PROBLEMS

Section 1-3: Traveling Waves

1.1 A 4-kHz sound wave traveling in the x-direction in air was observed to have a differential pressure \( p(x, t) = 5 \text{ N/m}^2 \) at \( x = 0 \) and \( t = 25 \mu s \). If the reference phase of \( p(x, t) \) is 42°, find a complete expression for \( p(x, t) \). The velocity of sound in air is 330 m/s.

1.2 For the pressure wave described in Example 1-1, plot the following:

(a) \( p(x, t) \) versus \( x \) at \( t = 0 \)

(b) \( p(x, t) \) versus \( t \) at \( x = 0 \)

Be sure to use appropriate scales for \( x \) and \( t \) so that each of your plots covers at least two cycles.

1.3 A harmonic wave traveling along a string is generated by an oscillator that completes 120 vibrations per minute. If it is observed that a given crest, or maximum, travels 250 cm in 10 s, what is the wavelength?

*Answer(s) available in Appendix D.

* Solution available in CD-ROM.
1.4 Two waves, $y_1(t)$ and $y_2(t)$, have identical amplitudes and oscillate at the same frequency, but $y_2(t)$ leads $y_1(t)$ by a phase angle of $60^\circ$. If

$$y_1(t) = 4 \cos(2\pi \times 10^3 t)$$

write the expression appropriate for $y_2(t)$ and plot both functions over the time span from 0 to 2 ms.

1.5a The height of an ocean wave is described by the function

$$y(x, t) = 1.5 \sin(0.5t - 0.6x) \quad (\text{m})$$

Determine the phase velocity and wavelength, and then sketch $y(x, t)$ at $t = 2s$ over the range from $x = 0$ to $x = 2\lambda$.

1.6 A wave traveling along a string in the $+x$-direction is given by

$$y_1(x, t) = A \cos(\omega t - \beta x)$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. 1.21. When wave $y_1(x, t)$ arrives at the wall, a reflected wave $y_2(x, t)$ is generated. Hence, at any location on the string, the vertical displacement $y_s$ is the sum of the incident and reflected waves:

$$y_s(x, t) = y_1(x, t) + y_2(x, t)$$

(a) Write an expression for $y_s(x, t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x, t), y_2(x, t)$ and $y_s(x, t)$ versus $x$ over the range $-2\lambda \leq x \leq 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

1.7 Two waves on a string are given by the following functions:

$$y_1(x, t) = 3 \cos(20t - 30x) \quad (\text{cm})$$
$$y_2(x, t) = -3 \cos(20t + 30x) \quad (\text{cm})$$

where $x$ is in centimeters. The waves are said to interfere constructively when their superposition $|y_s| = |y_1 + y_2|$ is a maximum, and they interfere destructively when $|y_s|$ is a minimum.

(a) What are the directions of propagation of waves $y_1(x, t)$ and $y_2(x, t)$?

(b) At $t = (\pi/50)$ s, at what location $x$ do the two waves interfere constructively, and what is the corresponding value of $|y_s|$?

(c) At $t = (\pi/50)$ s, at what location $x$ do the two waves interfere destructively, and what is the corresponding value of $|y_s|$?

1.8 Give expressions for $y(x, t)$ for a sinusoidal wave traveling along a string in the negative $x$-direction, given that $y_{\text{max}} = 20 \text{ cm}$, $\lambda = 30 \text{ cm}$, $f = 5$ Hz, and

(a) $y(x, 0) = 0$ at $x = 0$

(b) $y(x, 0) = 0$ at $x = 7.5 \text{ cm}$
An oscillator that generates a sinusoidal wave on a string completes 20 vibrations in 30 s. The wave peak is observed to travel a distance of 2.8 m along the string in 5 s. What is the wavelength?

The vertical displacement of a string is given by the harmonic function:

\[ y(x, t) = 5 \cos(12\pi t - 20\pi x) \quad \text{(m)} \]

where \( x \) is the horizontal distance along the string in meters. Suppose a tiny particle were attached to the string at \( x = 5 \text{ cm} \). Obtain an expression for the vertical velocity of the particle as a function of time.

Given two waves characterized by the following:

\[ y_1(t) = 6 \cos 6t \]
\[ y_2(t) = 6 \sin(6t + 30^\circ) \]

does \( y_2(t) \) lead or lag \( y_1(t) \) and by what phase angle?

The voltage of an electromagnetic wave traveling on a transmission line is given by

\[ v(z, t) = 3e^{-\alpha z} \sin(2\pi \times 10^9 t - 10\pi z) \quad \text{(V)} \]

where \( z \) is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.
(b) At \( z = 2 \text{ m} \), the amplitude of the wave was measured to be 1 V. Find \( \alpha \).

A certain electromagnetic wave traveling in seawater was observed to have an amplitude of 19.025 (V/m) at a depth of 10 m, and an amplitude of 12.13 (V/m) at a depth of 100 m. What is the attenuation constant of seawater?

Evaluate each of the following complex numbers and express the result in rectangular form:

\( z_1 = 3e^{j\pi/4} \)
\( z_2 = \sqrt{3} e^{j3\pi/4} \)
\( z_3 = 2 e^{-j\pi/3} \)
\( z_4 = j^3 \)
\( z_5 = j^{-4} \)
\( z_6 = (1 - j)^3 \)
\( z_7 = (1 - j)^{1/2} \)

Complex numbers \( z_1 \) and \( z_2 \) are given by the following:

\[ z_1 = 3 - j2 \quad \text{and} \quad z_2 = -4 + j2 \]

(a) Express \( z_1 \) and \( z_2 \) in polar form.
(b) Find \( |z_1| \) by first applying Eq. (1.41) and then by applying Eq. (1.43).
(c) Determine the product \( z_1z_2 \) in polar form.
(d) Determine the ratio \( z_1/z_2 \) in polar form.
(e) Determine \( z_1^2 \) in polar form.

If \( z = -2 + j3 \), determine the following quantities in polar form:

(a) \( 1/z \)
(b) \( z^3 \)
(c) \( |z|^2 \)
(d) \( 2m[z] \)
(e) \( 2m[z^*] \)

Find complex numbers \( t = z_1 + z_2 \) and \( s = z_1 - z_2 \), both in polar form, for each of the following pairs:

(a) \( z_1 = 2 + j3 \) and \( z_2 = 1 - j2 \)
(b) \( z_1 = 2 \) and \( z_2 = -j2 \)
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(c) \( z_1 = 3\angle 30^\circ \) and \( z_2 = 3\angle -30^\circ \)

^ (d) \( z_1 = 3\angle 50^\circ \) and \( z_2 = 3\angle 150^\circ \)

1.18 Complex numbers \( z_1 \) and \( z_2 \) are given by the following:

\[
\begin{align*}
z_1 &= 5\angle -60^\circ \\
z_2 &= 2\angle 60^\circ
\end{align*}
\]

(a) Determine the product \( z_1 z_2 \) in polar form.
(b) Determine the product \( z_1 z_2^* \) in polar form.
(c) Determine the ratio \( z_1/z_2 \) in polar form.
(d) Determine the ratio \( z_1^*/z_2^* \) in polar form.
(e) Determine \( \sqrt{z_1} \) in polar form.

1.19 If \( z = 3 - j4 \), find the value of \( \ln(z) \).

1.20 If \( z = 3 - j4 \), find the value of \( e^z \).

Section 1.6: Phasors

1.21 A voltage source given by

\[ v(t) = 10 \cos(2\pi \times 10^3 t - 30^\circ) \quad \text{(V)} \]

is connected to a series RC load as shown in Fig. 1.19. If \( R = 1 \, \text{M}\Omega \) and \( C = 100 \, \text{pF} \), obtain an expression for \( v_c(t) \), the voltage across the capacitor.

 vra Find the phasors of the following time functions:

(a) \( v(t) = 3 \cos(\omega t - \pi/4) \quad \text{(V)} \)

(b) \( v(t) = 12 \sin(\omega t + \pi/4) \quad \text{(V)} \)

(c) \( i(x, t) = 4e^{-3t} \sin(\omega t - \pi/6) \quad \text{(A)} \)

(d) \( i(t) = -2 \cos(\omega t + 3\pi/4) \quad \text{(A)} \)

(e) \( i(t) = 2 \sin(\omega t + \pi/3) + 3 \cos(\omega t - \pi/6) \quad \text{(A)} \)

1.23 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) \( V = -3e^{j\pi/3} \quad \text{(V)} \)

(b) \( V = j6e^{j\pi/4} \quad \text{(V)} \)

(c) \( I = (3 + j4) \quad \text{(A)} \)

^ (d) \( I = -3 + j2 \quad \text{(A)} \)

(e) \( I = j \quad \text{(A)} \)

(f) \( I = 2e^{j3\pi/4} \quad \text{(A)} \)

1.24 A series RLC circuit is connected to a generator with a voltage \( v(t) = V_0 \cos(\omega t + \pi/3) \quad \text{(V)} \).

(a) Write the voltage loop equation in terms of the current \( i(t) \), \( R \), \( L \), \( C \), and \( v(t) \).

(b) Obtain the corresponding phasor-domain equation.

(c) Solve the equation to obtain an expression for the phasor current \( I \).