Radioisotope Geochemistry

Learning objectives:

1) To describe what radioisotopes are and how they are different from stable isotopes.

2) To understand the 3 modes of decay and to be able to write equations for different types of decay.

3) To know how to use the decay equation, $N = N_0e^{-\lambda t}$, to find the half-lives of nuclides or to determine the age of sample materials.

4) To understand that different radioisotopes have unique half-lives and chemistry, making them suitable as tracers for different processes in the ocean.

5) To know how ^{14}C is used to determine ages/accumulation rates in sediment cores and what assumptions need to be made particularly those regarding initial ^{14}C content.

5) To be able to explain secular equilibrium and its application to determining particle flux in the upper ocean using ^{234}Th.

==MODE OF DECAY==

Classroom activities: provide the periodic table and ask students to help finding the equations for different nuclides.

* Write the equation for alpha decay of $^{254}_{100}Fm$.

Answer

$^{254}_{100}Fm \rightarrow {}^{250}_{98}Cf + {}^{4}_{2}He$

Explanation

In alpha decay a helium-4 nucleus is emitted, which has two protons and two neutrons. (Its symbol is $^{4}_{2}He$.) Thus the initial nucleus loses two protons -- its atomic number is reduced by 2. It loses 4 particles altogether and so its mass number is reduced by 4.

$^{254}_{100}Fm$ (fermium) decays to the element with atomic number 98 and mass number 250. The element number 98 is californium, chemical symbol Cf. So in addition to the helium-4 nucleus, we also get $^{250}_{98}Cf$.

* Write the equation for alpha decay of polonium-210.

Answer

$^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$

Explanation

The initial nucleus is polonium-210, with atomic number 84. After alpha decay we have a nucleus with atomic number 82 and mass number 206. This nucleus is lead, $^{206}_{82}Pb$.

* Write the equation for the beta decay of neptunium-239.

Answer

$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + e$

Explanation

In beta decay one neutron disappears and one proton is created. So the atomic number increases by 1. The mass number always remains the same in beta decay, because it is the sum of protons plus neutrons.

For this question, the initial nucleus is neptunium. The atomic number of neptunium is 93. It has 93 protons and 146 neutrons (239 - 93). Its symbol is $^{239}_{93}Np$. Hence the product nucleus has atomic number 94 and mass number 239. From the Table you find that plutonium is the element with mass number 94; its chemical symbol is Pu. So the products of the decay are $^{239}_{94}Pu$ plus an electron.

* Write the equation for beta decay of carbon-14.

Answer

$^{14}_{6}C \rightarrow ^{14}_{7}N + e$.

Explanation

The initial nucleus is carbon, atomic number 6. So the final nucleus has atomic number 7; it is nitrogen. So the final nucleus is $^{14}_{7}N$, which is the common isotope of nitrogen.

== FINDING DECAY CONSTANT AND HALF LIVES==

*** The activity of a radioisotope is found to decrease to 45% of its original value in 30 days.**

(a) What is the decay constant?

We are solving for the decay constant, λ. We know that the activity after t=30 days is now $0.45N_0$ of the original activity, N_0. Solving for λ:

$$N = N_0\, e^{-\lambda t}$$

$$N/N_0 = 0.45 = e^{-\lambda (30)}$$

$\therefore \lambda = 0.0266\ days^{-1}$

$= 38.3\ min^{-1}$

(b) What is the half-life?

By definition of half-life:

$t_{1/2} = \ln 2/\ \lambda\ =\ \ln 2/0.0266 = 26.06 days$

*** A sample of Se-83 registers 10^{12} disintegrations per second when first tested. What rate would you predict for this sample 3.5 hours later, if the half-life is 22.3 minutes?**

Solution:

210 min / 22.3 min = 9.42 half-lives (210 min is 3.5 hours)

$(1/2)^{9.42}$ = 0.00146 (the decimal fraction remaining)

10^{12} x 0.00146 = 1.46 x 10^9 disintegrations per second remaining

*** A certain isotope has a half-life of 20 years. If I had 2,000,000 of these nuclei in 1995, how many will I have in the year 2015? How many in 2035, 2055, 2075?**

Twenty years elapse from 1995 to 2015. Since the half-life is 20 years, the number of nuclei that decay in that period is half of the number that existed at the start. Hence 1,000,000 nuclei decay, and 1,000,000 nuclei remain in 2015.

(This reduction by half can be applied to numbers of nuclei, or equally well to total weight.)

The time from 2015 to 2035 is also 20 years. Hence, during that period the number of nuclei that decay is half of what there was at the beginning of that period , namely half of 1,000,000. So 500,000 nuclei decay in this 20-year period; 500,000 are left at the end, in 2035. Note that what happens between 2015 and 2035 is not calculated on the basis of what we have in 1995. It is calculated only on the basis of what we have in 2015.

In the same way, between 2035 and 2055, half of the initial number of nuclei decay. This initial number is 500,000. So 250,000 decay, and 250,000 remain. You can now see the progression. After another 20 years, the number is reduced again by half, to 125,000.

Year	Number
2015	1,000,000
2035	500,000
2055	250,000
2075	125,000

*** In a sample of radioactive waste there are 4 kg of ^{137}Cs. How much is there after 30 years has elapsed? after 60 years? Given that the half-life of Cesium-137 is 30 years.**

Thus, if I have 4 kg at first, I will have half, or 2 kg, at the end of a 30-year period.

When another 30 years goes by, the 2 kg is reduced by half again, and so I have 1 kg. After a total of 60 years, the initial 4 kg is reduced to 1 kg.

*** Strontium-90 is a dangerous radioactive isotope. Because of its similarity to calcium, it is easily absorbed into human bones. The half-life of strontium-90 is 28 years. If a certain amount is absorbed into the bones due to exposure to a nuclear explosion, what percentage will remain after (a) 84 years? (b) 100 years?**

Let P be the amount of strontium-90 at time t. Then

$$P = P_0 e^{kt}.$$

The half-life is 28 years, so when $t = 28$, $P = \frac{P_0}{2}$:

$$\frac{P_0}{2} = P_0 e^{28k}, \quad \frac{1}{2} = e^{28k}, \quad \ln\frac{1}{2} = \ln e^{28k}, \quad \ln\frac{1}{2} = 28k, \quad k = \frac{1}{28} \mathrm{l}$$

Therefore, $P = P_0 e^{(t \ln 0.5)/28}$.

When $t = 84$,

$$P = P_0 e^{(84 \ln 0.5)/28}.$$

The fraction of the initial amount is

$$\frac{P_0 e^{(84 \ln 0.5)/28}}{P_0} = e^{(84 \ln 0.5)/28} \approx 0.29003.$$

That is, 29.003% remains after 50 years.

When $t = 100$,

$$P = P_0 e^{(100 \ln 0.5)/28}.$$

The fraction of the initial amount is

$$\frac{P_0 e^{(100 \ln 0.5)/28}}{P_0} = e^{(100 \ln 0.5)/28} \approx 0.08412.$$

That is, 8.412% remains after 50 years.

==FINDING NUCLIDES' AGES==

* **Rn-222 has a half-life of 3.82 days. How long before only 1/16 of the original sample remains?**

Solution:

recognize 1/16 as a fraction associated with 4 half-lives (from $(1/2)^4 = 1/16$)

3.82 days x 4 = 15.3 days

* **The half-life of ^{14}C is 5730 years. Living things are made of carbon, and a small fraction of that carbon is ^{14}C. Suppose that in the shinbone of a living human being I ordinarily find 8 micrograms of ^{14}C. Now I find a fossil of an ancient human, and in the shinbone there are 4 micrograms of this isotope. When did this human being live? What assumptions are**

necessary to make this calculations?

(The assumption here is that a living thing is built out of carbon from the environment, and keeps building itself as long as it is alive. Therefore it always has the same fraction of ^{14}C in its body as there is in the carbon of the environment. But when it dies it stops taking anything in from the environment -- such as nutrients -- and whatever radioactive carbon it has decays.)

→6000 years ago.

Since the half-life of ^{14}C is 6000 years, it would take 6000 years for the quantity of this isotope in the shinbone to be reduced by half. Assuming that the human had 8 micrograms when it died, it would have 4 micrograms 6000 years later. If we see 4 micrograms at the present time, it must have been 6000 years ago when it died.

Reasoning backward like this is the way observation of isotopes today can be used to deduce when something occurred in the past.

* **How long will it take for a 64.0 g sample of Rn-222 (half-life = 3.8235 days) to decay to 8.00 g?**

Solution:

8.00 / 64.0 = 0.125 (the decimal fraction remaining)

$(1/2)^n = 0.125$

by experience, n = 3 (remember that 0.125 is 1/8)

3.8235 x 3 = 11.4705 days

* **The radioisotope potassium-40 decays to argon-40 by positron emission with a half life of 1.27 x 10^9 yr. A sample of moon rock was found to contain 78 argon-40 atoms for every 22 potassium-40 atoms. The age of the rock is . . .**

Solution:

Assume the sample was 100% K-40 at start. In the present day, the sample contains 78% Ar-40 and 22% K-40. We will use 0.22, the decimal percent of

K-40 remaining:

$(1/2)^n = 0.22$

where n is the number of half-lives.

n log 0.5 = log 0.22

n = 2.18

What is the total elapsed time?

$(2.18)\ (1.27 \times 10^9) = 2.77 \times 10^9$ yrs

* **What is the age of a rock in which the mass ratio of Ar-40 to K-40 is 3.8? K-40 decays to Ar-40 with a half-life of 1.27×10^9 yr.**

Solution:

Since, the sample is 3.8 parts by mass Ar and 1 part K, the orginal sample contained 4.8 parts K and zero parts Ar.

What is the decimal amount of K-40 that remains?

1 part divided by 4.8 parts = 0.20833

How many half-lives are required to reach 0.20833 remaining?

$(1/2)^n = 0.20833$

where n is the number of half-lives.

n log 0.5 = log 0.20833

n = 2.263

What is the total elapsed time?

$(2.263)\ (1.27 \times 10^9) = 2.87 \times 10^9$ yrs

* **Scientists dating a fossil estimate that 20% of the original amount of**

carbon-14 is present. Recalling that the half-life is 5730 years, approximately how old is the fossil?

Let P be the amount of carbon-14 present at time t, so

$$P = P_0 e^{kt}.$$

The half-life is 5730, so $P = \frac{1}{2} P_0$ when $t = 5730$:

$$\frac{1}{2} P_0 = P_0 e^{5730k}, \quad \frac{1}{2} = e^{5730k}, \quad \ln \frac{1}{2} = \ln e^{5730k}, \quad \ln \frac{1}{2} = 5730k,$$

Therefore, $P = P_0 e^{(t \ln 0.5)/5730}$.

Since 20% of the original amount is present, $P = 0.2 P_0$:

$$0.2 P_0 = P_0 e^{(t \ln 0.5)/5730}, \quad 0.2 = e^{(t \ln 0.5)/5730}, \ln 0.2 = \ln e^{(t \ln 0.5)/5730}$$

$$t = \frac{5730 \ln 0.2}{\ln 0.5} \approx 13304.64798 \text{ years.} \quad \square$$