92.305 Homework 1

YOUR NAME HERE

September 15, 2016

Exercise 1.2.1. (a) Prove that $\sqrt{3}$ is irrational.
Proof: Suppose that $\sqrt{3}$ is rational. Then ... ■

(b) Where does the proof of Theorem 1.1.1 break down if we try to use it to prove $\sqrt{4}$ is irrational?
Answer: Let us try to use the method of Theorem 1.1.1 to prove that $\sqrt{4}$ is irrational. We would suppose $\sqrt{4}$ is rational, so ....

Comment: I liked this problem.

Time spent on this problem: 15 minutes

Exercise 1.2.2. Decide which of the following ...

(a) If $A_1 \supseteq A_2 \supseteq A_3 \supseteq A_4 \cdots$ are all sets containing an infinite number of elements, then the intersection $\bigcap_{n=1}^{\infty} A_n$ is infinite as well.

(b) ...

(c) $A \cap (B \cup C) = ...$

Time spent on this problem: 5 minutes

Exercise 1.2.10. Let $y_1 = 1$, and for each $n \in \mathbf{N}$ define $y_{n+1} = (3y_n + 4)/4$.

(a) Use induction...

Time spent on this problem: 120 minutes

Comment: I think this problem is too hard.

Exercise 1.3.3.

Time spent on this problem: 20 minutes

Comment: I think this problem would be okay if you gave more guidance about how to get started. Once I knew how to get started, it actually wasn't that hard to solve.

Exercise 1.3.8.

Comment: Are you aware that you showed us how to prove this in class? Just wondering. Usually your problems are more challenging than that. Maybe you meant to change it a little?

Time spent on this problem: 5 minutes

$$\Diamond$$

Here are some LaTeX-isms you might find helpful:

No matter what integers p and q are chosen, it is never the case that $(p/q)^2 = 2$.

$$\left(\frac{p}{q}\right)^2 = 2$$

$x \in A \cup B$ provided that $x \in A$ or $x \in B$ (or potentially both)

$$S = \{r \in \mathbf{Q} \ : r^2 < 2\}$$

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbf{Q} \\ 0 & \text{if } x \notin \mathbf{Q} \end{cases}$$

$$\begin{aligned} m &\leq na + 1 \\ &< n\left(b - \frac{1}{n}\right) + 1 \\ &= nb. \end{aligned}$$

$$\begin{array}{lccccccc} \mathbf{N}: & 1 & 2 & 3 & 4 & \cdots & n & \cdots \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \cdots & \updownarrow & \\ E: & 2 & 4 & 6 & 8 & \cdots & 2n & \cdots \end{array}$$

$$\begin{array}{ccccccc} 1 & 3 & 6 & 10 & 15 & \cdots \\ 2 & 5 & 9 & 14 & \cdots \\ 4 & 8 & 13 & \cdots \\ 7 & 12 & \cdots \\ 11 & \cdots \\ \vdots \end{array}$$

If you have other ideas for LaTeX-isms I should include, let me know and I'll add them.