position is

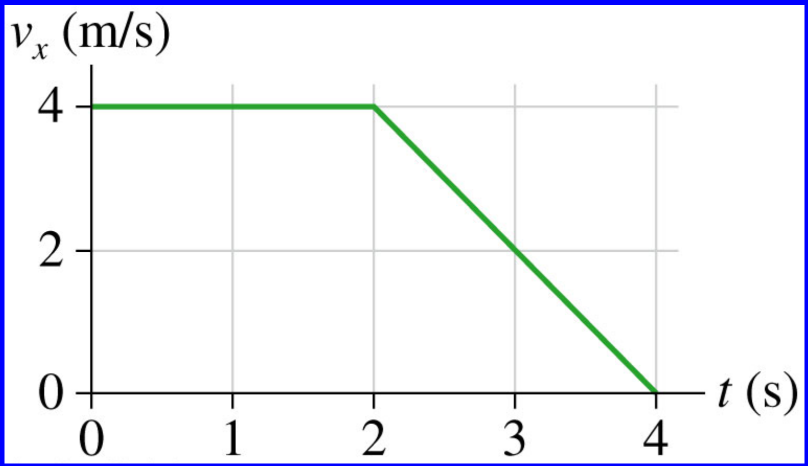
A) 20 m

B) 16 m

C) 12 m

D) 8 m

E) 4 m



$$x_f = x_i + Area\ under\ v-vs-t\ between\ t_i\ and\ t_f$$

Displacement = area under the curve