Section 2-10: Impedance Matching

Problem 2.45 A 50-Ω lossless line 0.6λ long is terminated in a load with $Z_L = (50 + j25) \, \Omega$. At 0.3λ from the load, a resistor with resistance $R = 30 \, \Omega$ is connected as shown in Fig. 2-43 (P2.45(a)). Use the Smith chart to find $Z_{in}$.

Figure P2.45: (a) Circuit for Problem 2.45.
Solution: Refer to Fig. P2.45(b). Since the 30-Ω resistor is in parallel with the input impedance at that point, it is advantageous to convert all quantities to admittances.

\[ z_L = \frac{Z_L}{z_0} = \frac{(50+j25)\ \Omega}{50\ \Omega} = 1 + j0.5 \]

and is located at point Z-LOAD. The corresponding normalized load admittance is at point Y-LOAD, which is at 0.394λ on the WTG scale. The input admittance of the load only at the shunt conductor is at 0.394λ + 0.300λ - 0.500λ = 0.194λ and is denoted by point A. It has a value of

\[ \gamma_{inA} = 1.37 + j0.45. \]
The shunt conductance has a normalized conductance

\[ g = \frac{50 \, \Omega}{30 \, \Omega} = 1.67. \]

The normalized admittance of the shunt conductance in parallel with the input admittance of the load is the sum of their admittances:

\[ y_{inB} = g + y_{inA} = 1.67 + 1.37 + j0.45 = 3.04 + j0.45 \]

and is located at point B. On the WTG scale, point B is at 0.242λ. The input admittance of the entire circuit is at 0.242λ + 0.300λ - 0.500λ = 0.042λ and is denoted by point Y-IN. The corresponding normalized input impedance is at Z-IN and has a value of

\[ z_{in} = 1.9 - j1.4. \]

Thus,

\[ Z_{in} = z_{in} Z_0 = (1.9 - j1.4) \times 50 \, \Omega = (95 - j70) \, \Omega. \]

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**Problem 2.47** Repeat Problem 2.46 for a load with \( Z_L = (100 + j50) \, \Omega. \)

**Solution:** Refer to Fig. P2.47(a) and Fig. P2.47(b), which represent two different solutions.

\[ z_L = \frac{Z_L}{Z_0} = \frac{100 + j50 \, \Omega}{50 \, \Omega} = 2 + j1 \]

and is located at point Z-LOAD in both figures. Since it is advantageous to work in admittance coordinates, \( y_L \) is plotted as point Y-LOAD in both figures. Y-LOAD is at 0.463λ on the WTG scale.
Figure P2.47: (a) First solution to Problem 2.47.

For the first solution in Fig. P2.47(a), point $Y$-LOAD-IN-1 represents the point at which $g = 1$ on the SWR circle of the load. $Y$-LOAD-IN-1 is at 0.162$\lambda$ on the WTG scale, so the stub should be located at $0.162\lambda - 0.463\lambda + 0.500\lambda = 0.199\lambda$ from the load (or some multiple of a half wavelength further). At $Y$-LOAD-IN-1, $b = 1$, so a stub with an input admittance of $y_{stub} = 0 - j1$ is required. This point is $Y$-STUB-IN-1 and is at 0.375$\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y$-SHT, located at 0.250$\lambda$. Therefore, the short stub must be $0.375\lambda - 0.250\lambda = 0.125\lambda$ long (or some multiple of a half wavelength longer).

For the second solution in Fig. P2.47(b), point $Y$-LOAD-IN-2 represents the point at which $g = 1$ on the SWR circle of the load. $Y$-LOAD-IN-2 is at 0.338$\lambda$ on the
WTG scale, so the stub should be located at $0.338\lambda - 0.463\lambda + 0.500\lambda = 0.375\lambda$ from the load (or some multiple of a half wavelength further). At $Y$-LOAD-IN-2, $b = -1$, so a stub with an input admittance of $y_{stub} = 0 + j1$ is required. This point is $Y$-STUB-IN-2 and is at $0.125\lambda$ on the WTG scale. The short circuit admittance is denoted by point $Y$-SHT, located at $0.250\lambda$. Therefore, the short stub must be $0.125\lambda - 0.250\lambda + 0.500\lambda = 0.375\lambda$ long (or some multiple of a half wavelength longer).

**Problem 2.49** Repeat Problem 2.48 for the case where all three transmission lines are $\lambda/4$ in length.

**Solution:** Since the transmission lines are in parallel, it is advantageous to express loads in terms of admittances. In the upper branch, which is a quarter wave line,

$$Y_{in} = \frac{y_0^2}{y_1} = \frac{Z_1}{Z_0^2},$$
and similarly for the lower branch,

\[ Y_{2\text{ in}} = \frac{Y_2}{Z_0} = \frac{Z_2}{Z_0^2} \]

Thus, the total load at the junction is

\[ Y_{\text{JCT}} = Y_{1\text{ in}} + Y_{2\text{ in}} = \frac{Z_1 + Z_2}{Z_0^2} \]

Therefore, since the common transmission line is also quarter-wave,

\[ Z_{\text{cm}} = \frac{Z_0^2}{Z_{\text{JCT}}} = \frac{Z_0^2}{Z_{\text{JCT}}} = Z_1 + Z_2 = (50 + j50) \Omega + (50 - j50) \Omega = 100 \Omega. \]

Section 2.11: Transients on Transmission Lines

**Problem 2.50** Generate a bounce diagram for the voltage \( V(z,t) \) for a 1-m long lossless line characterized by \( Z_0 = 50 \Omega \) and \( u_p = 2c/3 \) (where \( c \) is the velocity of light) if the line is fed by a step voltage applied at \( t = 0 \) by a generator circuit with \( V_g = 60 \text{ V} \) and \( R_g = 100 \Omega \). The line is terminated in a load \( Z_L = 25 \Omega \). Use the bounce diagram to plot \( V(t) \) at a point midway along the length of the line from \( t = 0 \) to \( t = 25 \text{ ns} \).

**Solution:**

\[ \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}, \]

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = -\frac{1}{3}. \]

From Eq. (2.124b),

\[ V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100 + 50} = 20 \text{ V}. \]

Also,

\[ T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \text{ ns}. \]

The bounce diagram is shown in Fig. P2.50(a) and the plot of \( V(t) \) in Fig. P2.50(b).
Figure P2.50: (a) Bounce diagram for Problem 2.50.

Figure P2.50: (b) Time response of voltage.
Problem 2.51 Repeat Problem 2.50 for the current \( I \) on the line.

**Solution:**

\[
\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3},
\]

\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}.
\]

From Eq. (2.124a),

\[
I_1^+ = \frac{V_g}{R_g + Z_0} = \frac{60}{100 + 50} = 0.4 \text{ A}.
\]

The bounce diagram is shown in Fig. P2.51(a) and \( I(t) \) in Fig. P2.51(b).

**Figure P2.51:** (a) Bounce diagram for Problem 2.51.
Problem 2.53: In response to a step voltage, the voltage waveform shown in Fig. 2.46 (P2.53) was observed at the sending end of a shorted line with $Z_0 = 50$ Ω and $\varepsilon_r = 4$. Determine $V_g$, $R_g$, and the line length.

![V(t)](image)

Figure P2.53: Observed voltage at sending end.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{4}} = 1.5 \times 10^8 \text{ m/s},$$

$$7 \mu s = 7 \times 10^{-6} \text{ s} = \frac{2l}{u_p} = \frac{2l}{1.5 \times 10^8}.$$ 

Hence, $l = 525$ m.
From the voltage waveform, $V_{1+} = 12 \text{ V}$. At $t = 7\mu s$, the voltage at the sending end is

$$V(z = 0, t = 7\mu s) = V_{1+}^+ + \Gamma_L V_{1+}^+ + \Gamma_g \Gamma_L V_{1+}^+ = -\Gamma_g V_{1+}^+ \quad \text{(because } \Gamma_L = -1).$$

Hence, $3 \text{ V} = -\Gamma_g \times 12 \text{ V}$, or $\Gamma_g = -0.25$. From Eq. (2.128),

$$R_g = Z_0 \left( \frac{1 + \Gamma_g}{1 - \Gamma_g} \right) = 50 \left( \frac{1 - 0.25}{1 + 0.25} \right) = 30 \Omega.$$ 

Also,

$$V_{1+}^+ = \frac{V_g Z_0}{R_g + Z_0}, \quad \text{or} \quad 12 = \frac{V_g \times 50}{30 + 50},$$

which gives $V_g = 19.2 \text{ V}$.