Lecture 11

Physics I

Chapter 8

Motion in a Plane
Uniform Circular Motion

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

IN THIS CHAPTER, you will learn to solve problems about motion in two dimensions.

Today we are going to discuss:

Chapter 8:

- ***Uniform Circular Motion:*** *Section 8.2*
- ***Circular Orbits:*** *Section 8.3*
- ***Reasoning about Circular Motion:*** *Section 8.4*

Let's recall circular motion

An object is undergoing circular motion

Instantaneous velocity is tangent to the path

Velocity is a vector and has magnitude/direction)

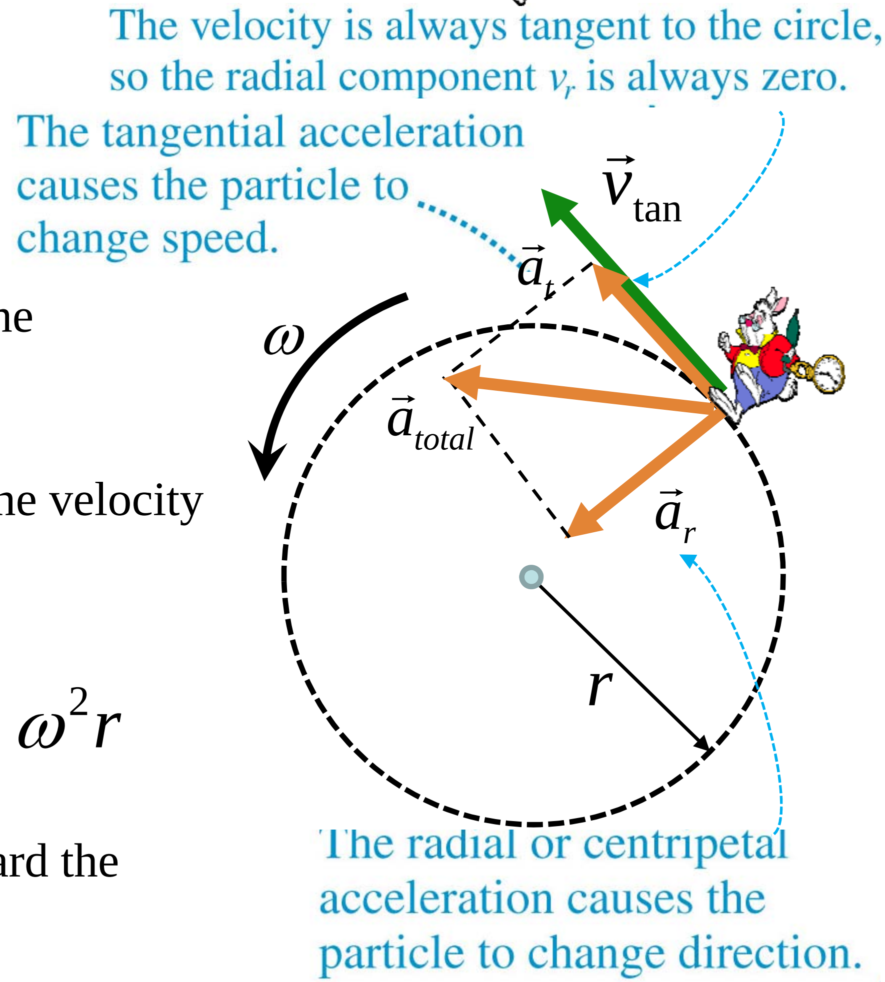
- ***tangential acceleration*** a_t changes magnitude of the velocity, which is always tangent to the circle.

- ***centripetal acceleration*** a_r changes direction of the velocity

$a_r = v_t^2/r$, where v_t is the tangential speed.

$$\boxed{a_r = \frac{v_t^{\,2}}{r} \quad \textit{(toward center of circle)}} \qquad a_r = \omega^2 r$$

The direction of the centripetal acceleration is toward the center of the circle.



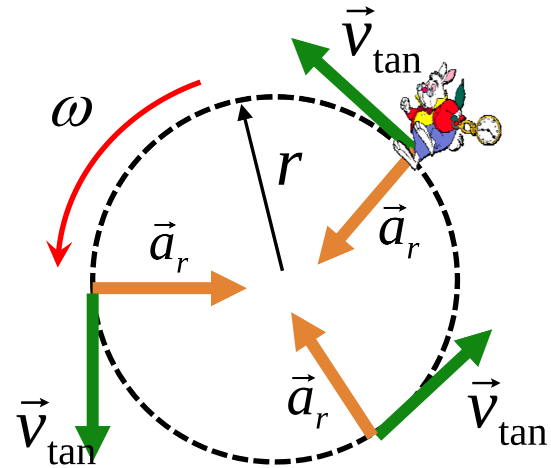
Uniform Circular Motion

Let's simplify "our life"

- In uniform circular motion
  the speed is constant $\omega=const$ or $\boxed{v_t=const}$

  Then, the tangential acceleration $a_t=0$

  Then, the centripetal acceleration is not zero
  because direction of the velocity changes



The velocity is tangent to the circle.

- The magnitude of the centripetal acceleration is constant for uniform circular motion:

$$a_r = \frac{v_t^2}{r} = const$$

The best coordinate system

for a Uniform Circular Motion

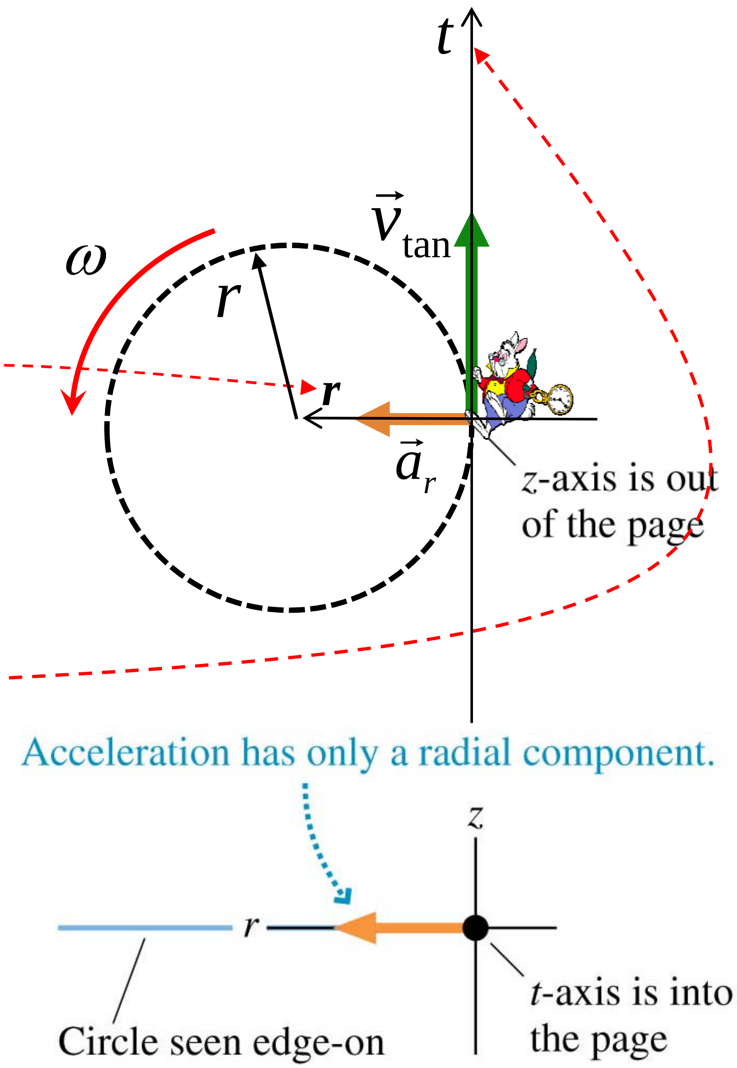
- When describing circular motion, it is convenient to define a moving *rtz*-coordinate system.

- The origin moves along with a certain particle moving in a circular path.

- The *r*-axis (radial) points *from* the particle *toward* the center of the circle.

- The *t*-axis (tangential) is tangent to the circle, pointing in the ccw direction.

- The *z*-axis is perpendicular to the plane of motion.



If there is an acceleration, there must be a force

Uniform Circular Motion

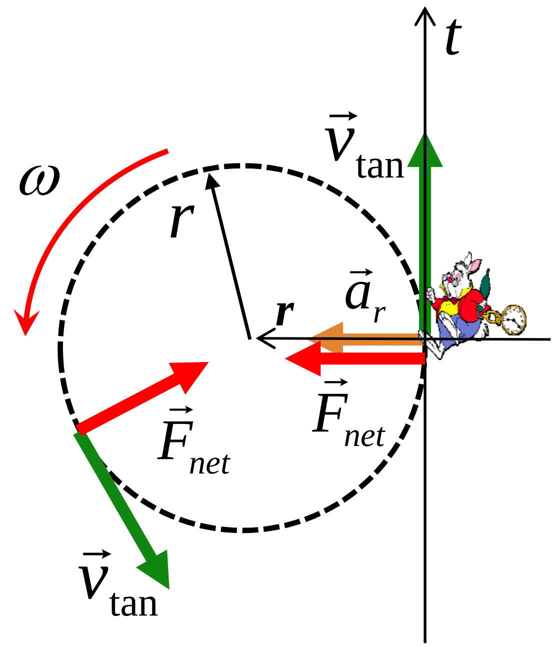
The figure shows a particle in uniform circular motion.

If there is an acceleration, there must be a force (N. 2nd law)

$$(F_{\text{net}})_z = \sum F_z = m\cancel{a_z}^{0} = 0$$

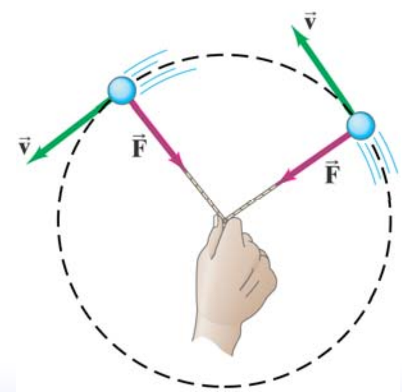
$$(F_{\text{net}})_t = \sum F_t = m\cancel{a_t}^{0} = 0$$

$$(F_{\text{net}})_r = \boxed{\sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r}$$



The net force must point in the radial direction, toward the center of the circle.

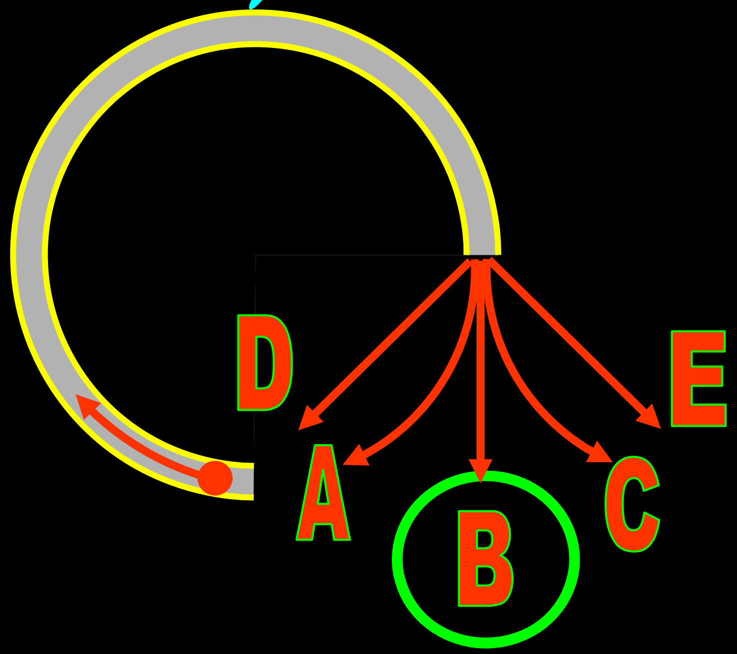
For an object to be in uniform circular motion, there must be a net force acting on it radially inwards.



This centripetal force is not a new force. This can be any one of the forces we have already encountered: tension, gravity, normal force, friction, …

ConcepTest Missing Link

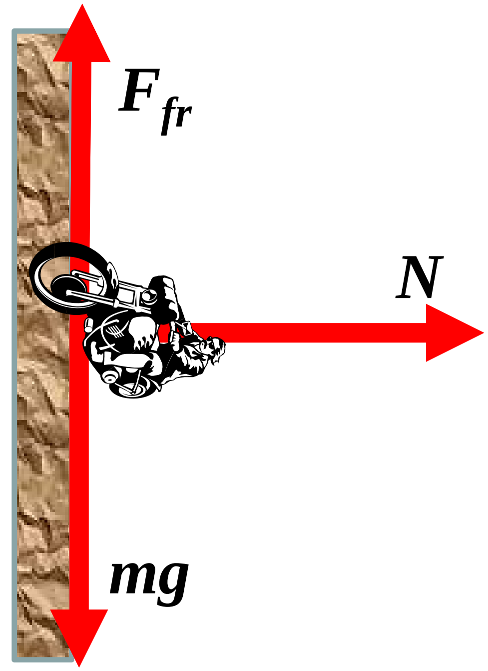
A Ping-Pong ball is shot into a circular tube that is lying flat (horizontal) on a tabletop. When the Ping-Pong ball leaves the track, which path will it follow?



Once the ball leaves the tube, there is no longer a force to keep it going in a circle. Therefore, it simply continues in a straight line, as Newton's First Law requires!

Follow-up: What physical force provides the centripetal acceleration?

Examples. Wall of death



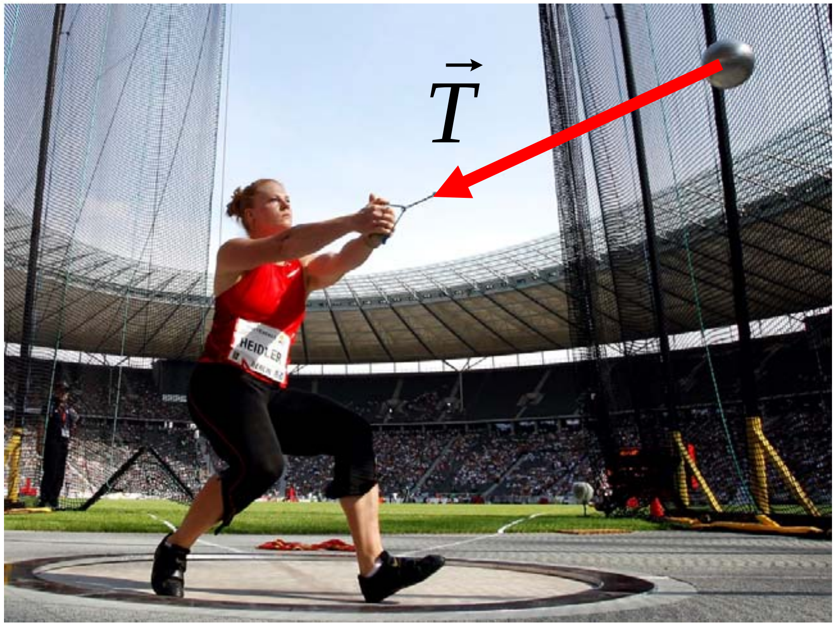
$$\sum F = m\overset{v^2/R}{\overset{=}{a_r}} \Rightarrow \boxed{N = \frac{mv^2}{R}}$$

v –velocity of the motorbike
R- radius of the circle

Normal force provides the centripetal acceleration

Bike going in a circle: the wall exerts an inward force (normal force) on a bike to make it move in a circle.

Examples. Hammer Thrower.



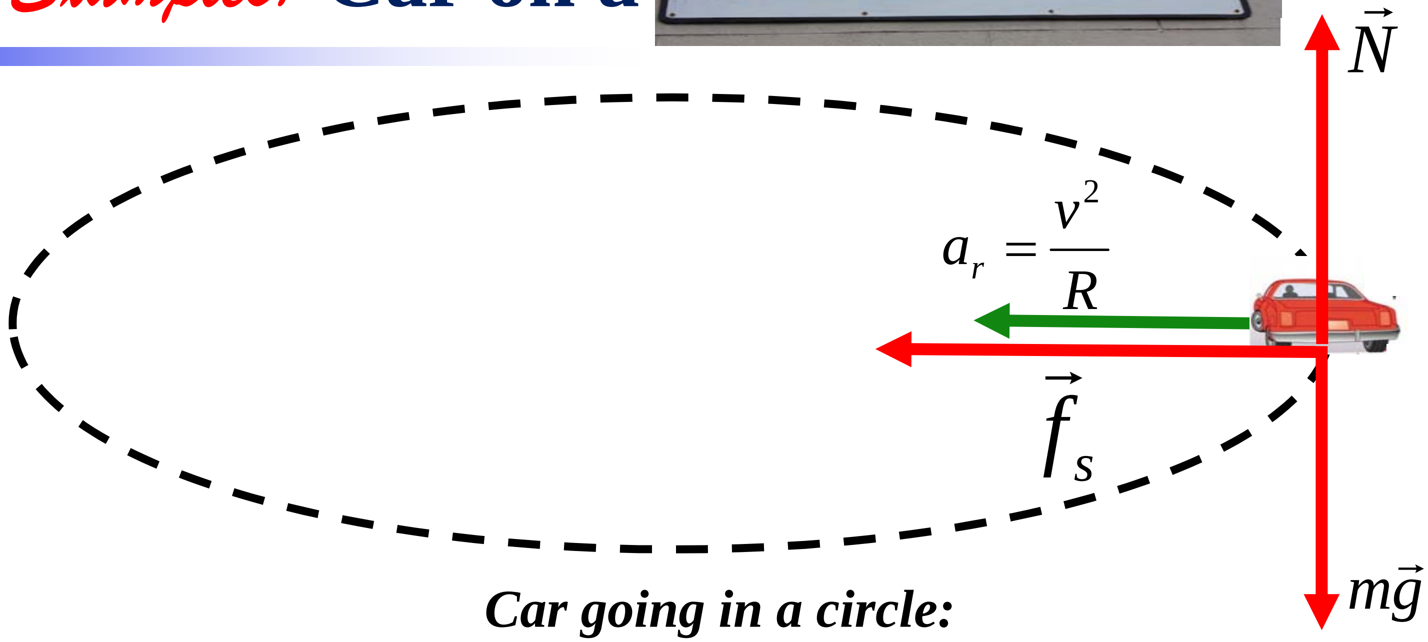
$$\sum F = m\underbrace{a_r}_{=v^2/R}$$

$$\boxed{T = \frac{mv^2}{R}}$$

A hammer going in a circle: the cord exerts an inward force (tension) on a hammer to make it move in a circle.

Tension provides the centripetal acceleration

Examples. Car on a CIRCULAR ROAD



Car going in a circle:

the road exerts an inward force (friction) on a car to make it move in a circle.

Friction provides the centripetal acceleration

$$\sum F = ma \quad a = \frac{v^2}{R}$$

$$\boxed{f_s = \frac{mv^2}{R}}$$

$$f_s = \mu_s N$$

v –velocity of the car

R- radius of the circle

Out of these two equations you can get anything you need

http://phys23p.sl.psu.edu/phys_anim/mech/car_Fc_new.avi

ConcepTest Around the Curve

You drive your car too fast around a curve and the car starts to skid. What is the correct description of this situation?

A) car's engine is not strong enough to keep the car from being pushed out

B) friction between tires and road is not strong enough to keep car in a circle

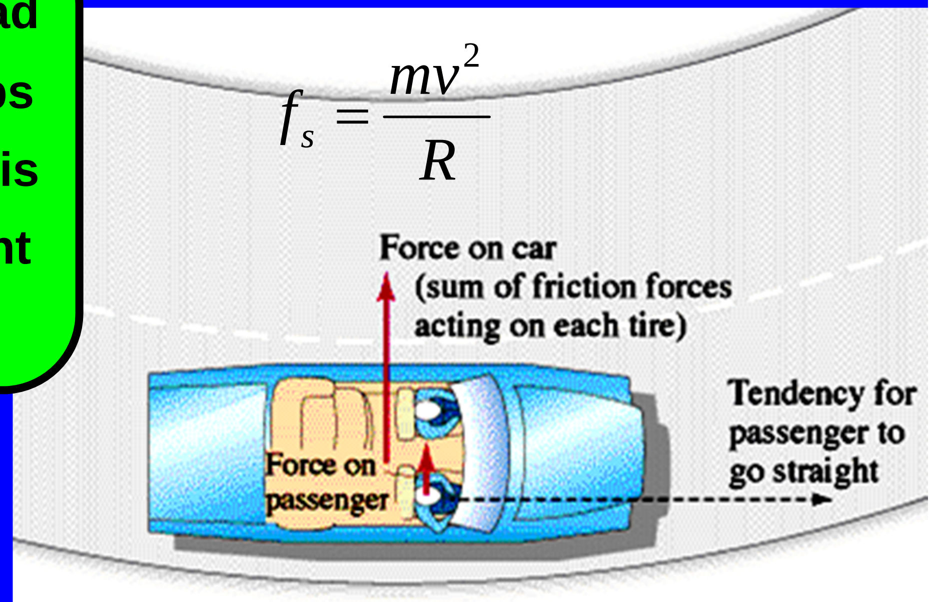
C) car is too heavy to make the turn

D) a deer caused you to skid

E) none of the above

The friction force between tires and road provides the centripetal force that keeps the car moving in a circle. If this force is too small, the car continues in a straight line!

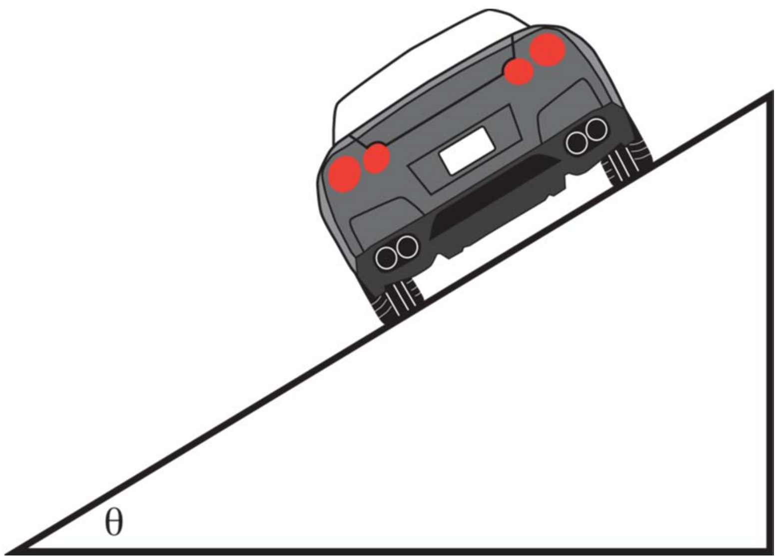
$$f_s = \frac{mv^2}{R}$$



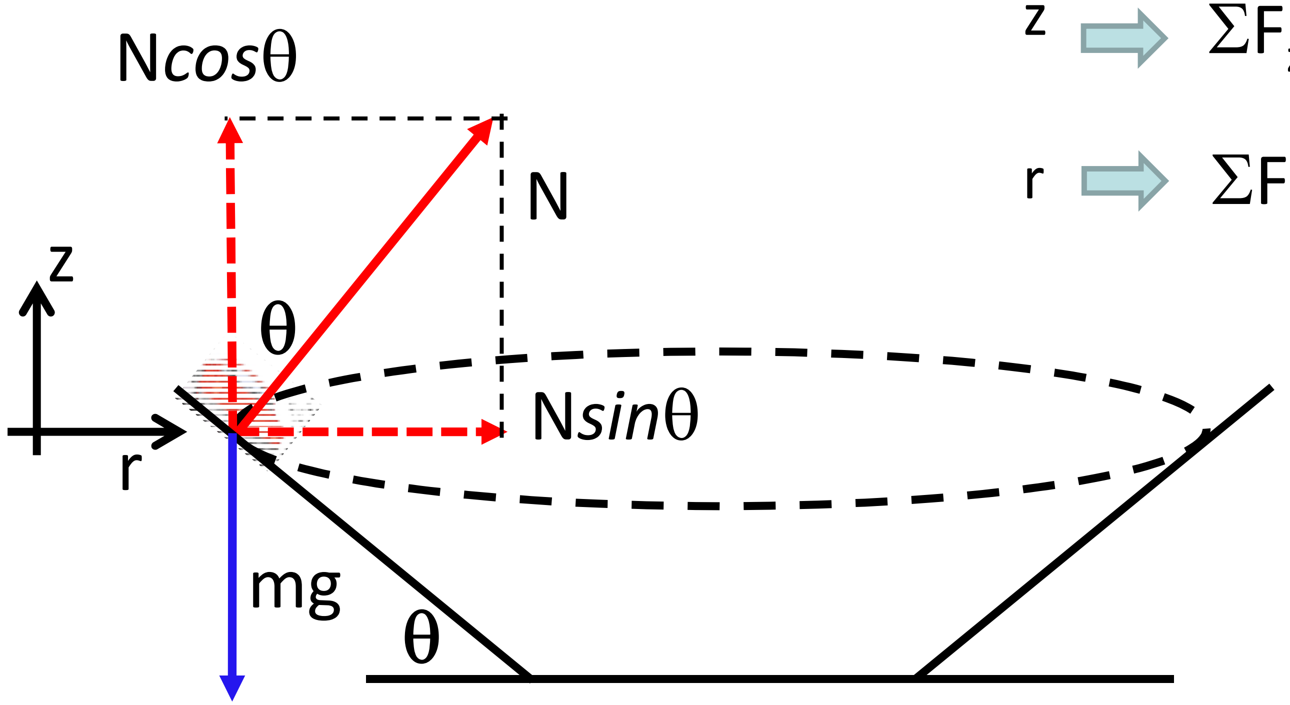
Follow-up: What could be done to the road or car to prevent skidding?

Examples. Banked curve

- ***But sometimes, friction force is not enough to keep a car on a circular road.***
- ***Banking the curve can help to keep cars from skidding.***



Banked Curves (solution)



z ⇨ $\Sigma F_z = N\cos\theta - mg = 0$

r ⇨ $\Sigma F_r = N\sin\theta = ma_r$

$a_r = v^2/R$

$$\begin{cases} N\sin\theta = mv^2/R \\ N\cos\theta = mg \end{cases}$$

Take a ratio

Independent of object mass !!!

$$\boxed{\tan\theta = v^2/gR}$$

r component of normal force provides the centripetal acceleration

http://phys23p.sl.psu.edu/phys_anim/mech/car_banked_new.avi

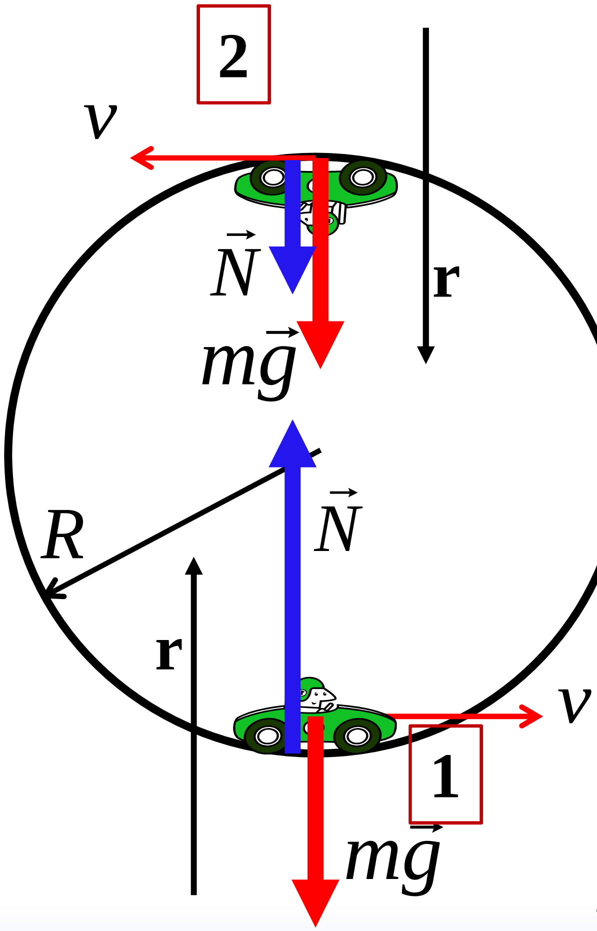
Example: Loop the Loop

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_fail.avi

Loop the Loop

To make the loop-the-loop at a constant speed, what minimum speed does the motorcycle need?



2 $$\sum F_r = ma_r \qquad a_r = \frac{v^2}{R}$$

$$N + mg = m\frac{v^2}{R}$$

$$\boxed{v = \sqrt{\frac{R}{m}(N + mg)}}$$

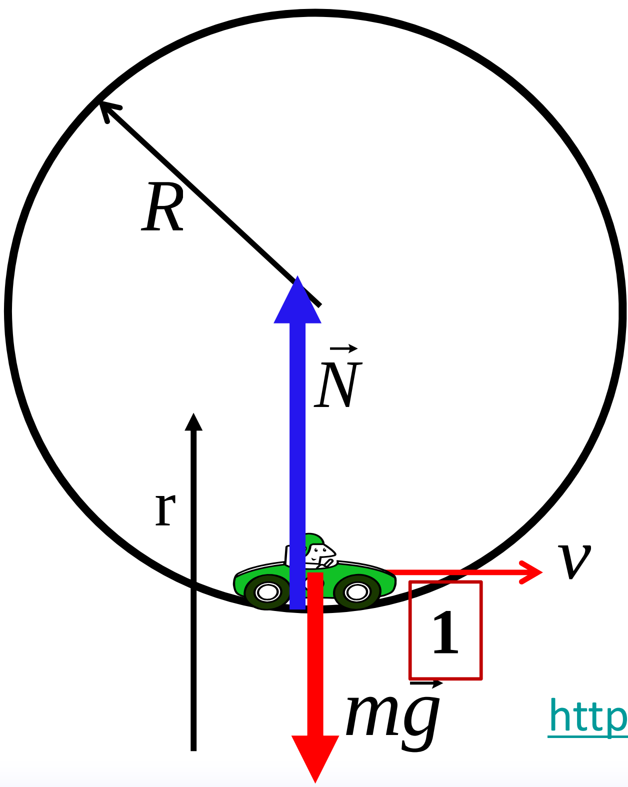
When N=0 (feels like no weight), then speed is minimum

$$\boxed{v = \sqrt{gR}}$$

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi

Loop the Loop

Apparent weight at the bottom, point 1?



$\boxed{1}$ $$\sum F_r = ma_r \quad \Leftarrow \quad a_r = \frac{v^2}{R}$$

$$N - mg = m\frac{v^2}{R}$$

$$N = m\frac{v^2}{R} + mg$$

Thus, N > mg. You would feel heavier (similar to an elevator)

http://phys23p.sl.psu.edu/phys_anim/mech/car_vert_bare.avi

ConcepTest Going in Circles

You're on a Ferris wheel moving in a vertical circle. When the Ferris wheel is at rest, the normal force N exerted by your seat is equal to your weight *mg*. How does N change at the top of the Ferris wheel when you are in motion?

A) N remains equal to *mg*

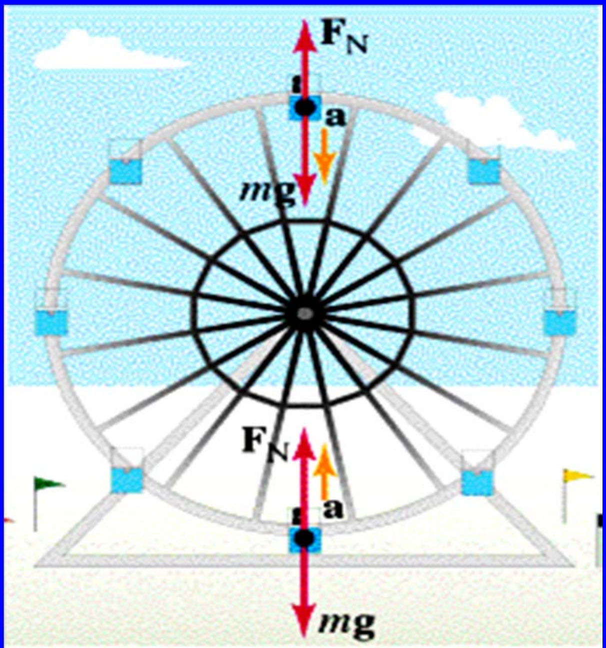
B) N is smaller than *mg*

C) N is larger than *mg*

D) none of the above

$$mg - N = m\frac{v^2}{R}$$

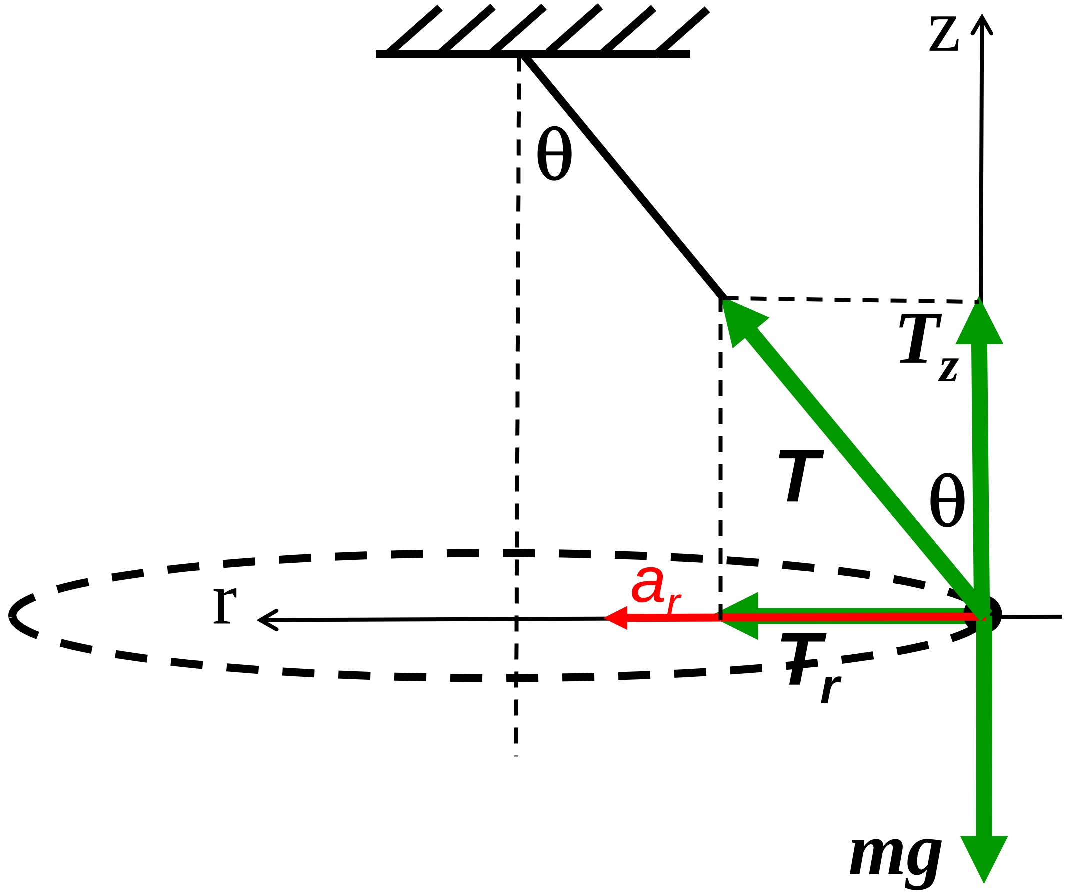
$$mg - m\frac{v^2}{R} = N$$



You are in circular motion, so there has to be a centripetal force pointing *inward*. At the top, the only two forces are *mg* (down) and *N* (up), so *N* must be smaller than *mg*.

Follow-up: Where is N larger than *mg*?

Example: Conical pendulum



$$\sum F_z = T_z - mg = 0 \quad (1)$$

$$\sum F_r = T_r = ma_r$$

$$T_r = T \cdot Sin\theta = \frac{mv^2}{R} \quad (2)$$

r component of tension provides the centripetal acceleration

Thank you
See you on Wednesday

ConcepTest 4 Going in Circles II

A skier goes over a small round hill with radius *R*. Because she is in circular motion, there has to be a ***centripetal force.*** At the top of the hill, what is F_c of the skier equal to?

A) $F_c = N + mg$

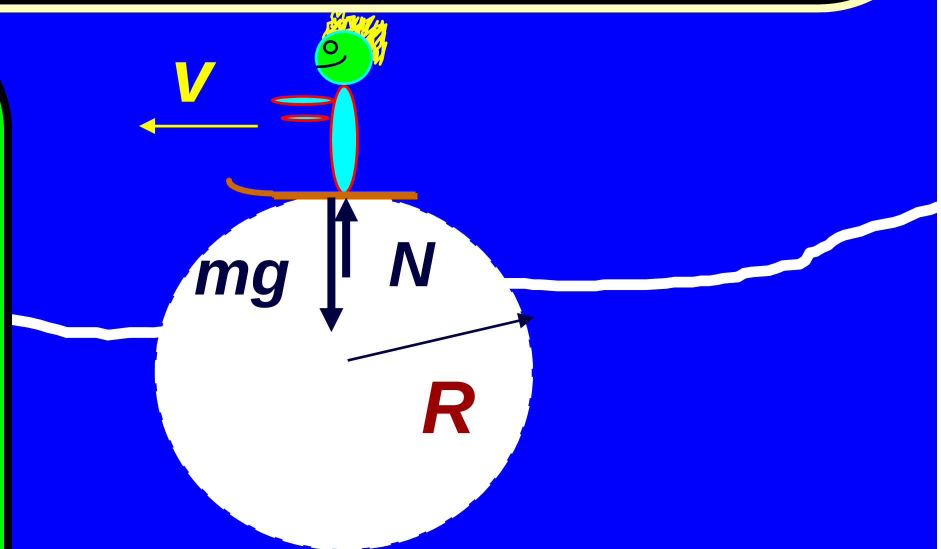
B) $F_c = mg - N$

C) $F_c = T + N - mg$

D) $F_c = N$

E) $F_c = mg$

F_c points toward the center of the circle (*i.e.*, downward in this case). The weight vector points down and the normal force (exerted by the hill) points up. The magnitude of the net force, therefore, is $F_c = mg - N$.



Follow-up: What happens when the skier goes into a small dip?