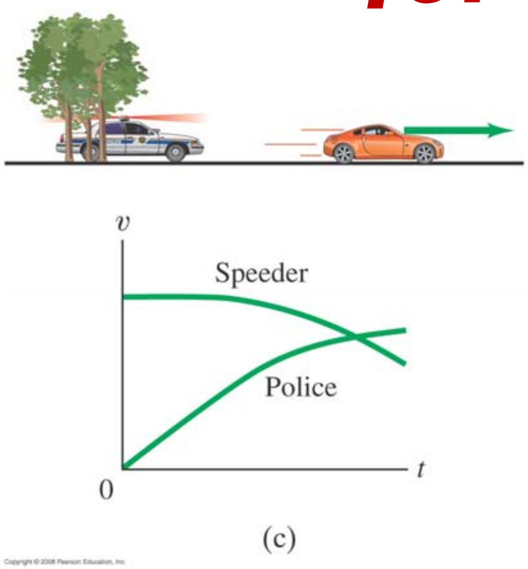
Lecture 3

Physics I

Chapter 2

Equations of motion for constant acceleration



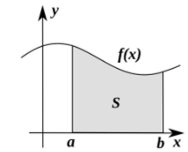
Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Today we are going to discuss:

Chapter 2:

- ***Motion with constant acceleration:*** *Section 2.4*
- ***Free fall (gravity):*** *Section 2.5*

Finding Position from a Velocity Graph



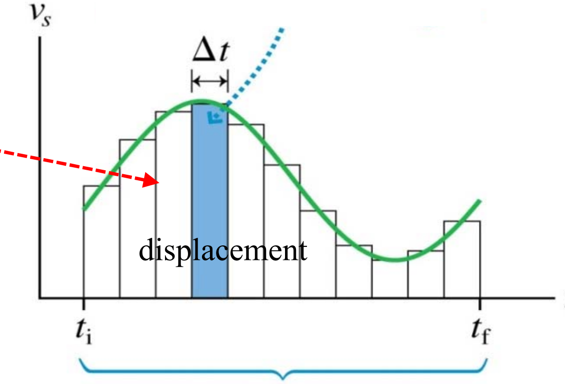
$v(t) = \frac{dx}{dt} \Rightarrow dx = v \cdot dt \Rightarrow$ Let's integrate it: $\int_{t_1}^{t_2} dx = \int_{t_1}^{t_2} v \cdot dt \Rightarrow x(t_2) - x(t_1) = \int_{t_1}^{t_2} v \cdot dt$

$$\boldsymbol{x(t_2) = x(t_1) + \int_{t_1}^{t_2} v \cdot dt}$$

Initial position

Total displacement

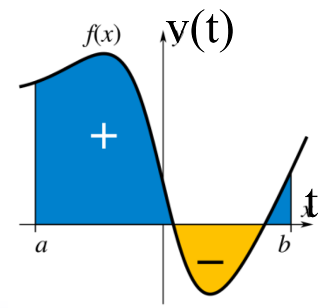
Geometrical meaning of an integral is an area



During the interval t_i to t_f, the total displacement Δs is the "area under the curve."

The total displacement Δs is called the "area under the velocity curve." (the total area enclosed between the t-axis and the velocity curve).

$$x_f = x_i + Area\ under\ v - vs - t\ between\ t_i\ and\ t_f$$



ConcepTest

Position from velocity

Here is the velocity graph of an object that is at the origin (x = 0 m) at t = 0 s.

At t = 4.0 s, the object's position is

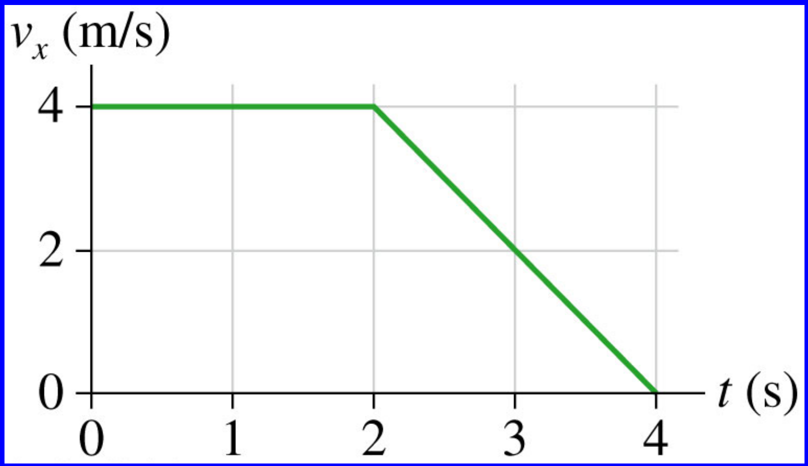
A) 20 m

B) 16 m

C) 12 m

D) 8 m

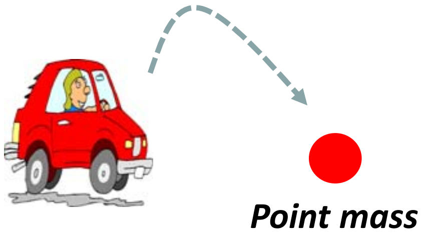
E) 4 m



Displacement = area under the curve

Simplifications

- ***Objects are point masses:*** ***have mass, no size***



- ***In a straight line:*** ***one dimension***

Consider a special, important type of motion:

- ***Acceleration is constant (a = const)***

$t=0$ $t>0$

a = const.

we need to predict future, $x(t)$, $v(t)$

NEED: Equations ???

The Kinematic Equations of Constant Acceleration

Velocity equation. Equation 1.

(constant acceleration)

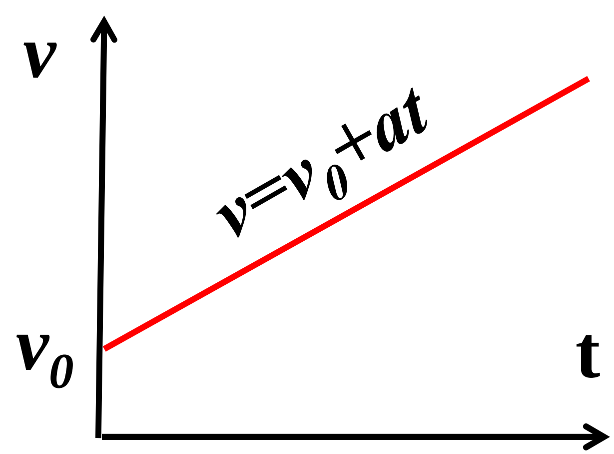
by definition, acceleration $$a = \frac{v(t) - v_o}{t - t_0} \text{ and } t_0 = 0$$

$$a = \frac{v(t) - v_o}{t} \quad \Rightarrow$$

Velocity equation

$$v(t) = v_o + at \qquad (1)$$

the velocity is increasing at a constant rate

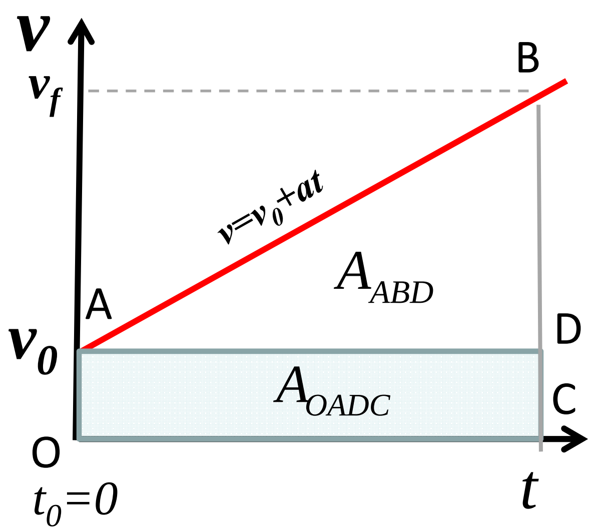


Position equation. Equation 2

(constant acceleration)

Recall Eq (2.11)

$$x_f = x_0 + Area\ under\ v - vs - t\ between\ t_0\ and\ t_f$$



$$x_f = x_0 + A_{OADC} + A_{ABD}$$

$$a = \frac{v(t) - v_o}{t}$$

$$x_f = x_0 + v_0 t + \tfrac{1}{2}(v_f - v_0)t \Leftarrow v(t) - v_o = at$$

Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad (2)$$

No time equation. Equation 3

(constant acceleration)

We can also combine these two equations so as to *eliminate t*:

Velocity equation

$$v(t) = v_o + at$$

Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

No time equation

$$v^2 = {v_0}^2 + 2a(x - x_0) \qquad (3)$$

It's useful when time information is not given.

Motion at Constant Acceleration (all equations)

We now have all the equations we need to solve constant-acceleration problems.

Velocity equation

$$v(t) = v_o + at \qquad (1)$$

Position equation

$$x_f = x_0 + v_0 t + \tfrac{1}{2} a t^2 \qquad (2)$$

No time equation

$$v^2 = v_0{}^2 + 2a(x - x_0) \qquad (3)$$

Problem Solving

<u>How to solve:</u>

– Divide problem into "knowns" and "unknowns"
– Determine best equation to solve the problem
– Input numbers

Example 

A plane, taking off from rest, needs to achieve a speed of 28 m/s in order to take off. If the acceleration of the plane is constant at 2 m/s^2, what is the minimum length of the runway which can be used?

Example

○ which eq-n to use?

~~(1)~~ $v = v_0 + at$ (no time info)

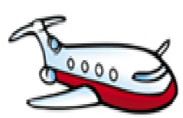
~~(2)~~ $x = x_0 + v_0 t + at^2/2$ (no time info)

(3) $v^2 = v_0^2 + 2a(x - x_0)$ ✓ (our eq-n)

$$v^2 = v_0^2 + 2a(x - x_0), \quad v_0 = 0, \quad x_0 = 0$$

v ↑ known, a ↑ known, x ?

$$x = \frac{v^2}{2a} = \frac{(28\,m/s)^2}{2 \cdot 2\,m/s^2} = 196\,m$$

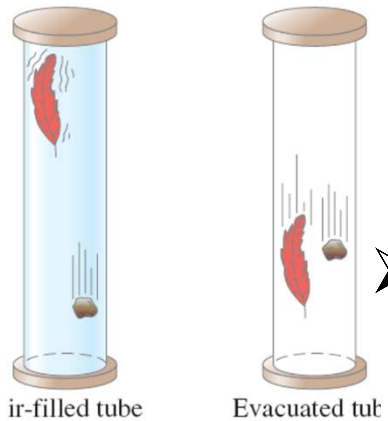


The runway **must** be 196 m long.

Free

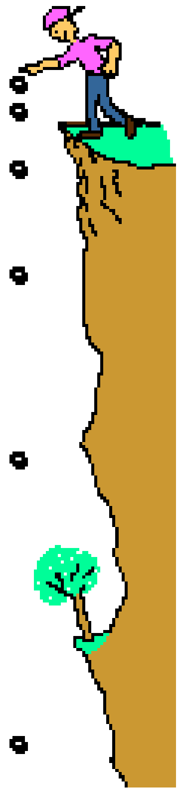
Freely Falling Objects

One of the most common examples of motion with <u>constant acceleration</u> is freely falling objects.



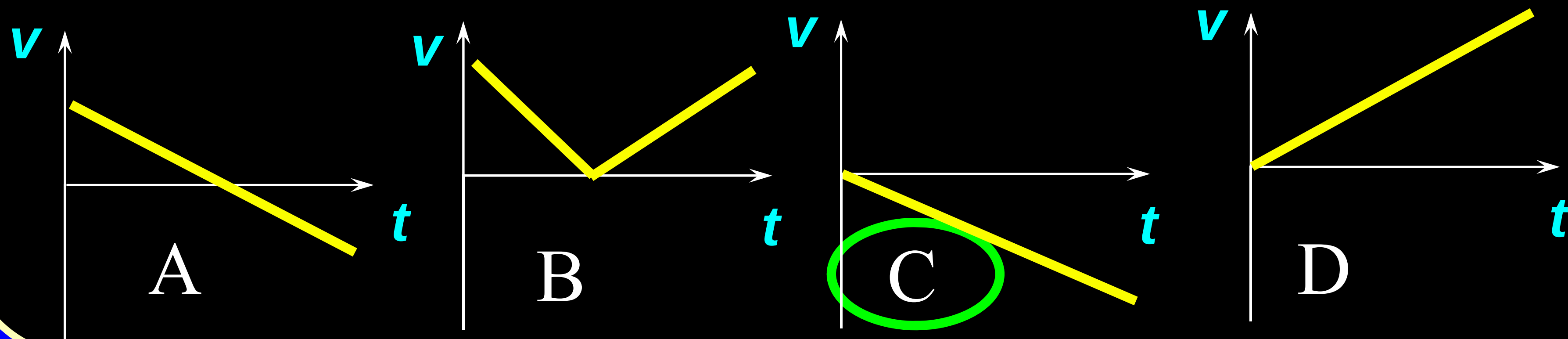
Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.

- ***All free-falling objects (on Earth) accelerate downwards at a rate of 9.8 m/s²***
- ***Air resistance is neglected***

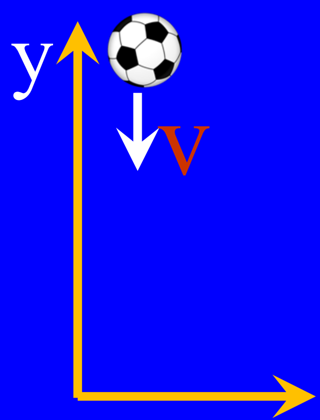


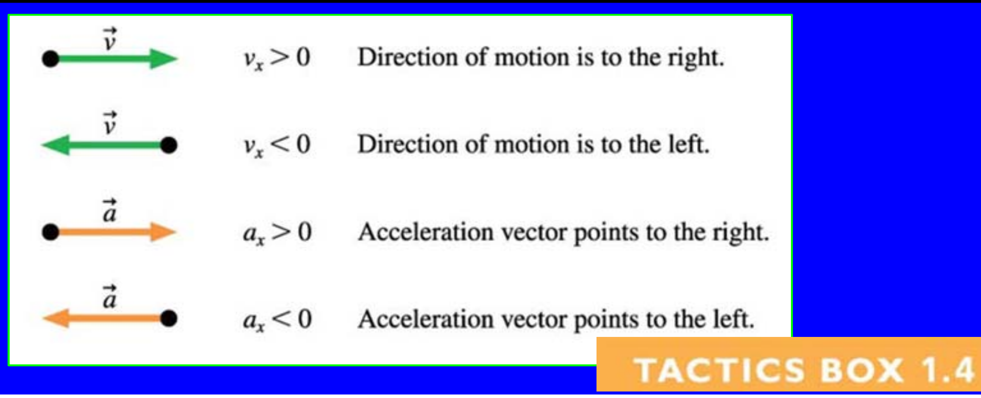
ConcepTest — Free Fall

You drop a ball. Right after it leaves your hand and before it hits the floor, which of the above plots represents the v vs. t graph for this motion? (Assume your y-axis is pointing up).



The ball is dropped from rest, so its initial velocity is zero. Because the y-axis is pointing upward and the ball is falling downward, its velocity is negative and becomes more and more negative as it accelerates downward.





$\vec{v}$	$v_x > 0$	Direction of motion is to the right.
$\vec{v}$	$v_x < 0$	Direction of motion is to the left.
$\vec{a}$	$a_x > 0$	Acceleration vector points to the right.
$\vec{a}$	$a_x < 0$	Acceleration vector points to the left.

TACTICS BOX 1.4

Freely Falling Objects

Velocity equation

$v(t) = v_o + at$ (1)

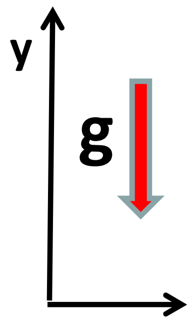
Position equation

$x_f = x_0 + v_0 t + \frac{1}{2} a t^2$ (2)

No time equation

$v^2 = v_0{}^2 + 2a(x - x_0)$ (3)

if

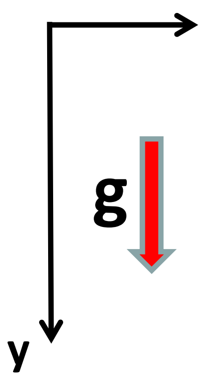


then ***a = -g***

$$\begin{cases} v = v_0 - gt \\ y = y_0 + v_0 t - {gt^2}/{2} \\ v^2 = v_0{}^2 - 2g(y - y_0) \end{cases}$$

- -

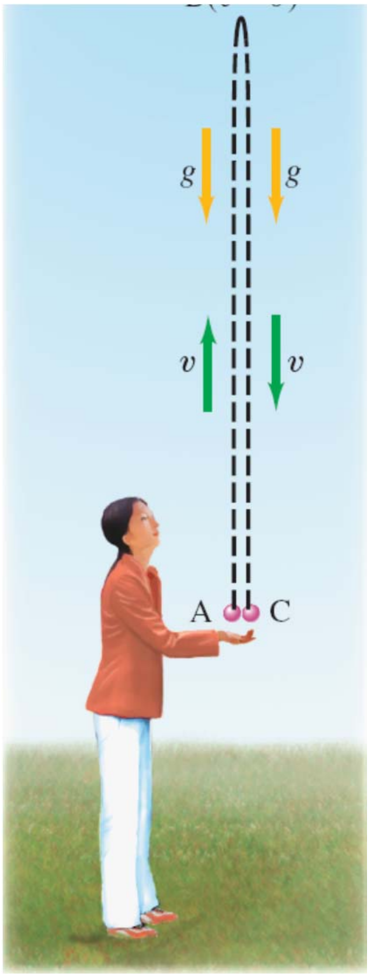
if



then ***a = g***

$$\begin{cases} v = v_0 + gt \\ y = y_0 + v_0 t + {gt^2}/{2} \\ v^2 = v_0{}^2 + 2g(y - y_0) \end{cases}$$

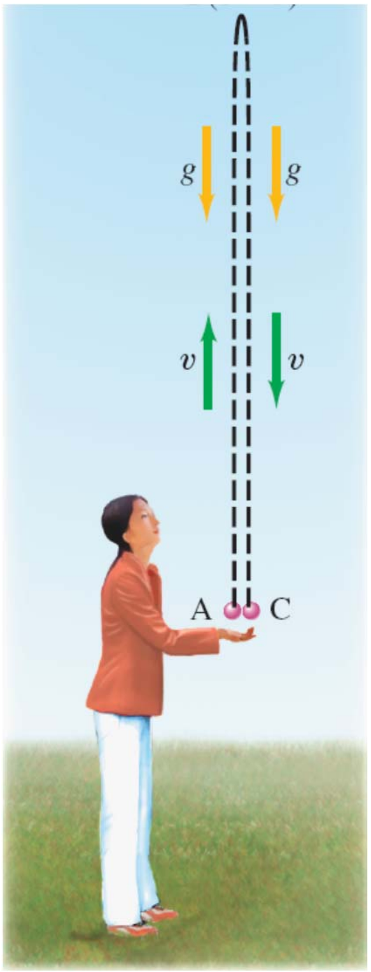
Example: Ball thrown upward.

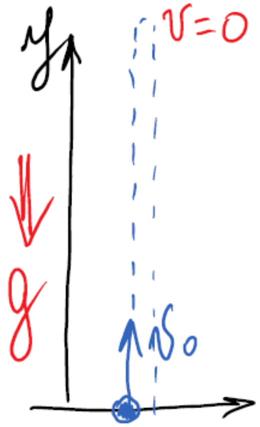


A person throws a ball upward into the air with an initial velocity of 10.0 m/s.
Calculate

(a) how high it goes, and
(b) how long the ball is in the air before it comes back to the hand.
(Ignore air resistance.)

Example





<u>Given:</u> $v_0 = 10\ m/s$; $y_0 = 0$

<u>Calculate how high it goes: y - ?</u>

1. Choose a coord. system:
   y - upward,
   g - downward (always)
   so $a = -g$

$$\begin{cases} y = y_0 + v_0 t + \frac{at^2}{2} \\ v = v_0 + at \\ v^2 = v_0^2 + 2a(y - y_0) \end{cases}$$

$a = -g$

$$\begin{cases} y = y_0 + v_0 t - \frac{gt^2}{2} & \times \text{ (no t info)} \\ v = v_0 - gt & \times \text{ (no t info)} \\ v^2 = v_0^2 - 2g(y - y_0) & \checkmark \end{cases}$$

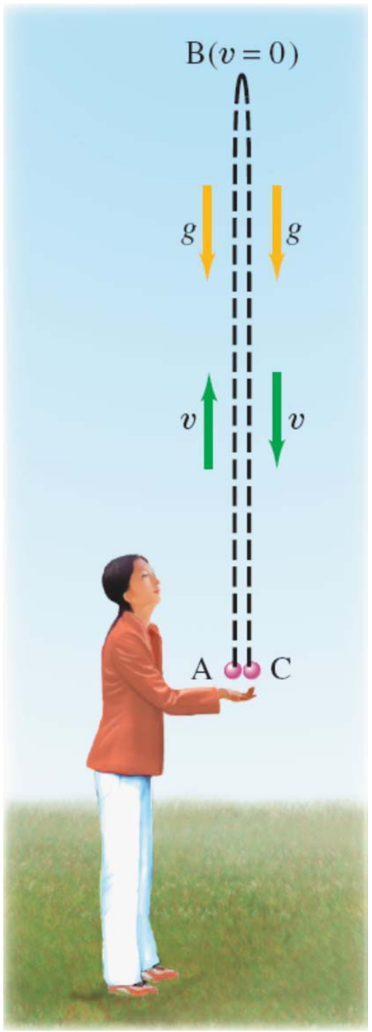
($v^2 \to 0$, $y_0 \to 0$)

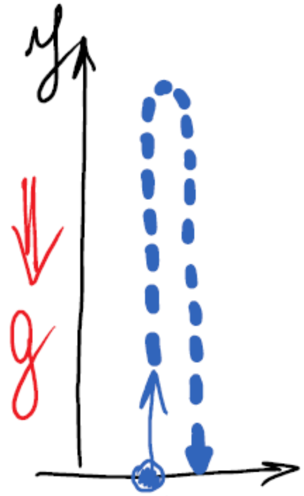
at max. hight, $v = 0$

$$0 = v_0^2 - 2 \cdot g \cdot y$$

$$y = \frac{v_0^2}{2 \cdot g} = \frac{(10\ m/s)^2}{2 \cdot 9.8\ m/s^2} \approx \frac{100}{20} m = 5m$$

Example





initial and final points

- How long the ball is in the air?

1. $y = y_0 + V_0 t - \frac{gt^2}{2}$ ✓
2. $v = v_0 - gt$ ✓
3. $v^2 = v_0^2 - 2g(y - y_0)$ ✗

both OK!!

- At the final point: $y = 0$

let's use eq-n 1.

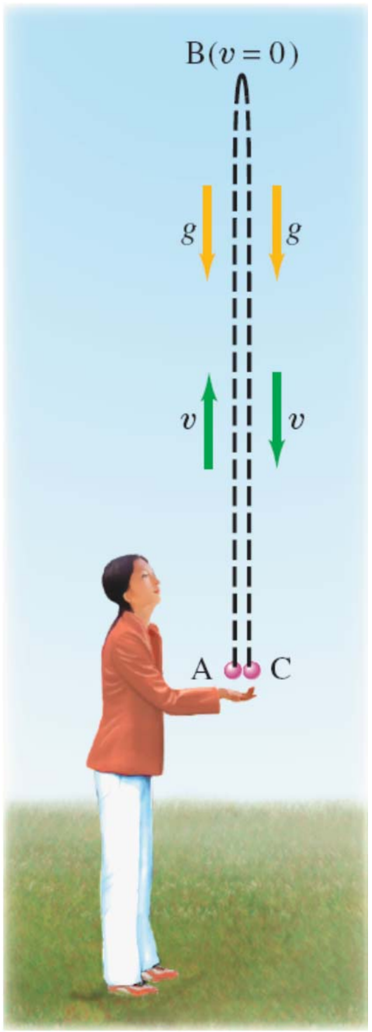
$$y^{\nearrow 0} = y_0^{\nearrow 0} + V_0 t - \frac{gt^2}{2}$$

$$0 = t\left(v_0 - \frac{gt}{2}\right) \quad \text{there are two solution.}$$

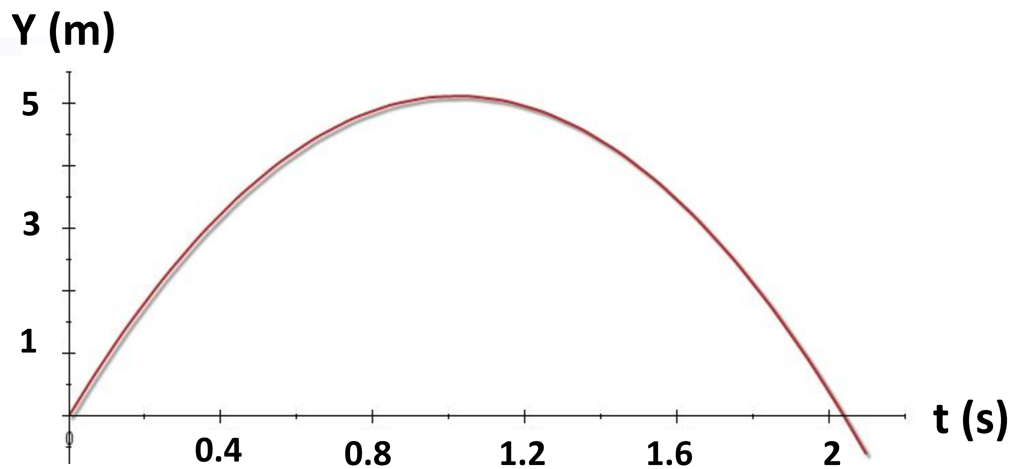
$$t_1 = 0; \quad V_0 - \frac{gt}{2} = 0$$

$$t_2 = \frac{2V_0}{g} = \frac{2 \cdot 10\,m/s}{9.8\,m/s^2} \simeq 2\,s$$

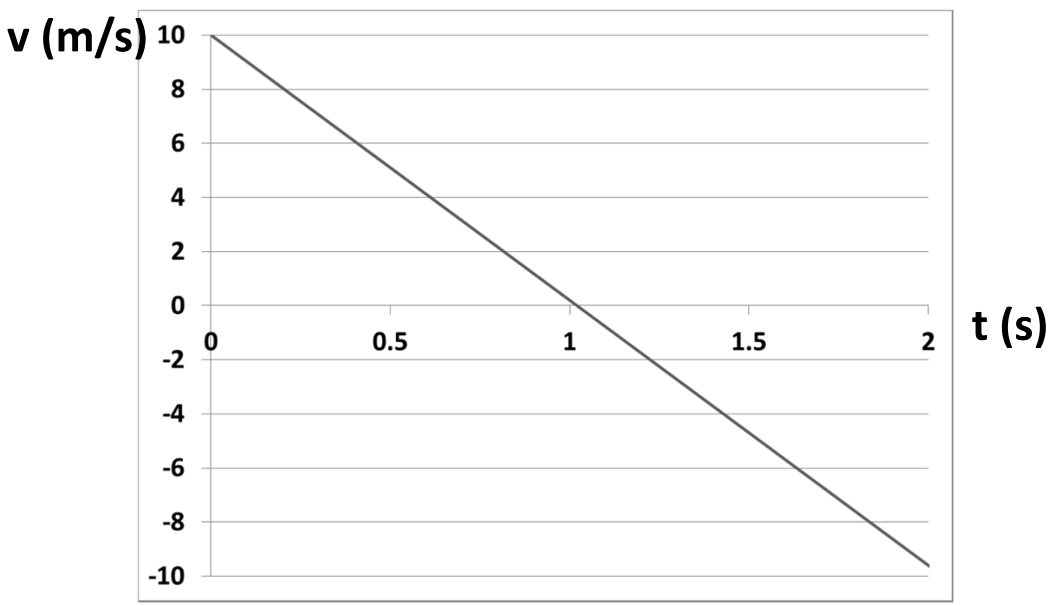
Example



B(v = 0)
g
g
v
v
A
C



Y (m)
5
3
1
0
0.4
0.8
1.2
1.6
2
t (s)

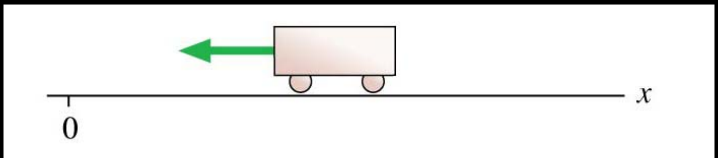


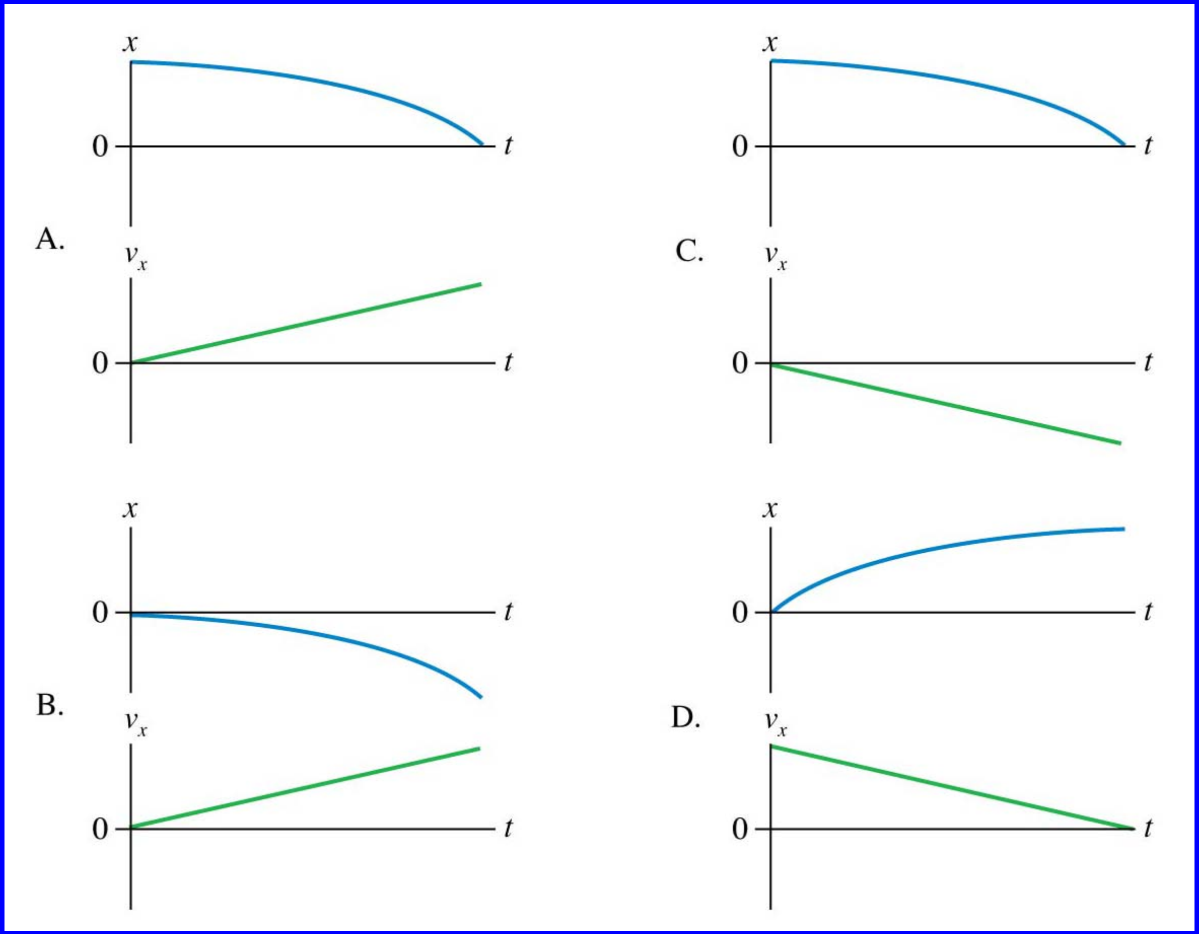
v (m/s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.5
1
1.5
2
t (s)

Thank you
See you on Monday

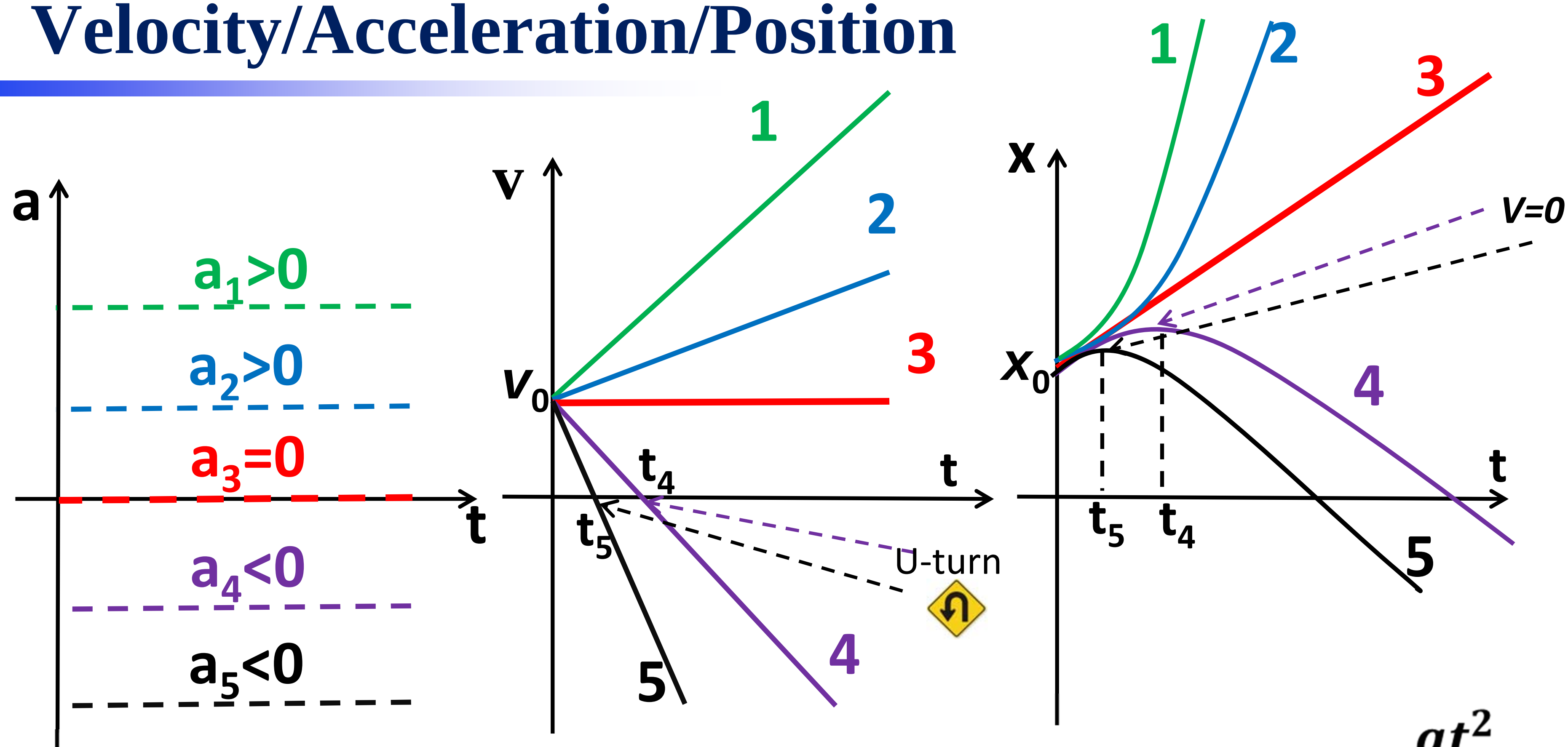
ConcepTest 1 Roller Coaster

A cart speeds up toward the origin. What do the position and velocity graphs look like?





Velocity/Acceleration/Position



$a = const$ $\qquad$ $v = v_0 + at$ $\qquad$ $x = x_0 + v_0 t + \frac{at^2}{2}$

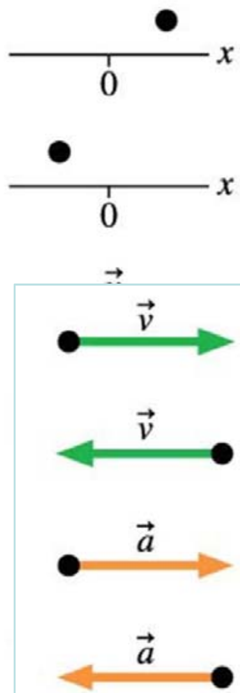
- 4,5 – negative acceleration,

  but from $0<t<t_4$ or t_5 – decceleration

  but for $t> t_4$ or t_5 – acceleration

Determining the Sign of the Position, Velocity, and Acceleration

TACTICS BOX 1.4



$x > 0$	Position to right of origin.
$x < 0$	Position to left of origin.
$v_x > 0$	Direction of motion is to the right.
$v_x < 0$	Direction of motion is to the left.
$a_x > 0$	Acceleration vector points to the right.
$a_x < 0$	Acceleration vector points to the left.

- The sign of position (x or y) tells us *where* an object is.
- The sign of velocity (v_x or v_y) tells us *which direction* the object is moving.
- The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, *not* whether the object is speeding up or slowing down.