Joint Member Stiffness

- In the previous lecture we discussed the stiffness of the fasteners in the clamped zone. Now we can address the stiffness of the members that are gripped. The pressure from the bolt stays high to \( \leq 1.5 \) bolt radius. Further away from the bolt the pressure falls off. The stiffness is determined by modeling the clamped material by a frustum of a hollow cone.

**Figure 8-15**

Compression of a member with the equivalent elastic properties represented by a frustum of a hollow cone. Here, \( l \) represents the grip length.

- The stiffness of the frustum is given by:

\[
K_m = \frac{0.5774 \pi E d}{2 \ln \left\{ 5 \left( \frac{0.5774 \ell + 0.5 d}{0.5774 \ell + 2.5 d} \right) \right\}}
\]

assumes \( \ell = \) grip length, \( d = \) hole diameter

or

\[
K_m = (Ed) A e \left( \frac{Bd}{8} \right)
\]

where \( A + B \) are from Table 8-8

- Table 8-9 and 8-10 and 8-11 show specifications for bolts

proof load - The maximum load that a bolt can withstand without acquiring a permanent set.

proof strength - Is the stress at the proof load \( \left( \frac{P_{\text{proof}}}{\text{Area}} \right) \)

\( F_i = \) pre load

\( P = \) external tensile load
\[ P_b = \text{Portion of } P \text{ taken by the bolt} \]
\[ P_m = \text{Portion of } P \text{ taken by the members} \]
\[ F_b = \text{Resultant bolt load} = P_b + F_i \]
\[ F_m = \text{Resultant load on members} = P_m - F_i \]
\[ C = \text{Fraction of the external load } P \text{ carried by the bolt} \]
\[ C = \frac{K_b}{K_b + K_m} = \text{"Stiffness constant" or "joint constant"} \]

\[ P_b = CP \quad ; \quad P_m = P - P_b = (1 - C)P \]

For compressive load on the members (no separation \( F_m < 0 \))
\[ F_b = P_b + F_i = CP + F_i \]
\[ F_m = P_m - F_i = (1 - C)P - F_i \]

**Relationship between Bolt Torque and Tension**

We can use the equations for power screws that we derived on p. 15-2 to relate the torque to the load on the bolt. (see p 423).

\[ T = K F_i d \]

where \( K = \text{torque factor from Table 8-15} \)

\[ \text{OR} \quad K = \left( \frac{d_m}{2d} \right) \left( \frac{\tan \alpha + \mu \sec \alpha}{1 - \mu \tan \alpha \sec \alpha} \right) + 0.625 \mu \]

**Example**

A \( \frac{3}{4} \) in - 16 UNF x 2 \( \frac{1}{2} \) in. SAE grade 5 bolt is subjected to a load \( P = 6000 \) lbs in a tension joint. The initial bolt tension is \( F_i = 25000 \) lbs and the bolt and joint stiffness is \( K_b = 6.5 \text{ M/ft} \) and \( K_m = 13.8 \text{ M/ft} \).
a) Determine the preload and service load stresses

\[ \sigma_i = \frac{F_i}{A_t} \rightarrow A_t = 0.373 \text{ in}^2 \]  (Table 8-2)

\[ \sigma_i = \frac{25,000 \text{ lbs}}{0.373 \text{ in}^2} = 67.02 \text{ Ksi} = \sigma_i \]

Under service load

\[ \sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = C \frac{P}{A_t} + \sigma_i \]

\[ C = \frac{K_b}{K_b + K_m} = \frac{6.5K}{6.5K + 13.8K} = 0.320 \]

\[ \sigma_b = C \frac{P}{A_t} + \sigma_i = 0.320 \frac{6000}{0.373} + 67.02 \text{ Ksi} = 72.17 \text{ Ksi} = \sigma_b \]

b) Table 8-9 \rightarrow Minimum Proof Strength = 85 Ksi

The preload is 21% less and the service load is 15% than the proof strength.

c) The torque necessary to develop the preload = K F_i d

From Table 8-15 \rightarrow Torque Factor K = 0.2

\[ T = (0.2) (2500 \text{ lb}) (0.75 \text{ in}) = 3750 \text{ lb-in} = T \]

Tension Joint with a Preload

\[ \sigma_b = \frac{F_b}{A_t} = \frac{CP}{NA_t} + \frac{F_i}{A_t} \]

\[ \Rightarrow \# \text{ of bolts} \]

The limiting value of \( \sigma_b \) is \( \sigma_p \) (proof strength).
Thus we can include a load factor "n"

\[
\frac{C_n P}{NA_t} + \frac{F_i}{A_t} = S_p = \text{proof strength}
\]

Solving for \( n \) \( \rightarrow \) \( n = \frac{N(S_p A_t - F_i)}{C P} = \text{load factor} \)

When \( n > 1 \) the bolt stress is less than the proof strength.

- For bolts we want the preload to be larger than the external load so joint separation does not occur. Joint separation occurs when the resultant load on the members = 0, \( F_m = 0 \) or the external load exceeds the preload. If the full bolt strength is not used to develop the preload, the joint is not as strong as it could be.

When \( F_m = 0 \) \( \Rightarrow \) \( F_m = 0 = (1-c)P_o - F_i \)

where \( P_o \) is the load that causes joint separation

The factor of safety against joint separation = \( N_o = \frac{P_o}{P} \)

Combining the two eq's

\[ N_o = \frac{F_i}{P(1-c)} = SF_{\text{joint separation}} \]

- Generally speaking a bolt will fracture during tightening or will not otherwise fail.

- The following is a guideline

\[ F_i = \text{Preload} = \begin{cases} 
0.75 A_t S_p & \text{for reused connections} \\
0.19 A_t S_p & \text{for permanent connections} 
\end{cases} \]

\( \Rightarrow \) Table 8-9 to 8-11 (Proof Strength)

\( \Rightarrow \) Table 8-1 + 8-2 (Tensile Area)
Example

It is required to resist a 36000 lb external force using a number of 5/8"-11 UNC x 2 1/4" grade 5 bolts. The grip is 1.5".

a) Calculate the joint constant

b) The # of bolts required if the design factor is two and the connection will be reused.

Solution

\[ C = \frac{Kb}{Kb + Km} = \frac{Ad At E}{Ad At + Ad \cdot \frac{At - Ed}{Ad}} \]

\[ Ad = \frac{\pi}{4} \left( \frac{5}{8} \right)^2 = 0.3068 \text{in}^2 \]

\[ At = 0.226 \text{in}^2 \quad \text{(Table 8-2)} \]

\[ E = 30 \times 10^6 \text{psi} \quad l_t = 0.75'' \quad l_d = 0.75'' \]

\[ Kb = \frac{0.3068 \text{in}^2 \cdot 0.226 \text{in}^2 \cdot (30 \times 10^6)}{(0.3068 \text{in}^2) \cdot (0.75'') + (0.226 \text{in}^2) \cdot (0.75'')} = 5.21 \times 10^6 \text{lb/in} \]

\[ Km = \frac{0.5774 \pi Ed}{2 \ln \left\{ 5 \frac{0.5774 \cdot l + 0.5d}{0.5774 \cdot l + 2.5d} \right\}} = \frac{0.5774 \pi (1.5) (12 \times 10^6) \cdot 0.625''}{2 \ln \left\{ 5 \frac{0.5774 (1.5) + 0.5 (1.625)}{0.5774 (1.5) + 2.5 (1.625)} \right\}} \]

\[ Km = 7.67 \times 10^6 \text{lb/in} \rightarrow C = \frac{5.21}{5.21 + 7.67} = 0.404 = C \]

b) \[ n = \frac{N (S_p At - F_i)}{C P} \rightarrow N = \frac{C n P}{S_p At - F_i} \]

\[ S_p = 85 \text{ksi} \quad \text{(Table 8-9 pp. 418)} \]

\[ F_i = 0.75 At S_p = 0.75 (0.226 \text{in}^2) (85 \text{ksi}) = 14.4 \times 10^3 \text{lbs} \]

\[ n = \frac{(0.404) (2) (36 \text{ksi})}{(85 \text{ksi}) (0.226 \text{in}^2)} - 14.4 = 6.05 = \# \text{ of bolts} \]

So we need 7 bolts.