Lecture 3

University of Massachusetts
UMASS Lowell

Equations of motion for constant acceleration

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsI

Lecture Capture:
http://echo360.uml.edu/danylov2013/physics1fall.html

Outline

Chapter 2: Sections 5-7

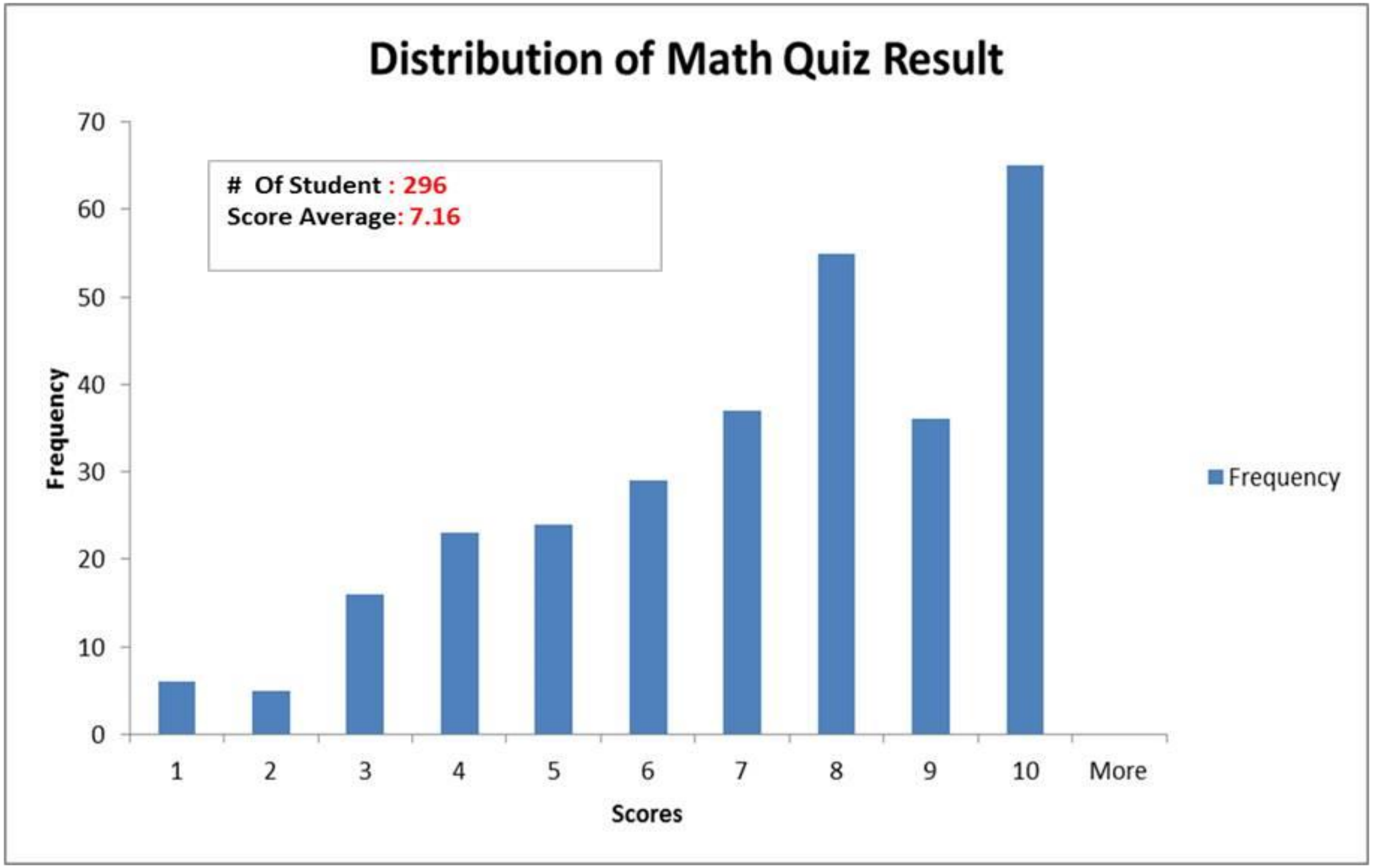
Constant acceleration

Equations of motion

Free fall (gravity)

Problem solving

Math test results



Motion at ***CONSTANT*** acceleration

Consider a special, important type of motion:

- Objects are point masses; have mass, no size
- In a straight line (one dimension)
- **Acceleration is constant** (**a=const**)

Motion at Constant Acceleration

- Assume:
  At $t_0 = 0$, initial values $x = x_0, v = v_0$ **and** $a = const$.
- Need to find:
  At $t > 0$, final values $x(t), v(t)$.

$t=0$ $\qquad$ $t>0$

$a = const.$

we need to predict <u>future</u>, $x(t)$, $v(t)$

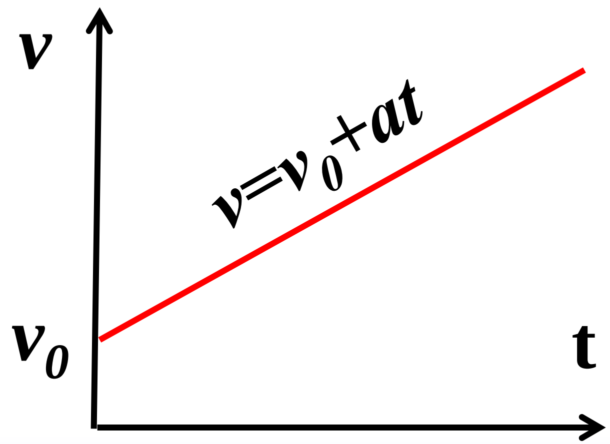
NEED: Equations ???

Motion at Constant Acceleration (equation 1)

$$\text{by definition, acceleration} \qquad a = \frac{v(t) - v_o}{t - t_0} \text{ and } t_0 = 0$$

$$a = \frac{v(t) - v_o}{t} \quad \Rightarrow \quad \mathbf{v(t) = v_0 + at} \qquad \textbf{(1)}$$

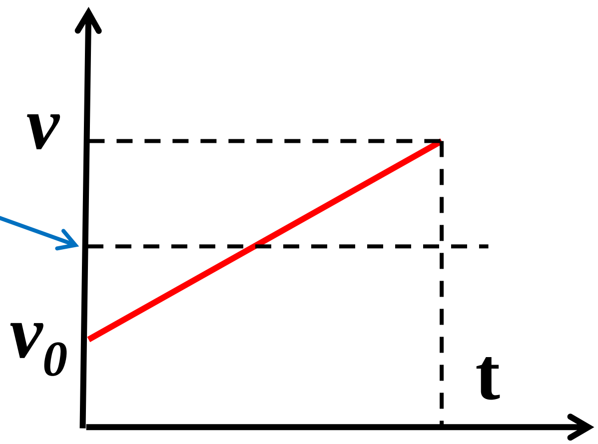
the velocity is increasing at a constant rate



Motion at Constant Acceleration (equation 4)

average velocity

$$\bar{v} = \frac{v(t) + v_o}{2}$$



When acceleration is constant, the average velocity is midway between the initial and final velocities.

Motion at Constant Acceleration (equation 2)

The average velocity of an object during a time interval *t* is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(t) - x_o}{t - t_0} = \frac{x(t) - x_o}{t}$$

$$\bar{v} = \frac{v(t) + v_o}{2}$$

$$v = v_0 + at$$

Combining these two equations

$$x(t) = x_0 + v_0 t + \frac{at^2}{2} \quad (2)$$

Equation 2. Derivation

Definition of average velocity ($t_0 = 0$)

$$\bar{v} \equiv \frac{x - x_0}{t} \Rightarrow x = x_0 + \bar{v} \cdot t$$

let's use Eq. 4. $\bar{v} = \dfrac{v_0 + v}{2}$, so

$$x = x_0 + \left(\frac{v_0 + v}{2}\right) \cdot t = \left\| \begin{array}{l} Eq.\ 1 \\ v = v_0 + at \end{array} \right\| =$$

$$= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right) \cdot t = x_0 + \frac{\cancel{2} v_0 t}{\cancel{2}} + \frac{at^2}{2}, \text{ so}$$

$$\boxed{x = x_0 + v_0 t + \frac{at^2}{2}} \quad Eq. 2.$$

$$v(t) = v_0 + at \qquad (1)$$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2} \qquad (2)$$

We need an equation without *time*

Motion at Constant Acceleration (equation 3)

We can also combine these equations so as to *eliminate t*:

$$\bar{v} = \frac{x(t) - x_o}{t}$$

$$\bar{v} = \frac{v(t) + v_o}{2}$$

$$a = \frac{v(t) - v_o}{t}$$

$$\boldsymbol{v^2(t) = v_0^2 + 2a(x - x_0)} \quad \textit{(3)}$$

Motion at Constant Acceleration (equation 3)

$$v^2(t) = v_0^2 + 2a(x - x_0) \quad (3)$$

Let's derive Eq. #3.

- Average velocity

$$\bar{v} = \frac{x - x_0}{t} \Rightarrow x = x_0 + \bar{v}t = \left\| \bar{v} = \frac{v + v_0}{2} \right\| =$$

$$= x_0 + \left(\frac{v + v_0}{2}\right) \cdot t = \left\| \begin{array}{l} a = \frac{v - v_0}{t} \\ t = \frac{v - v_0}{a} \end{array} \right\| =$$

$$= x_0 + \left(\frac{v + v_0}{2}\right) \cdot \left(\frac{v - v_0}{a}\right) = x_0 + \frac{1}{2a}\left(v^2 - v_0^2\right)$$

so $x = x_0 + \frac{1}{2a} \cdot \left(v^2 - v_0^2\right)$

let's invert it:

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

we managed to eliminate time.

Motion at Constant Acceleration (all equations)

We now have all the equations we need to solve constant-acceleration problems.

$$v(t) = v_0 + at \qquad (1)$$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2} \qquad (2)$$

$$v^2(t) = v_0^2 + 2a(x - x_0) \qquad (3)$$

$$\overline{v} = \frac{v(t) + v_0}{2} \qquad (4)$$

Problem Solving

- How to solve:
  - Divide problem into "knowns" and "unknowns"
  - Determine best equation to solve the problem
  - Input numbers

Example 2-9

- A plane taking off from rest a runway needs to achieve a speed of 28 m/s in order to take off. If the acceleration of the plane is constant at 2 m/s^2, what is the minimum length of the runway which can be used?

Example 2-9

initial $\qquad$ final

$x_0 = 0$ $\qquad$ x - ?

$v_0 = 0$ $\qquad$ $v = 28\,m/s$

o which eq-n to use?

$\cancel{(1)}$ $v = v_0 + at$ (no time info)

$\cancel{(2)}$ $x = x_0 + v_0 t + at^2/2$ (no time info)

(3) $v^2 = v_0^2 + 2a(x - x_0)$ ✓ (our eq-n)

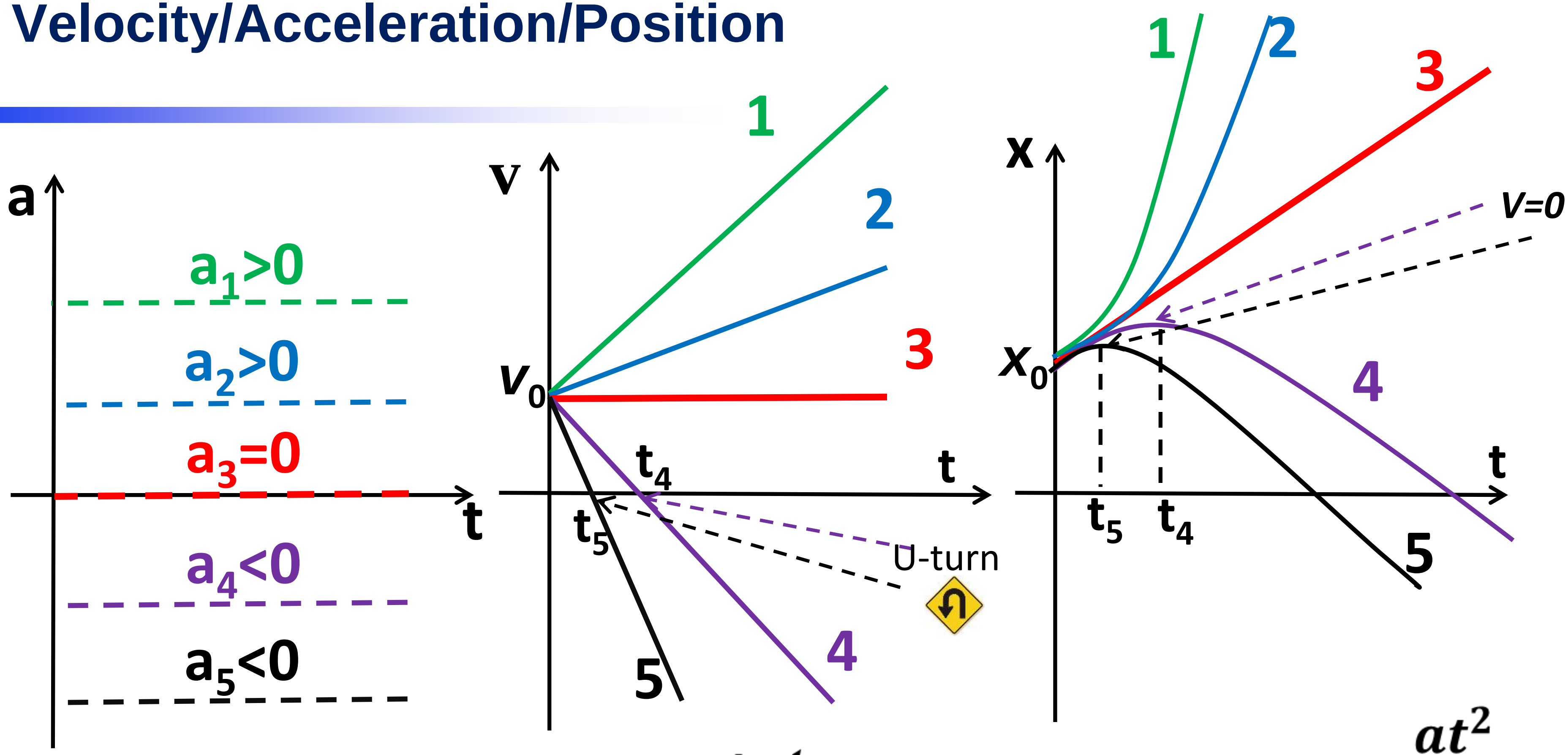
$$v^2 = \cancel{v_0^2}^{\,0} + 2a(x - \cancel{x_0}^{\,0})$$

known $\qquad$ known $\qquad$?

$$x = \frac{v^2}{2a} = \frac{(28\,m/s)^2}{2 \cdot 2\,m/s^2} = 196\,m$$

The runway must be 196m long.

Velocity/Acceleration/Position



$a = const$ $\quad v = v_0 + at$ $\quad x = x_0 + v_0 t + \frac{at^2}{2}$

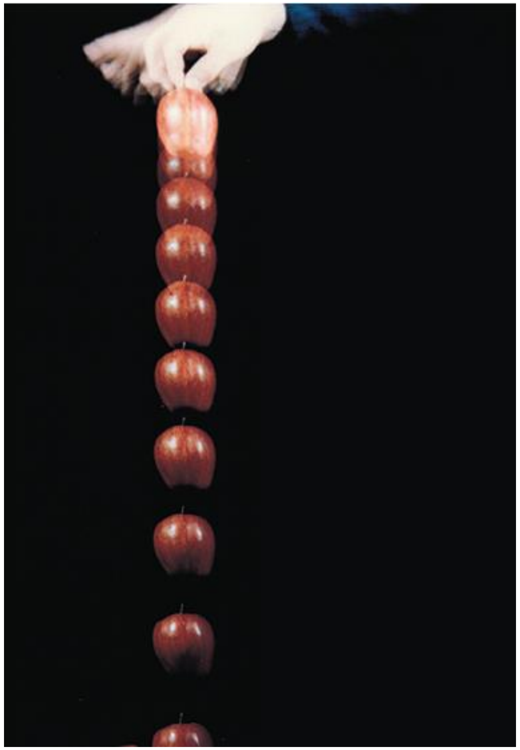
- 4,5 – negative acceleration,

  but from $0<t<t_4$ or t_5 – decceleration

  but for $t> t_4$ or t_5 – acceleration

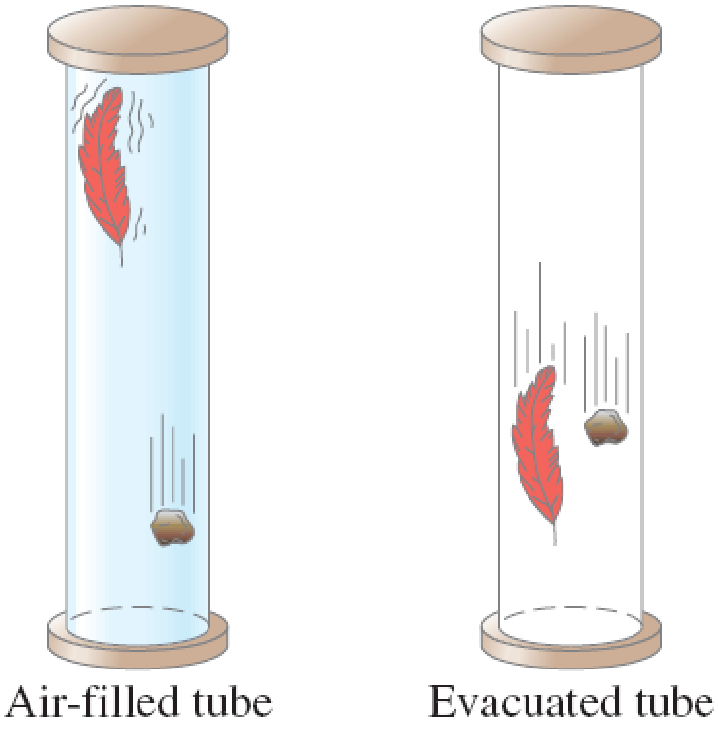
Freely Falling Objects

Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity.



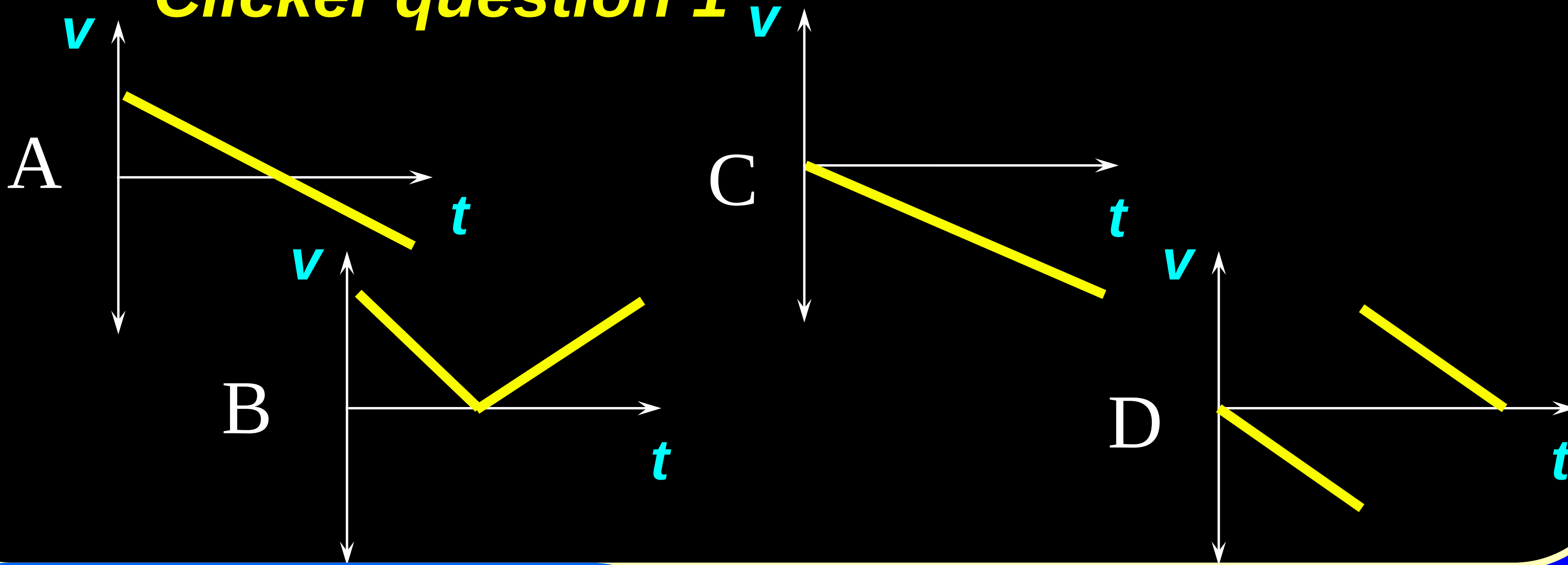
This is one of the most common examples of motion with constant acceleration.

Freely Falling Objects

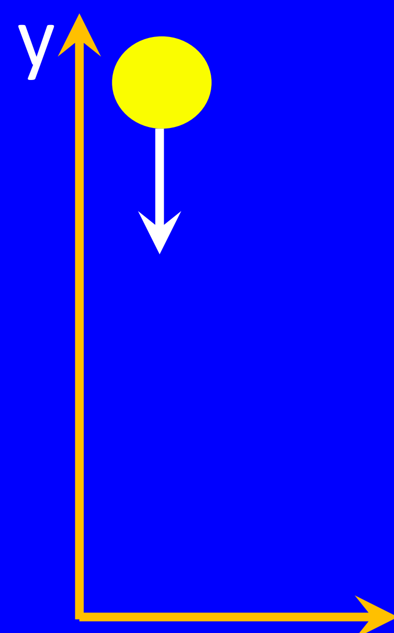


The acceleration due to gravity at the Earth's surface is approximately 9.80 m/s^2. At a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

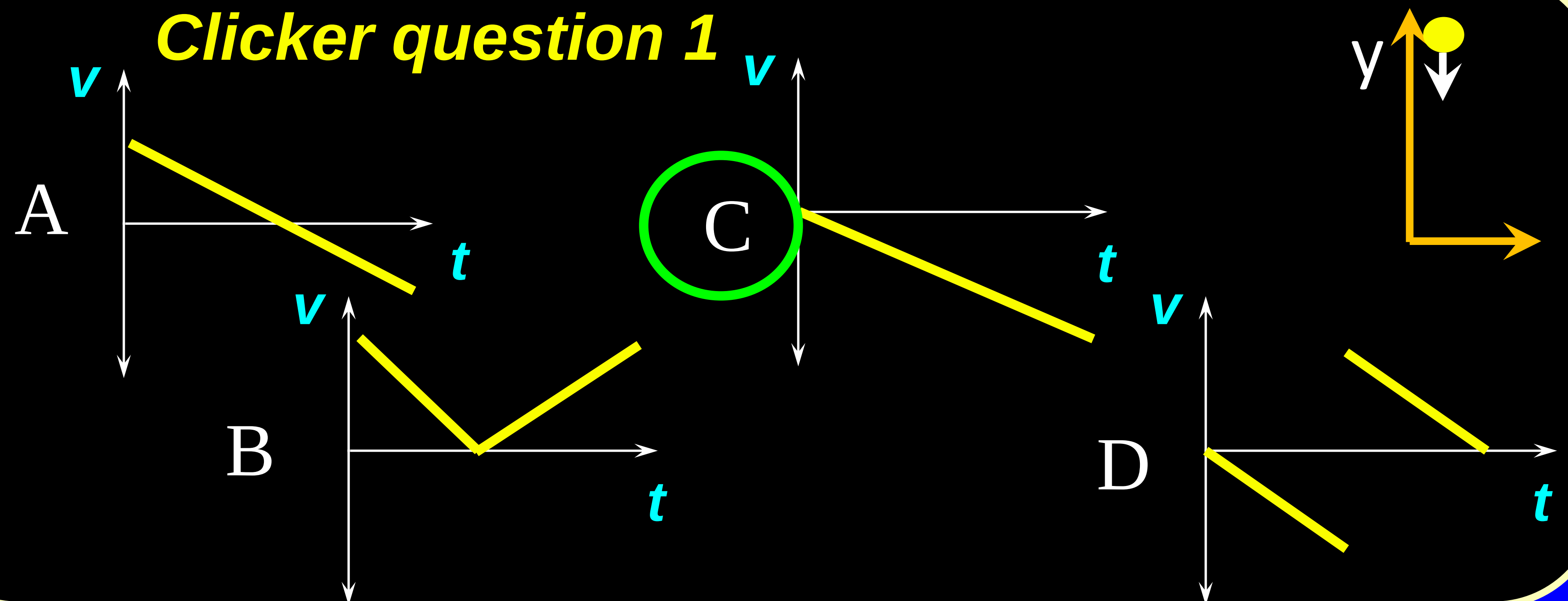
Clicker question 1



You drop a rubber ball. Right after it leaves your hand and before it hits the floor, which of the above plots represents the *v* vs. *t* graph for this motion? (Assume your y-axis is pointing up).



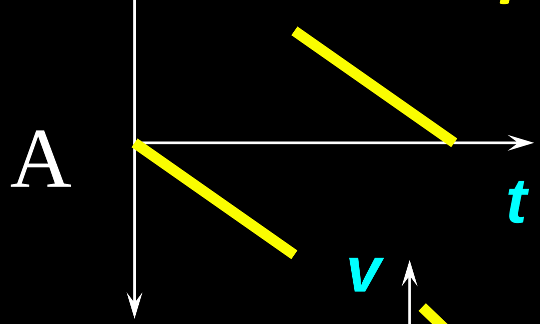
Clicker question 1

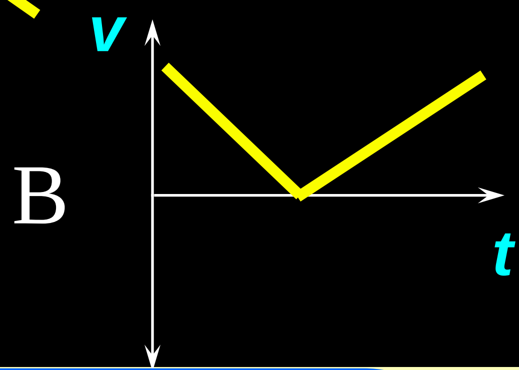


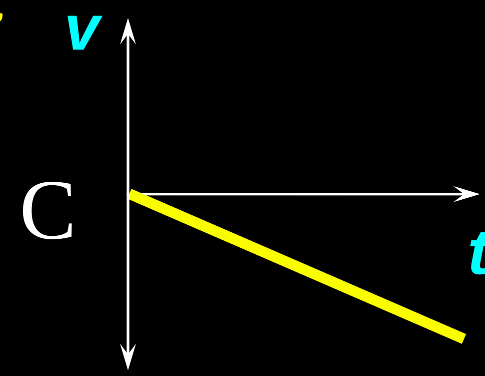
You drop a rubber ball. Right after it leaves your hand and before it hits the floor, which of the above plots represents the *v* vs. *t* graph for this motion? (Assume your y-axis is pointing up).

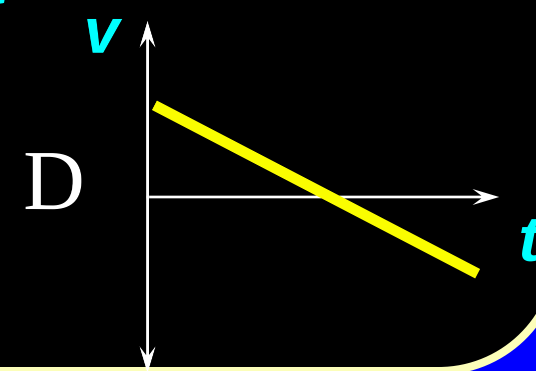
The ball is dropped from rest, so its initial velocity is zero. Because the y-axis is pointing upward and the ball is falling downward, its velocity is negative and becomes more and more negative as it accelerates downward.

Clicker question 2

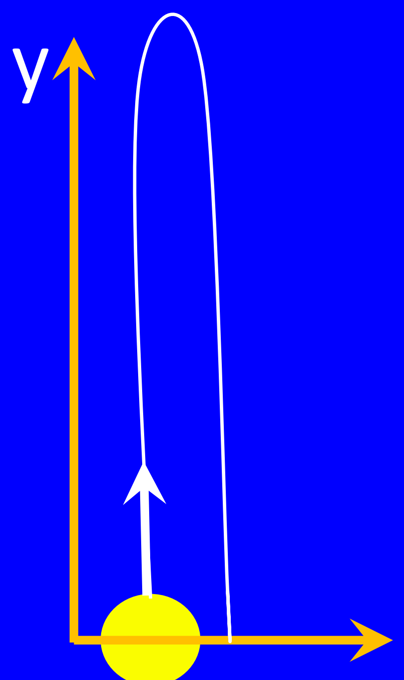




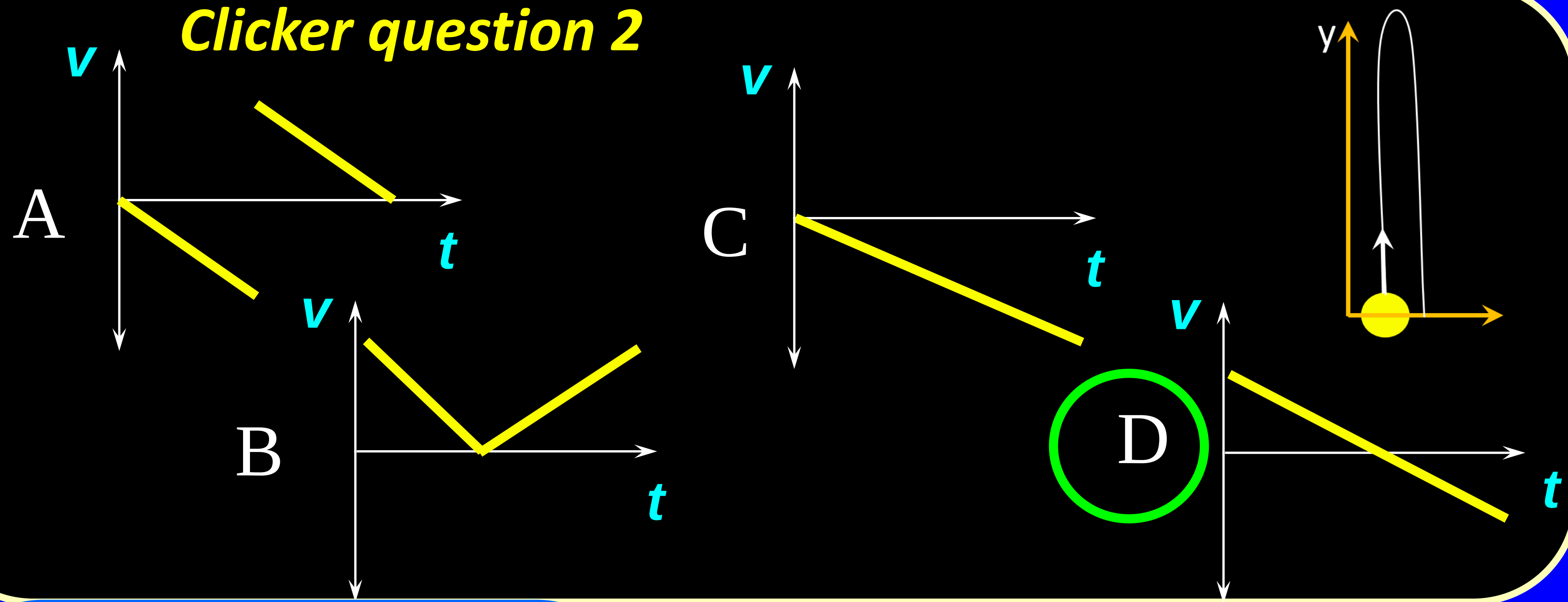




You toss a ball straight up in the air and catch it again. Right after it leaves your hand and before you catch it, which of the above plots represents the *v* vs. *t* graph for this motion? (Assume your y-axis is pointing up).



Clicker question 2



You toss a ball straight up in the air and catch it again. Right after it leaves your hand and before you catch it, which of the above plots represents the *v* vs. *t* graph for this motion? (Assume your y-axis is pointing up).

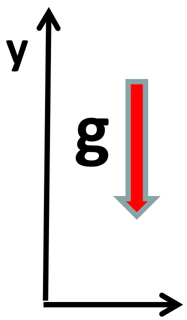
The ball has an initial velocity that is positive but diminishing as it slows. It stops at the top ($v = 0$), and then its velocity becomes negative and becomes more and more negative as it accelerates downward.

Freely Falling Objects

$$v(t) = v_0 + at \qquad (1)$$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2} \qquad (2)$$

$$v^2(t) = v_0^2 + 2a(x - x_0) \qquad (3)$$

if 

then $a = -g$

$$\begin{cases} v = v_0 - gt \\ y = y_0 + v_0 t - {gt^2}/{2} \\ v^2 = {v_0}^2 - 2g(y - y_0) \end{cases}$$

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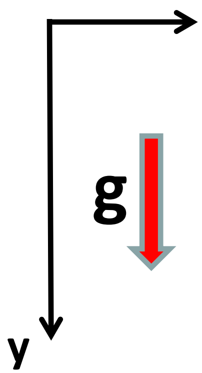
if 

then $a = g$

$$\begin{cases} v = v_0 + gt \\ y = y_0 + v_0 t + {gt^2}/{2} \\ v^2 = {v_0}^2 + 2g(y - y_0) \end{cases}$$

Question 3 Up in the Air I

You throw a ball upward with an initial speed of 10 m/s. Assuming that there is no air resistance, what is its speed when it returns to you?

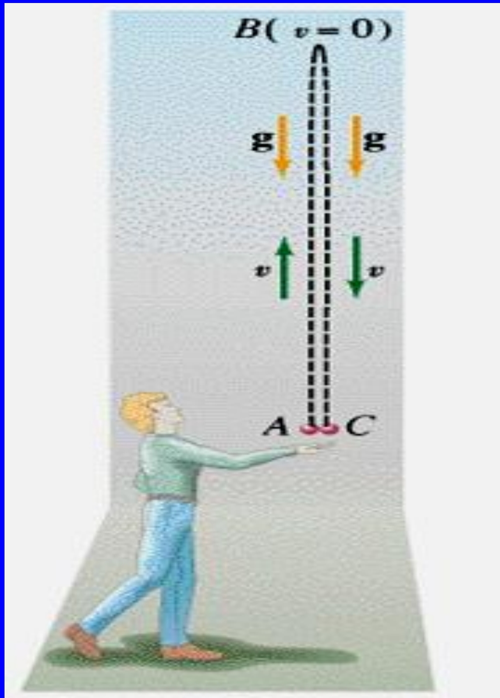
1) more than 10 m/s
2) 10 m/s
3) less than 10 m/s
4) zero
5) need more information

ConcepTest 2.10a Up in the Air I

You throw a ball upward with an initial speed of 10 m/s. Assuming that there is no air resistance, what is its speed when it returns to you?

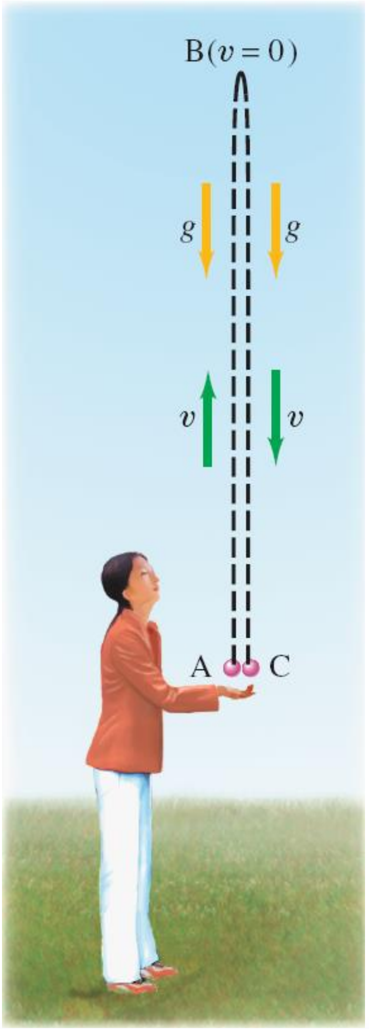
1) more than 10 m/s
2) 10 m/s
3) less than 10 m/s
4) zero
5) need more information

The ball is slowing down on the way up due to gravity. Eventually it stops. Then it accelerates downward due to gravity (again). Because $a = g$ on the way up and on the way down, the ball reaches the same speed when it gets back to you as it had when it left.



Example 2-16

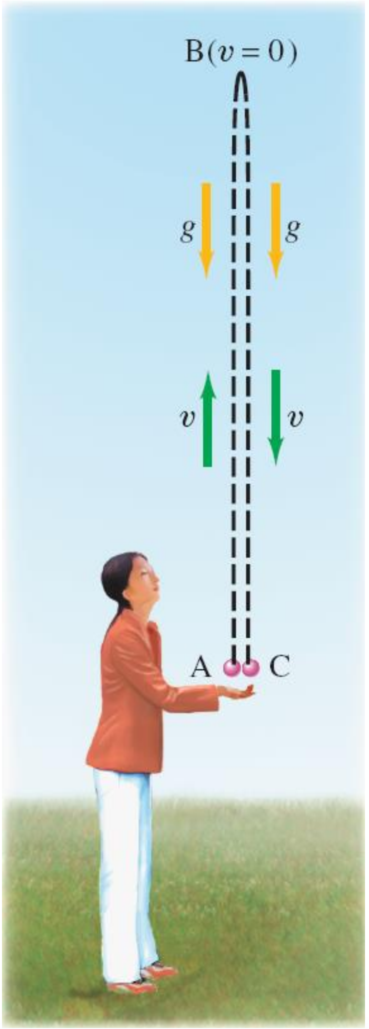
Freely Falling Objects

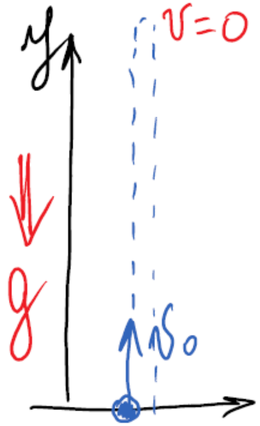


Example 2-16:
Ball thrown upward.

A person throws a ball upward into the air with an initial velocity of 10.0 m/s. Calculate (a) how high it goes, and (b) how long the ball is in the air before it comes back to the hand. Ignore air resistance.

Example 2-16(a)





Given: $v_0 = 10\ m/s$; $y_0 = 0$

Calculate how high it goes: y - ?

1. Choose a coord. system:
   y - upward,
   g - downward (always)
   so $a = -g$

$$\begin{cases} y = y_0 + v_0 t + \frac{at^2}{2} \\ v = v_0 + at \\ v^2 = v_0^2 + 2a(y - y_0) \end{cases}$$

$a = -g$

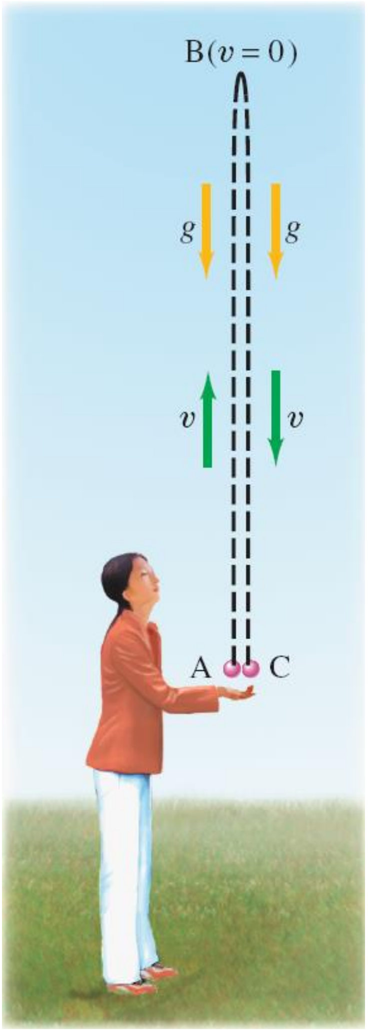
$$\begin{cases} y = y_0 + v_0 t - \frac{gt^2}{2} & \times \text{ (no } t \text{ info)} \\ v = v_0 - gt & \times \text{ (no } t \text{ info)} \\ \cancel{v^2}^{\,0} = v_0^2 - 2g(y - \cancel{y_0}^{\,0}) & \checkmark \end{cases}$$

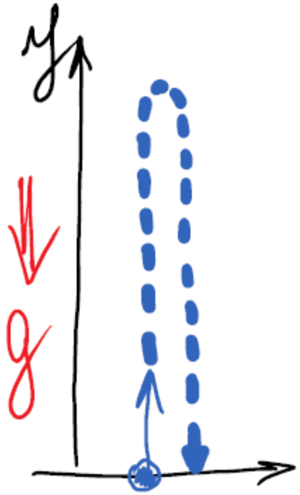
at max. hight, $v = 0$

$$0 = v_0^2 - 2 \cdot g \cdot y$$

$$y = \frac{v_0^2}{2g} = \frac{(10\ m/s)^2}{2 \cdot 9.8\ m/s^2} \approx \frac{100}{20}\ m = 5m$$

Example 2-16 (b)





initial and final points

- How long the ball is in the air?

$$\begin{aligned}
&1.\ y = y_0 + v_0 t - \frac{g t^2}{2} \quad \checkmark \\
&2.\ v = v_0 - g t \quad \checkmark \\
&3.\ v^2 = v_0^2 - 2g(y - y_0) \quad \times
\end{aligned}$$

both OK!!

- At the final point: $y = 0$

let's use eq-n 1.

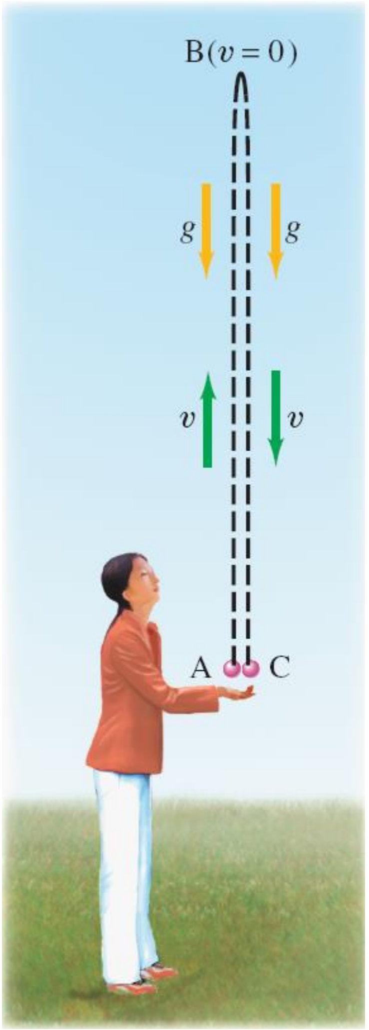
$$\cancel{y}^{0} = \cancel{y_0}^{0} + v_0 t - \frac{g t^2}{2}$$

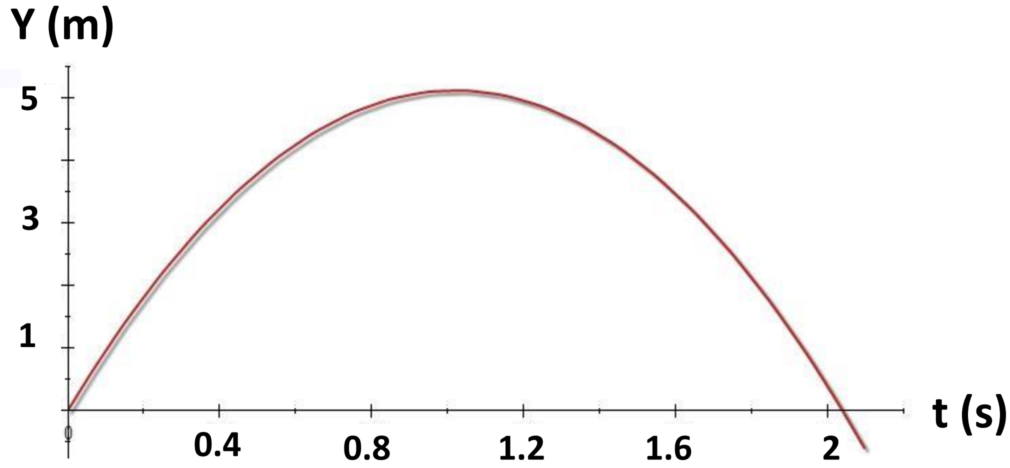
$$0 = t\left(v_0 - \frac{g t}{2}\right)$$ there are two solution.

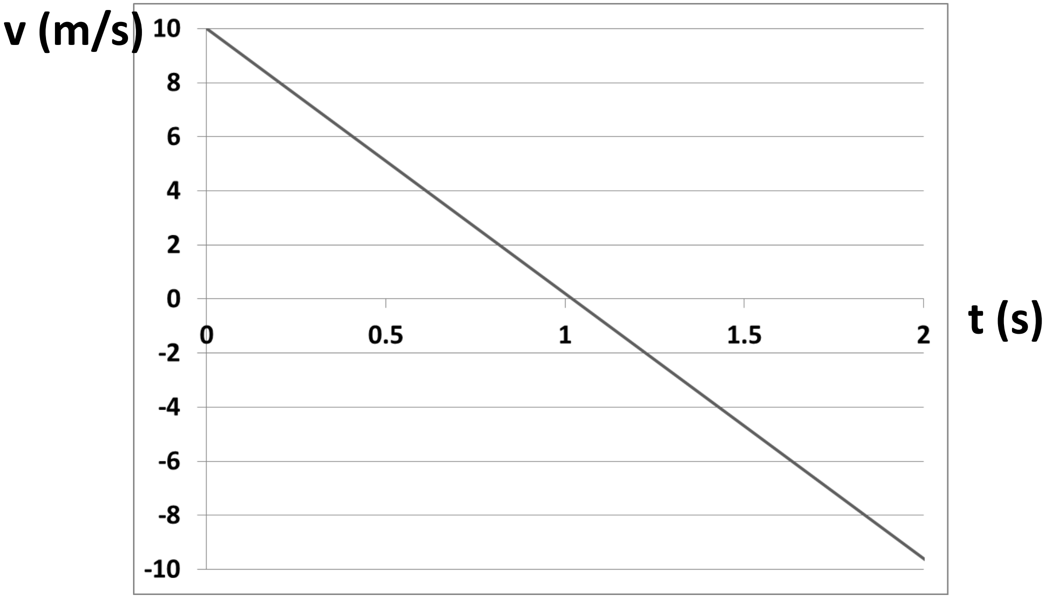
$$t_1 = 0; \quad v_0 - \frac{g t}{2} = 0$$

$$t_2 = \frac{2 v_0}{g} = \frac{2 \cdot 10\ m/s}{9.8\ m/s^2} \simeq 2\ s$$

Example 2-16 (b)







The End

See you on Monday.

HW2 is due to on Sunday (6pm)