

The n -tuple (x_1, x_2, \dots, x_n) is a *Markov n -tuple* if $\sum_{i=1..n} x_i^2 = n \prod_{i=1..n} x_i$. Notice that the n -tuple $(1, 1, \dots, 1)$ satisfies this relation. Notice also that if we replace x_k by $x'_k = \frac{1}{x_k} \cdot \sum_{i \neq k} x_i^2 = \frac{1}{x_k} \cdot (n(\prod_{i=1..n} x_i) - x_k^2) = n \prod_{i \neq k} x_i - x_k$, then the new n -tuple also satisfies the relation:

$$\begin{aligned}
\sum_{i \neq k} x_i^2 + x_k'^2 &= \sum_{i \neq k} x_i^2 + (n \prod_{i \neq k} x_i - x_k)^2 \\
&= \sum_{i \neq k} x_i^2 + x_k^2 - 2n \prod_{i=1..n} x_i + n^2 (\prod_{i \neq k} x_i)^2 \\
&= \sum_{i=1..n} x_i^2 - 2n \prod_{i=1..n} x_i + n^2 (\prod_{i \neq k} x_i)^2 \\
&= n \prod_{i=1..n} x_i - 2n \prod_{i=1..n} x_i + n^2 (\prod_{i \neq k} x_i)^2 \\
&= n^2 (\prod_{i \neq k} x_i)^2 - n \prod_{i=1..n} x_i \\
&= n \prod_{i \neq k} x_i \cdot (n \prod_{i \neq k} x_i - x_k) \\
&= n \prod_{i \neq k} x_i \cdot x_k'
\end{aligned}$$