Math 491, Problem Set #4
(due 9/23/03)

1. (a) Does there exist a polynomial $p(t)$ of degree 3 such that the linear operator $p(T)$ annihilates the sequence whose nth term (for $n \geq 0$) is $3^n + 2^n + 1^n$? Exhibit such a polynomial or explain why none exists.

   (b) Same as (a), but with "degree 3" replaced by "degree 4".

   (c) Same as (a), but with "degree 3" replaced by "degree 2".

2. Let F_n be the nth Fibonacci number, as Wilf indexes them (with $F_0 = F_1 = 1$, $F_2 = 2$, etc.). Give a simple homogeneous linear recurrence relation satisfied by the sequence whose nth term is

   (a) nF_n;

   (b) $1F_1 + 2F_2 + ... + nF_n$;

   (c) $nF_1 + (n-1)F_2 + ... + 2F_{n-1} + F_n$;

   (d) F_n when n is odd, and 2^n when n is even.

   In each case, an explanation should be included.

Please be sure to write down how many hours you spent working on the problems, and whom you worked with.