Chapter 8

Motion in a Plane
Uniform Circular Motion

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI
IN THIS CHAPTER, you will learn to solve problems about motion in two dimensions.

Today we are going to discuss:

Chapter 8:

- Uniform Circular Motion: Section 8.2
- Circular Orbits: Section 8.3
- Reasoning about Circular Motion: Section 8.4
Let’s recall circular motion

An object is undergoing circular motion

**Instantaneous velocity** is tangent to the path

Velocity is a vector and has magnitude/direction

- **tangential acceleration** $a_t$ changes magnitude of the velocity, which is always tangent to the circle.

- **centripetal acceleration** $a_r$ changes direction of the velocity $a_r = \frac{v_t^2}{r}$, where $v_t$ is the tangential speed.

\[
\omega = \frac{v_t}{r}
\]

(toward center of circle)

$a_r = \omega^2 r$

The direction of the centripetal acceleration is toward the center of the circle.

The velocity is always tangent to the circle, so the radial component $v_r$ is always zero.

The tangential acceleration causes the particle to change speed.

The radial or centripetal acceleration causes the particle to change direction.
Uniform Circular Motion

Let’s simplify “our life”

- In uniform circular motion the speed is constant \( \omega = \text{const} \) or \( v_t = \text{const} \)

Then, **the tangential acceleration** \( a_t = 0 \)

Then, **the centripetal acceleration is not zero** because direction of the velocity changes

- The magnitude of the centripetal acceleration is constant for uniform circular motion:

\[
a_r = \frac{v_t^2}{r} = \text{const}
\]

The velocity is tangent to the circle.
The best coordinate system
for a Uniform Circular Motion

- When describing circular motion, it is convenient to define a moving $rtz$-coordinate system.

- The origin moves along with a certain particle moving in a circular path.

  - The $r$-axis (radial) points *from* the particle *toward* the center of the circle.

  - The $t$-axis (tangential) is tangent to the circle, pointing in the ccw direction.

  - The $z$-axis is perpendicular to the plane of motion.

Acceleration has only a radial component.
Circle seen edge-on
$t$-axis is into the page  

$\omega$
$\vec{V}_{\text{tan}}$
$\vec{a}_r$
If there is an acceleration, there must be a force

Uniform Circular Motion

The figure shows a particle in uniform circular motion.

If there is an acceleration, there must be a force (N. 2nd law)

$$(F_{net})_z = \sum F_z = m\alpha^0_z = 0$$

$$(F_{net})_t = \sum F_t = m\alpha^0_t = 0$$

$$(F_{net})_r = \sum F_r = m\alpha_r = \frac{mv^2}{r} = m\omega^2 r$$

The net force must point in the radial direction, toward the center of the circle.

For an object to be in uniform circular motion, there must be a net force acting on it radially inwards.

This centripetal force is not a new force. This can be any one of the forces we have already encountered: tension, gravity, normal force, friction, …
A Ping-Pong ball is shot into a circular tube that is lying flat (horizontal) on a tabletop. When the Ping-Pong ball leaves the track, which path will it follow?

Once the ball leaves the tube, there is no longer a force to keep it going in a circle. Therefore, it simply continues in a straight line, as Newton’s First Law requires!

Follow-up: What physical force provides the centripetal acceleration?
Examples. Wall of death

Bike going in a circle: the wall exerts an inward force (normal force) on a bike to make it move in a circle.

\[ \sum F = m \ddot{a}_r \]

\[ N = \frac{mv^2}{R} \]

\[ v \] – velocity of the motorbike

R - radius of the circle

Normal force provides the centripetal acceleration
A hammer going in a circle: the cord exerts an inward force (tension) on a hammer to make it move in a circle.

**Tension provides the centripetal acceleration**

$T = \frac{mv^2}{R}$

$
\sum F = ma_r
$
Examples. Car on a circular road.

Car going in a circle: the road exerts an inward force (friction) on a car to make it move in a circle. Friction provides the centripetal acceleration.

\[ a_r = \frac{v^2}{R} \]

Friction force:
\[ f_s = \mu_s N \]
\[ f_s = \frac{mv^2}{R} \]

Out of these two equations you can get anything you need.

http://phys23p.sl.psu.edu/phys_anim/mech/car_Fc_new.avi
The friction force between tires and road provides the centripetal force that keeps the car moving in a circle. If this force is too small, the car continues in a straight line!

A) car’s engine is not strong enough to keep the car from being pushed out

B) friction between tires and road is not strong enough to keep car in a circle

C) car is too heavy to make the turn

D) a deer caused you to skid

E) none of the above

Follow-up: What could be done to the road or car to prevent skidding?
Examples. Banked curve

- But sometimes, friction force is not enough to keep a car on a circular road.
- Banking the curve can help to keep cars from skidding.
Banked Curves (solution)

\[ \sum F_z = N\cos\theta - mg = 0 \]

\[ \sum F_r = N\sin\theta = ma_r \]

\[ a_r = \frac{v^2}{R} \]

\[ N\sin\theta = \frac{mv^2}{R} \]
\[ N\cos\theta = mg \]

Take a ratio

\[ \tan\theta = \frac{v^2}{gR} \]

Independent of object mass !!!

\textit{r component of normal force provides the centripetal acceleration}

http://phys23p.sl.psu.edu/phys_anim/mech/car_banked_new.avi
Example: Loop the Loop

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_fail.avi
Loop the Loop

To make the loop-the-loop at a constant speed, what minimum speed does the motorcycle need?

\[ 2 \sum F_r = ma_r \]

\[ a_r = \frac{v^2}{R} \]

\[ N + mg = m \frac{v^2}{R} \]

\[ v = \sqrt{\frac{R}{m} (N + mg)} \]

When \( N=0 \) (feels like no weight), then speed is minimum

\[ v = \sqrt{gR} \]

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi
Loop the Loop

Apparent weight at the bottom, point 1?

1 \[ \sum F_r = ma_r \]

\[ a_r = \frac{v^2}{R} \]

\[ N - mg = m \frac{v^2}{R} \]

\[ N = m \frac{v^2}{R} + mg \]

Thus, \( N > mg \). You would feel heavier (similar to an elevator)

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi
You’re on a Ferris wheel moving in a vertical circle. When the Ferris wheel is at rest, the normal force $N$ exerted by your seat is equal to your weight $mg$. How does $N$ change at the top of the Ferris wheel when you are in motion?

You are in circular motion, so there has to be a centripetal force pointing inward. At the top, the only two forces are $mg$ (down) and $N$ (up), so $N$ must be smaller than $mg$.

Follow-up: Where is $N$ larger than $mg$?
**Example:** Conical pendulum

\[ \sum F_z = T_z - mg = 0 \quad (1) \]

\[ \sum F_r = T_r = ma_r \]

\[ T_r = T \cdot \sin \theta = \frac{mv^2}{R} \quad (2) \]

The \( r \) component of tension provides the centripetal acceleration.
Thank you
See you on Wednesday
A skier goes over a small round hill with radius $R$. Because she is in circular motion, there has to be a centripetal force. At the top of the hill, what is $F_c$ of the skier equal to?

ConcepTest 4 Going in Circles II

A) $F_c = N + mg$
B) $F_c = mg - N$
C) $F_c = T + N - mg$
D) $F_c = N$
E) $F_c = mg$

Follow-up: What happens when the skier goes into a small dip?

$F_c$ points toward the center of the circle (i.e., downward in this case). The weight vector points down and the normal force (exerted by the hill) points up. The magnitude of the net force, therefore, is $F_c = mg - N$. 

Fv points toward the center of the circle (i.e., downward in this case). The weight vector points down and the normal force (exerted by the hill) points up. The magnitude of the net force, therefore, is $F_c = mg - N$. 

Follow-up: What happens when the skier goes into a small dip?