Math 491, Problem Set #19
(due 12/9/03)

1. Let $P(n)$ and $Q(n)$ denote the numerator and denominator obtained when the continued fraction

$$x_1 + (y_1/(x_2 + (y_2/(x_3 + (y_3/ \cdots + (y_{n-2}/(x_{n-1} + (y_{n-1}/x_n))) \cdots)))))$$

is expressed as an ordinary fraction. Thus $P(n)$ and $Q(n)$ are polynomials in the variables $x_1, ..., x_n$ and $y_1, ..., y_{n-1}$.

   (a) By examining small cases, give a conjectural bijection between the terms of the polynomial $P(n)$ and domino tilings of the 2-by-n rectangle, and a similar bijection between the terms of the polynomial $Q(n)$ and domino tilings of the 2-by-$(n-1)$ rectangle, as well as a conjecture that gives all the coefficients.

   (b) Prove your conjectures from part (a) by induction on n.

2. Let $R(n)$ denote the determinant of the n-by-n matrix M whose i, jth entry is equal to

$$\begin{cases} x_i & \text{if } j = i, \\ y_i & \text{if } j = i+1, \\ z_{i-1} & \text{if } j = i-1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) By examining small cases, give a conjectural bijection between the terms of the polynomial $R(n)$ and domino tilings of the 2-by-n rectangle, and a conjecture for the coefficients.

   (b) Prove your conjectures from part (a) by induction on n.