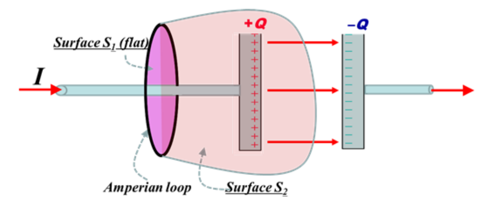
Lecture 19

Physics II

Chapter 34



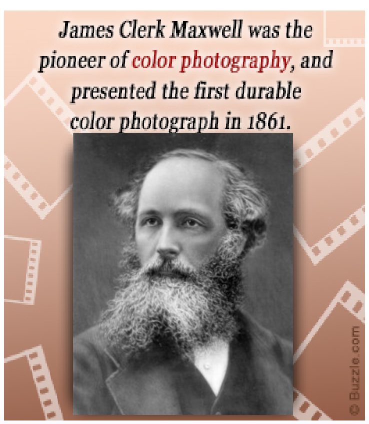
Inductors
Maxwell's equations

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

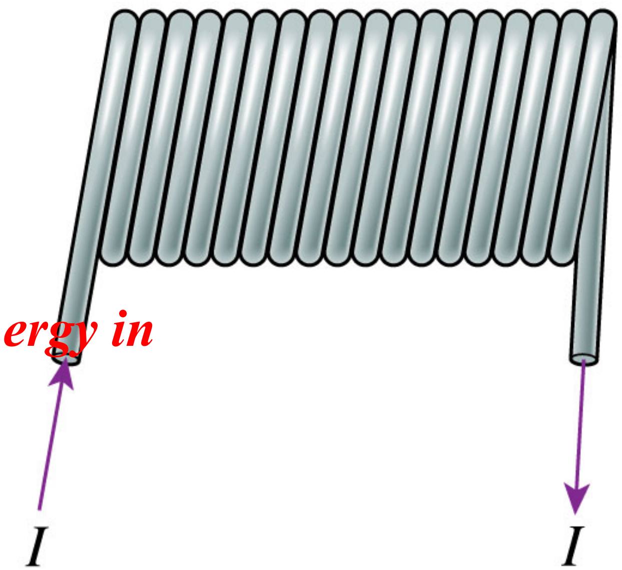
$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$



Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Inductors



Inductors (solenoids) store potential energy in a form of a magnetic field.

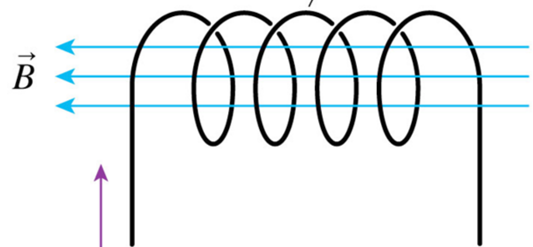
Inductance (definition)

Consider a solenoid of N turns with current I.

The solenoid's magnetic field passes through the coils, establishing a flux.

The total magnetic flux is ~ *I*

$$\Phi_m \sim I$$

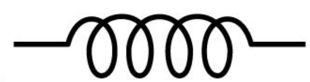


The coefficient of proportionality is called inductance, L

$$\boldsymbol{L = \frac{\Phi_m}{I}}$$

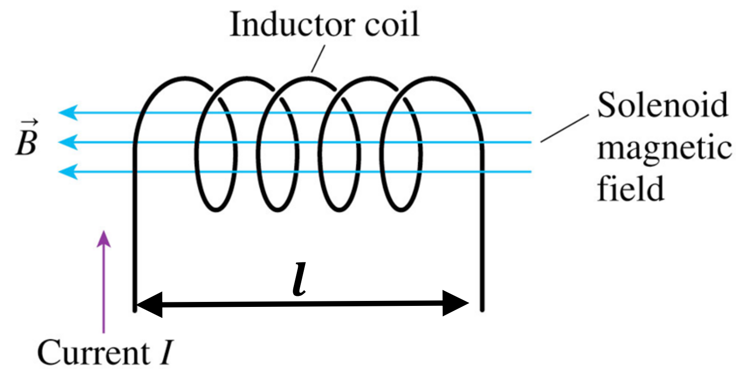
The SI unit of inductance is the henry, defined as:

1 henry = 1 H = 1 Wb/A = 1 T m^2/A

The circuit symbol for an ideal inductor is 

Let's find solenoid inductance

Consider a solenoid of ***N*** turns with current ***I***.



$$\Phi_m = N\Phi_1 = N(\vec{B}\cdot\vec{A}) = \|\vec{B} \parallel \vec{A}\| = NBA$$

Recall (Lecture 14) $B_{solenoid} = \mu_0 nI = \mu_0 \frac{N}{l} I$

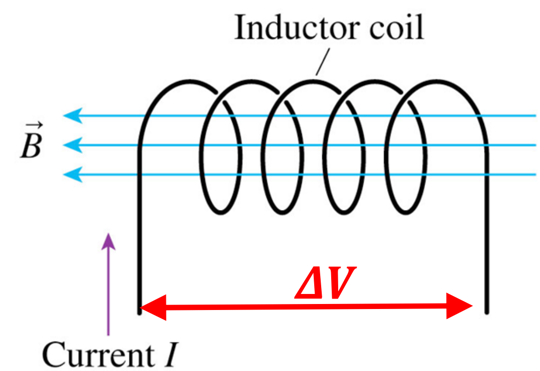
$$\Phi_m = N(\mu_0 \frac{N}{l} I)A = \frac{\mu_0 N^2 A}{l} I$$

$$L = \frac{\Phi_m}{I} \Rightarrow L = \frac{\mu_0 N^2 A}{l}$$

You see, solenoid inductance depends only on its geometry

Potential Difference across an Inductor

$$\Delta V = \mathcal{E} = -\frac{d\Phi_m}{dt} = \left\| \begin{matrix} L = \dfrac{\Phi_m}{I} \\ \Phi_m = LI \end{matrix} \right\| = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

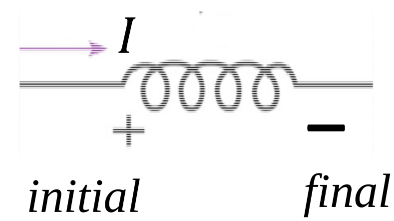


$$\Delta V = \boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

Potential difference across an inductor

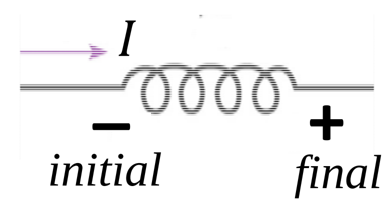
Note ! The magnitude of *I* has no effect on ΔV, only the rate of change of *I* counts.

<u>If current increases,</u> $\frac{dI}{dt} > 0$ ⇨ $\Delta V < 0$ ⇨ $V_f < V_i$ ⇨



The induced ΔV *decreases if* the current is increasing

<u>If current decreases,</u> $\frac{dI}{dt} < 0$ ⇨ $\Delta V > 0$ ⇨ $V_f > V_i$ ⇨

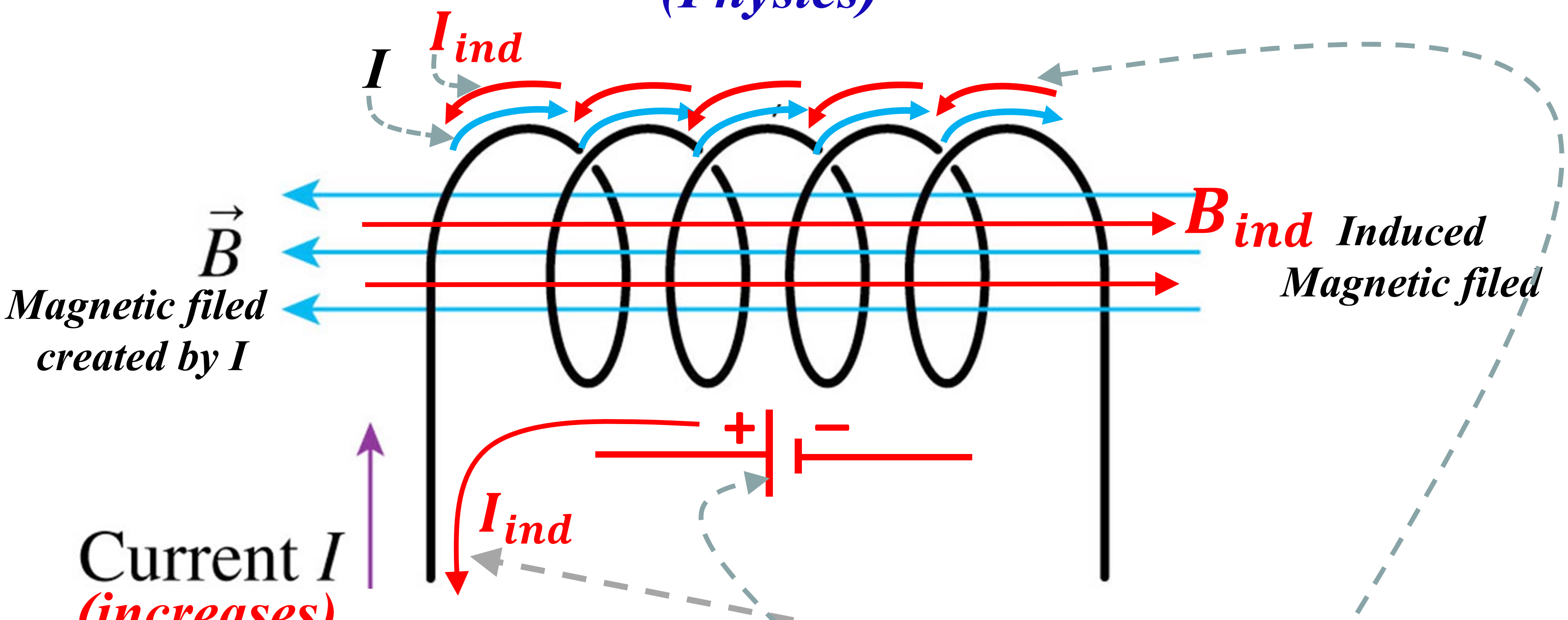


The induced ΔV *increases if* the current is decreasing

If current is constant, *I*=const ⇨ $\Delta V = 0$

$\varepsilon = \Delta V$ across a solenoid when the current increase

(Physics)

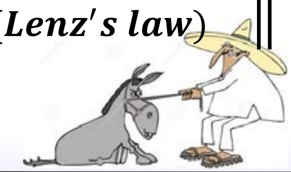


$I \nearrow \Rightarrow B \nearrow \Rightarrow \Phi_B \nearrow \Rightarrow B_{ind} \uparrow\downarrow B_{ext} \Rightarrow$ I_{ind} is in the opposite direction to a direction of I

B_{ind} opposes B_{ext} inside the loop (Lenz's law)

This is an "imaginary battery" which can create this induced current

Thus $V_i > V_f \Rightarrow \Delta V < 0$



ConcepTest ΔV Inductor

- Which current is changing more rapidly?

A. Current I_1

B. Current I_2

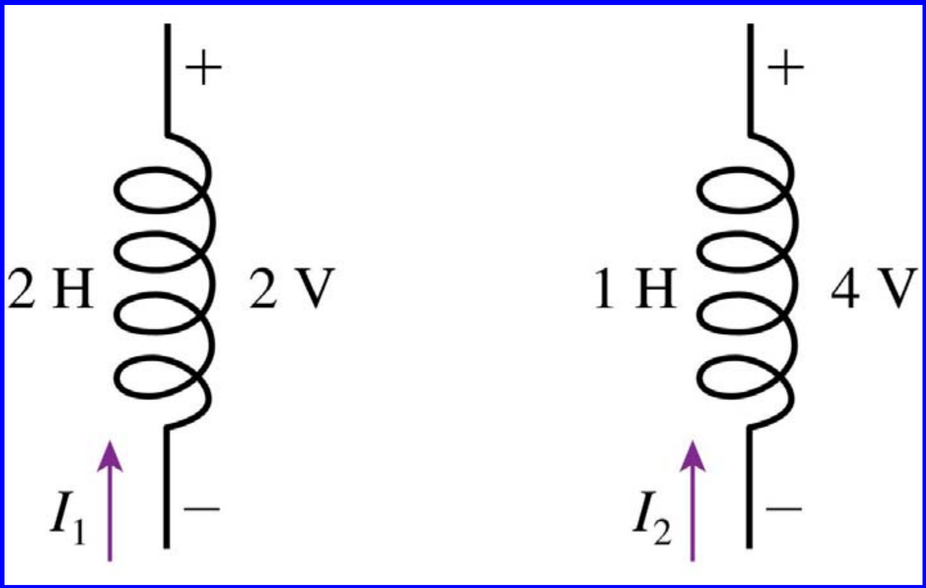
C. They are changing at the same rate

D. Not enough information to tell

$$\Delta V = -L\frac{dI}{dt}$$

$$\left(\frac{dI}{dt}\right)_1 = -\frac{\Delta V_1}{L_1} = -\frac{2V}{2H}$$

$$\left(\frac{dI}{dt}\right)_2 = -\frac{\Delta V_2}{L_2} = -\frac{4V}{1H}$$



Maxwell's equations

Let’s revisit Ampere’s Law *a straight wire with current I*

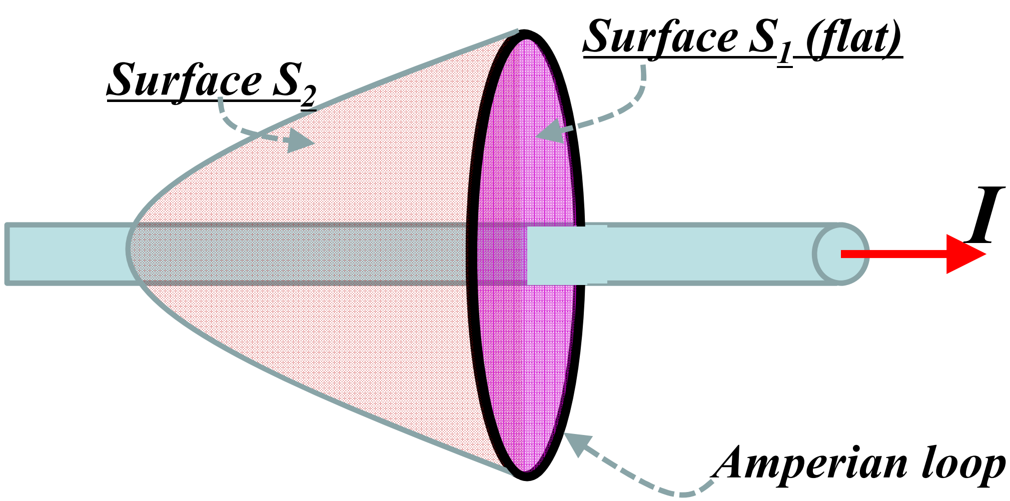
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

The line integral of the magnetic field around the curve is given by Ampère’s law:

Any closed loop (Amperian loop)

Current which goes through ANY surface enclosed by an amperian loop

Let’s consider a straight wire with current I:

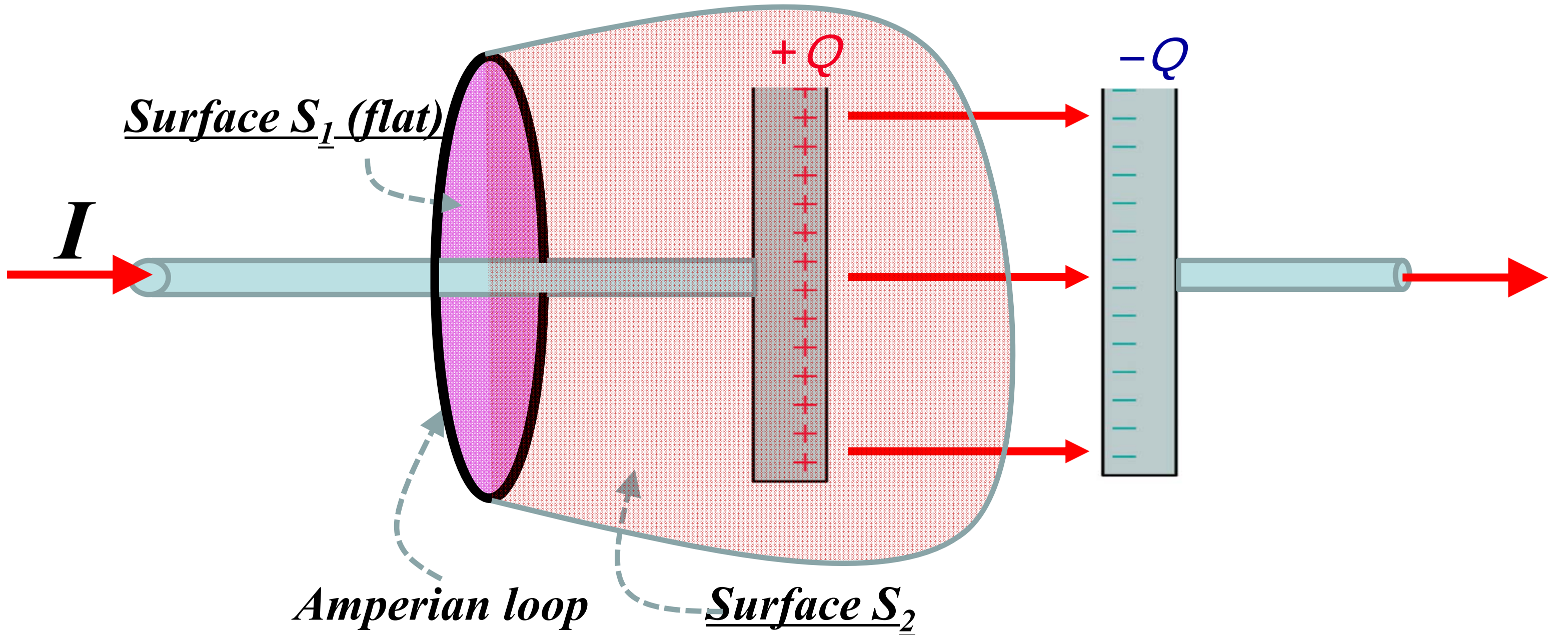


In this example both surfaces (S1 and S2) give us the same enclosed current, as it should be since Ampere’s law must work for any possible situation.

Great! Ampere’s Law works!

Let's revisit Ampere's Law for <u>*current I and a capacitor*</u>

Let's consider a wire with current I and a capacitor:



Let's apply Ampere's law for both surfaces (S1 and S2):

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow I_{in} = I \Rightarrow \oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

Amperian loop ***<u>Surface S_1 (flat)</u>***

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow I_{in} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{s} = 0$$

Amperian loop ***<u>Surface S_2 (curved)</u>***

The LH sides are the same, but the RH sides are different!!??
Something is missing in Ampere's law.

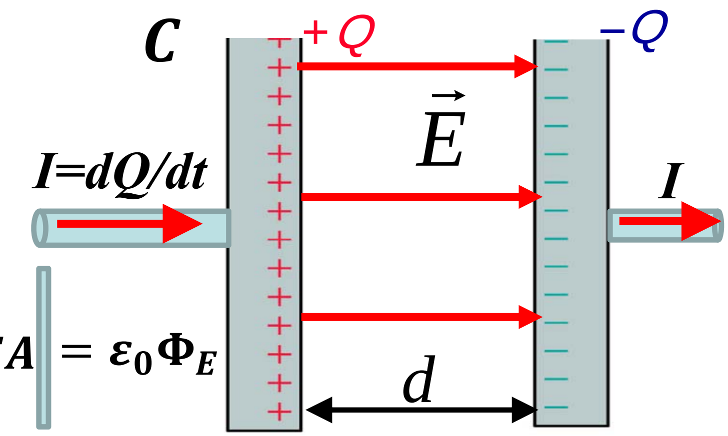
So!

<u>Ampere's Law needs to be adjusted!</u>

Displacement current/ *Ampere-Maxwell Law*

Let's get somehow an additional term with units of current and use it to generalize Ampere's Law

$$C \stackrel{\text{def}}{=} \frac{Q}{\Delta V_C}$$



$$Q = C\Delta V = \left| \begin{array}{l} C = \varepsilon_0 \frac{A}{d} \\ \Delta V = Ed \end{array} \right| = (\varepsilon_0 \frac{A}{d})Ed = \varepsilon_0 EA = \left| \Phi_E = EA \right| = \varepsilon_0 \Phi_E$$

But we need something which has units of current. So let's take a derivative:

$$I = \frac{dQ}{dt} = \frac{d(\varepsilon_0 \Phi_E)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

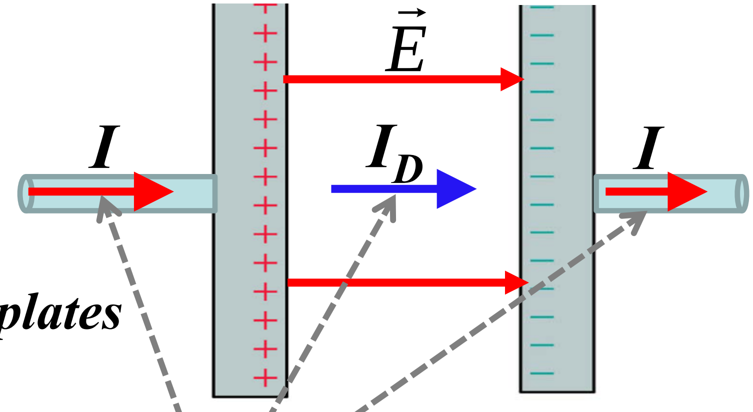
Maxwell interpreted as being equivalent current and called it

a Displacement current $$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{in} + I_D) = \mu_0 (I_{in} + \varepsilon_0 \frac{d\Phi_E}{dt})$$ ***Ampere-Maxwell Law***

End. Spring 2016

Displacement current



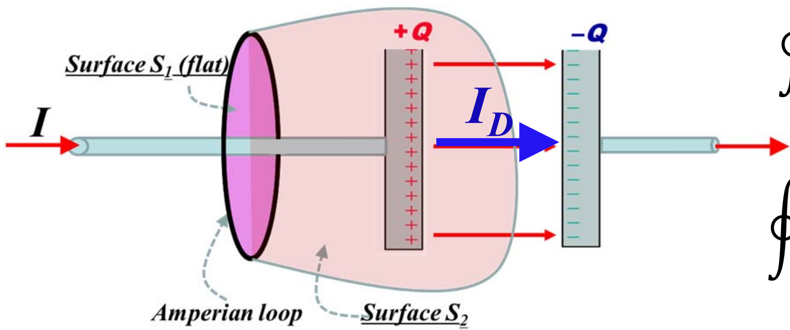
Displacement current $I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$

1) The displacement current is only between the plates since $\Phi_E = EA$ is zero outside

2) The way I_D was introduced allows us to say that numerically I_D=I (real current in the wire charging the capacitor). In some sense “current” is conserved all the way through the capacitor

3) I_D is not a flow of charge. It is equivalent to a real current in that it creates the same magnetic field $\oint \vec{B}\cdot d\vec{s} = \mu_0(I_{in}+I_D)$

Let’s apply Ampere-Maxwell Law for the “capacitor system”



$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I_{in}+I_D) = \left\| \begin{matrix} I_{in} = I \\ I_D = 0 \end{matrix} \right\| = \mu_0 I$$

Amperian Surface S_1

$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I_{in}+I_D) = \left\| \begin{matrix} I_{in} = 0 \\ I_D = I \end{matrix} \right\| = \mu_0 I$$

Amperian Surface S_2

Now it works. Each surface gives us the same answer as it should be.

Induced Magnetic Field

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(I_{in} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Thus, the magnetic field B can be generated by:

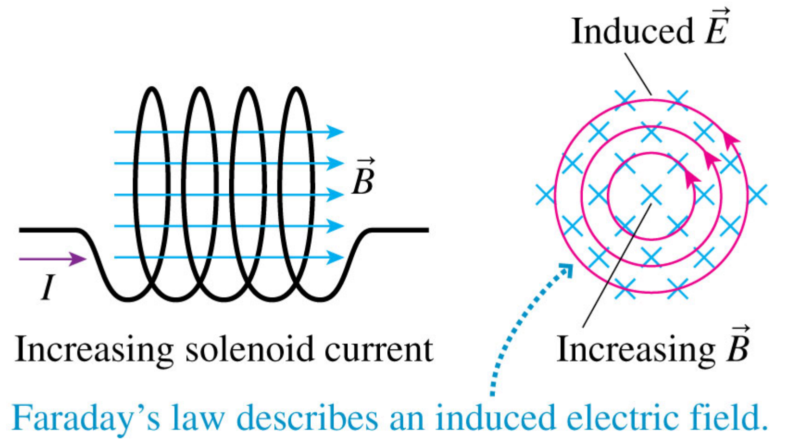
1) ***An ordinary electric current, I_{in}***
2) ***Changing electric flux (particularly, changing electric field)***

Another amazing thing!!!

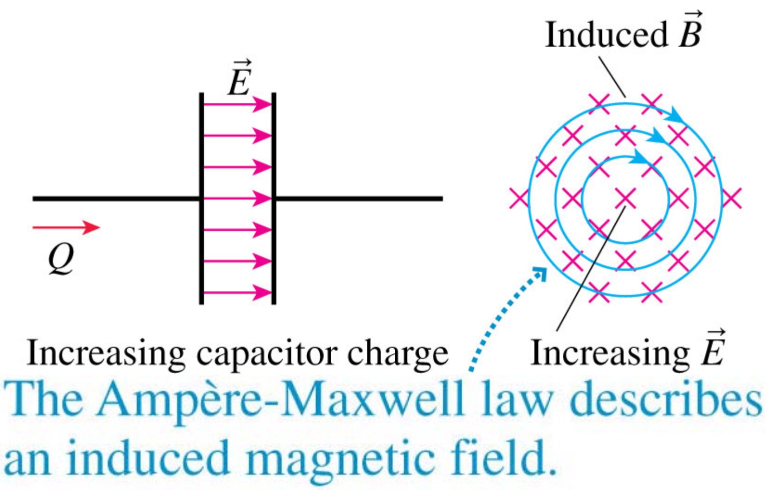
Changing electric field inside a capacitor produces a magnetic field

Induced Fields

- ***An increasing solenoid current causes an increasing magnetic field, which induces a circular electric field.***



- ***An increasing capacitor charge causes an increasing electric field, which induces a circular magnetic field.***



What you should read

Chapter 34 (Knight)

Sections

- ***34.1 (skip)***
- ***34.2***
- ***34.3***
- ***34.4***

Thank you

See you on Tuesday