Chapter 13:
RC & RL Circuits

Instructor: Jean-François MILLITHALER

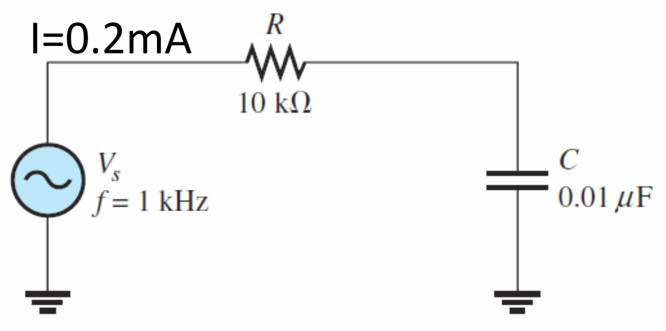
http://faculty.uml.edu/JeanFrancois_Millithaler/FunElec/Spring2017

Impedance & Admittance

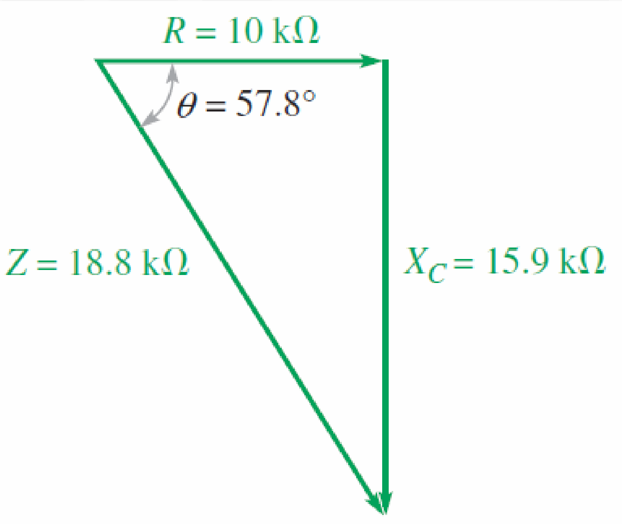
Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

RC Circuit

- Determine the source voltage and the phase angle.
- Draw the impedance triangle.



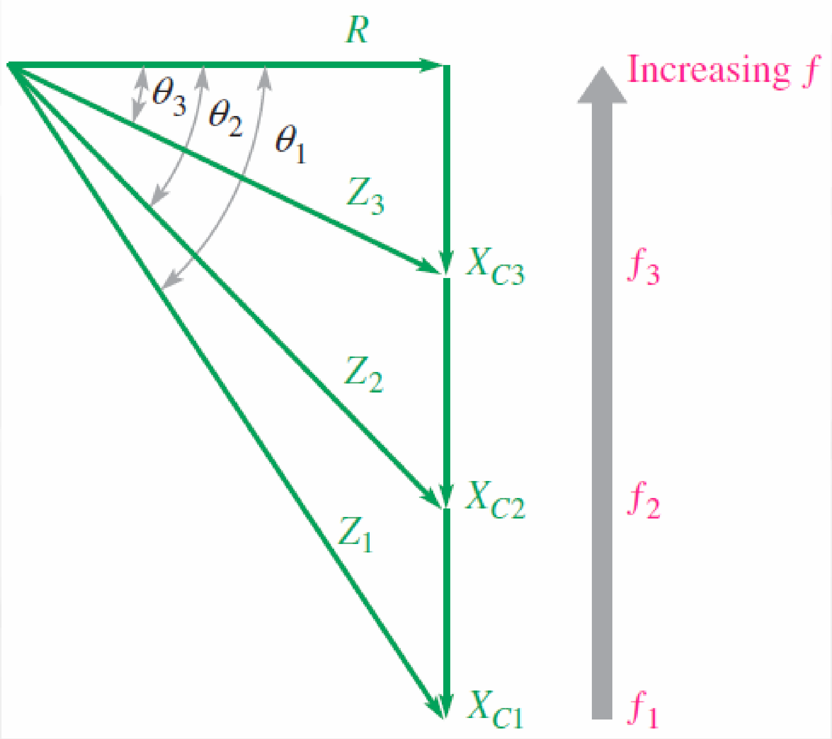
- $Z = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} = 10^4 - j\frac{1}{2\pi * 10^3 * 10^{-8}} = 10^4 - j(15.9 * 10^3)$
- $Z = A + jB \quad A = 10\text{k}\Omega \qquad B = 15.9\text{k}\Omega$
- $Z = \sqrt{A^2 + B^2} = 18.8\text{k}\Omega$
- $\theta = tan^{-1}\left(\frac{-15.9\text{ k}\Omega}{10\text{k}\Omega}\right) = 57.8°$
- $V_S = IZ = 0.2 * 18.8 = 3.76$ V



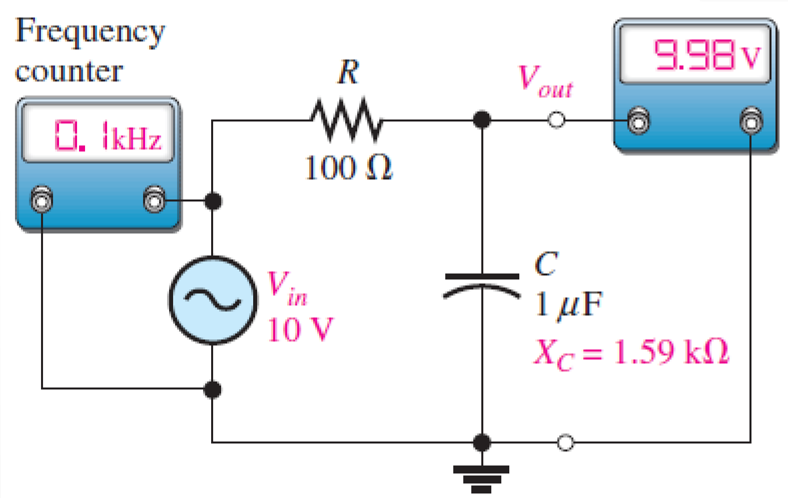
Variation of phase angle with frequency

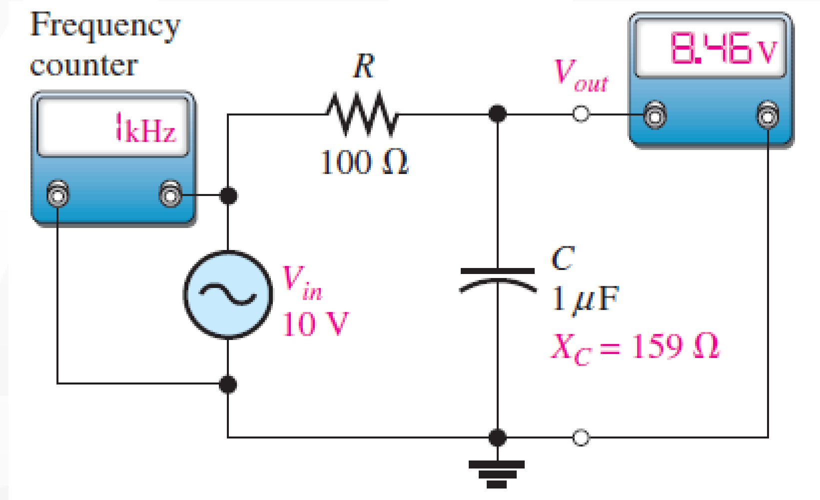
- Phasor diagrams that have reactance phasors can only be drawn for a single frequency because $X_C = \frac{1}{\omega C}$ is a function of frequency.

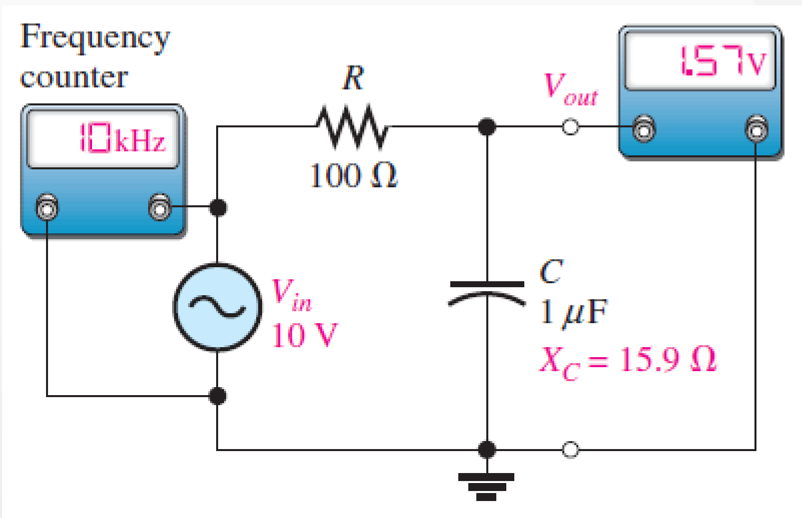
- As frequency changes, the impedance triangle for an *RC* circuit changes as illustrated here because X_C decreases with increasing f. This determines the *frequency response* of RC circuits.

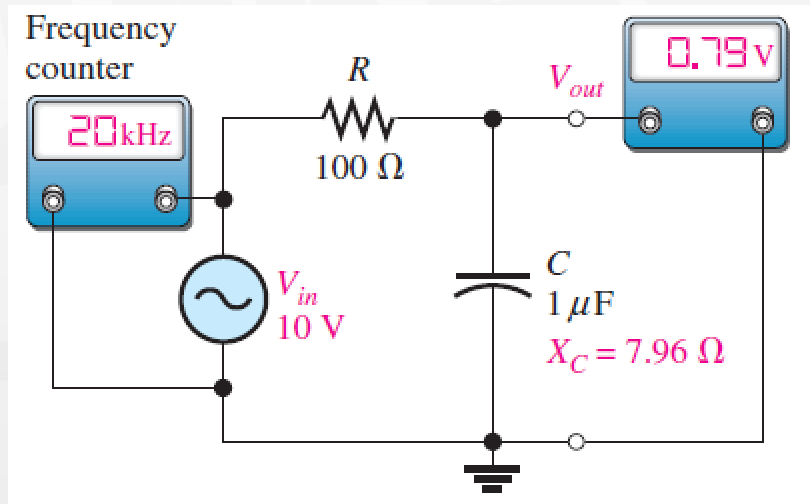


Frequency response



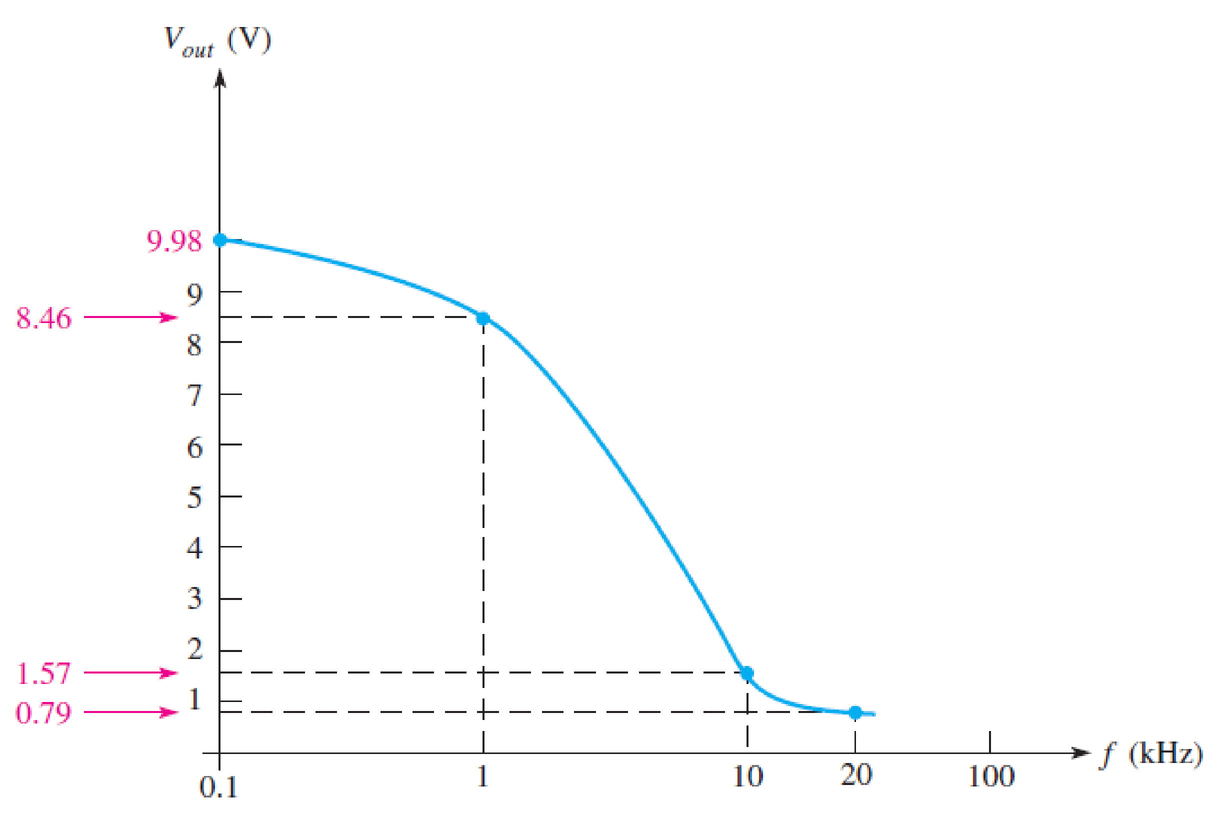






Frequency response

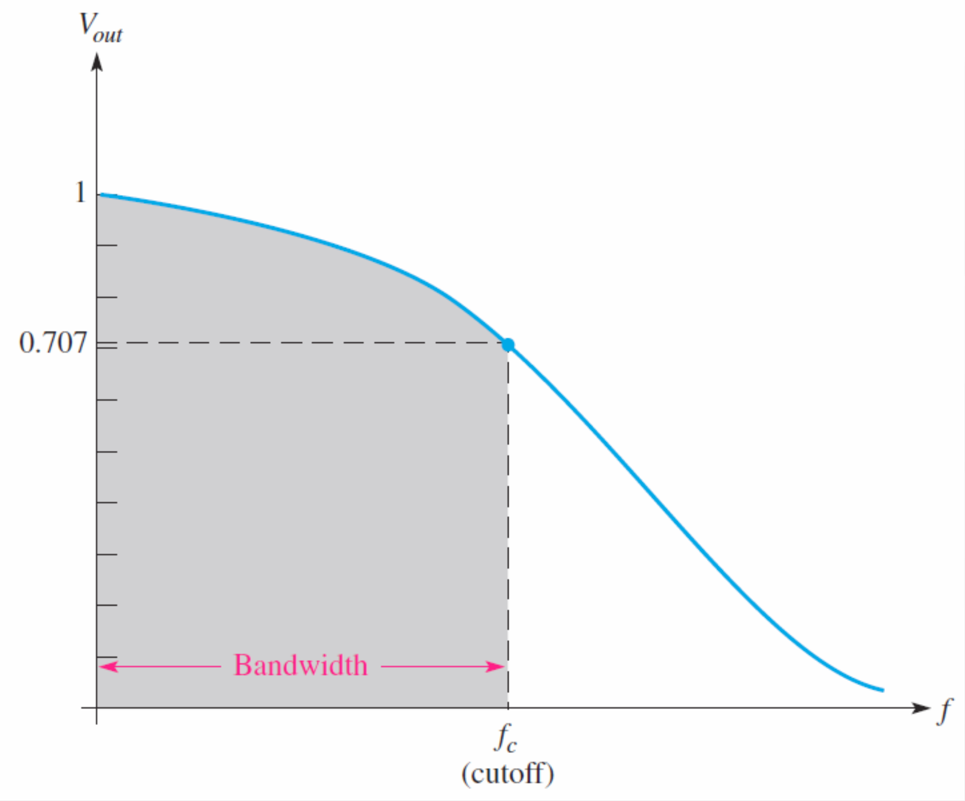
- **Frequency response** of the low-pass *RC* circuit



Frequency response

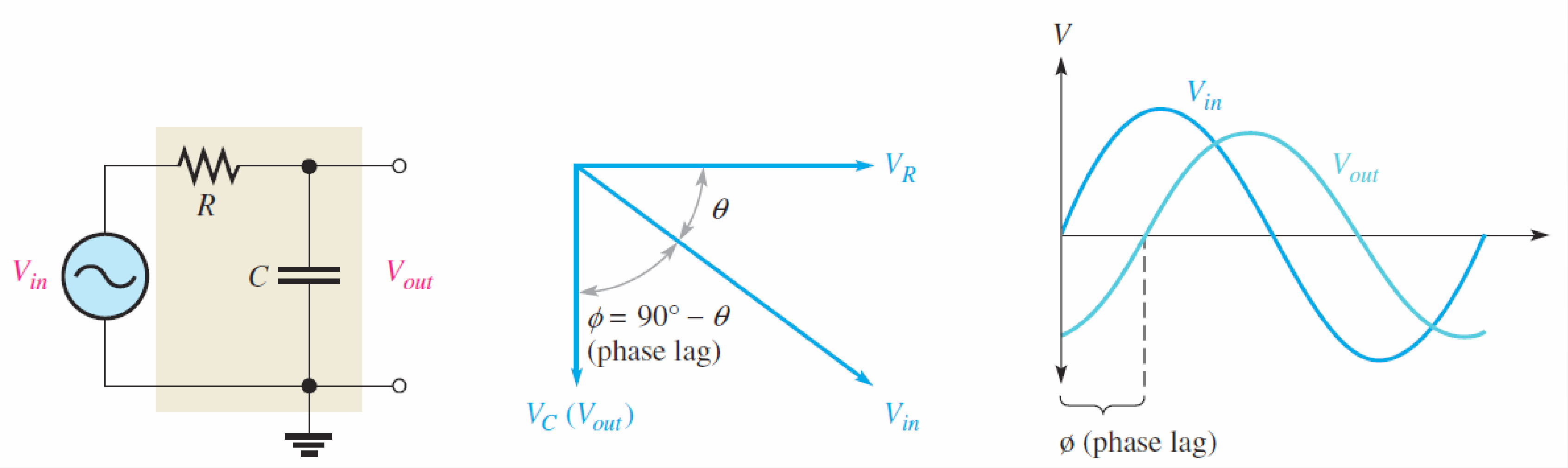
Cutoff Frequency

$$f_C = \frac{1}{2\pi RC}$$



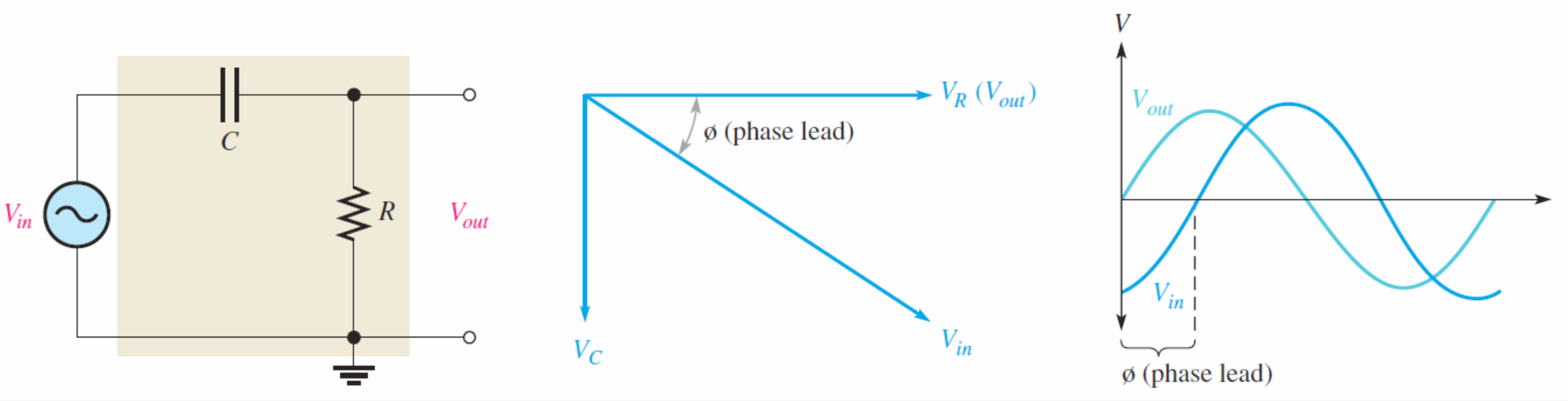
Applications

- For a given frequency, a series *RC* circuit can be used to produce a phase lag by a specific amount between an input voltage and an output by taking the output across the capacitor. This circuit is also a basic *low-pass filter*, a circuit that passes low frequencies and rejects all others.



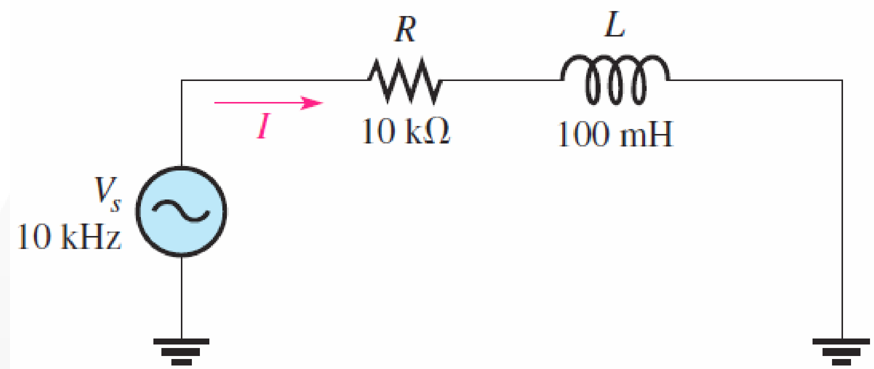
Applications

- Reversing the components in the previous circuit produces a circuit that is a basic lead network. This circuit is also a basic *high-pass filter*, a circuit that passes high frequencies and rejects all others. This filter passes high frequencies down to a frequency called the *cutoff* frequency.



RL Circuit

- The current is 200 μA.
  Determine the source voltage.



- $2\pi f L = 2\pi(10\text{ kHz})(100\text{ mH}) = 6.28\text{ k}\Omega$

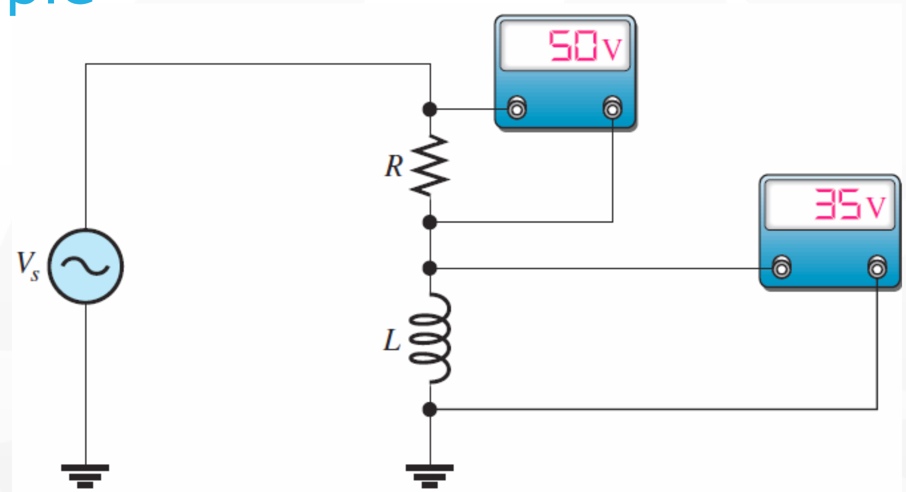
- The impedance is
- $|Z| = \sqrt{(10\text{ k}\Omega)^2 + (6.28\text{ k}\Omega)^2} = 11.8\text{ k}\Omega$
- $\theta = tan^{-1}\left(\frac{6.28\text{ k}\Omega}{10\text{ k}\Omega}\right) = 32.1°$

- Applying Ohm's law yields
- $V_S = IZ = (200\text{ μA})(11.8\text{ k}\Omega) = 2.36$ V

Phase Relationships of the Current and Voltages

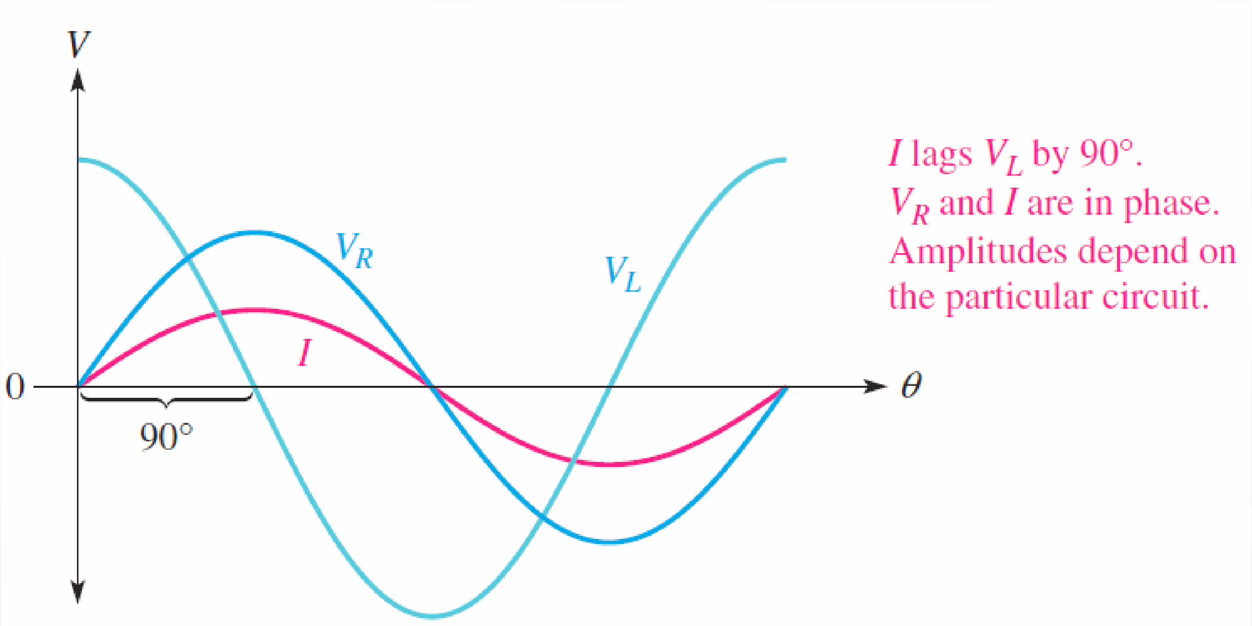
Example

- Determine the source voltage and the phase angle



- The source voltage is the phasor sum of V_R and V_L.
- $V_S = \sqrt{{V_R}^2 + {V_L}^2} = \sqrt{50^2 + 35^2} = 61$ V
- The phase angle between the resistor voltage and the source voltage is
- $\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{35}{50}\right) = 35^{o}$

Phase Relationships of the Current and Voltages



$$V_S = \sqrt{{V_R}^2 + {V_L}^2} \qquad \theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

Variation of Impedance and Phase Angle with Frequency

- Phasor diagrams that have reactance phasors can only be drawn for a single frequency because *X* is a function of frequency.

- As frequency changes, the impedance triangle for an *RL* circuit changes as illustrated here because $j\omega L$ increases with increasing *f*.
- This determines the *frequency response* of *RL* circuits.