Chapter 13: RC & RL Circuits

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## Impedance & Admittance

<table>
<thead>
<tr>
<th>Element</th>
<th>Impedance</th>
<th>Admittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$Z = R$</td>
<td>$Y = \frac{1}{R}$</td>
</tr>
<tr>
<td>$L$</td>
<td>$Z = j\omega L$</td>
<td>$Y = \frac{1}{j\omega L}$</td>
</tr>
<tr>
<td>$C$</td>
<td>$Z = \frac{1}{j\omega C}$</td>
<td>$Y = j\omega C$</td>
</tr>
</tbody>
</table>
Determine the source voltage and the phase angle.

- **Draw the impedance triangle.**

\[
Z = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} = 10^4 - j\frac{1}{2\pi \times 10^3 \times 10^{-8}} = 10^4 - j(15.9 \times 10^3)
\]

- \(Z = A + jB\) \(A = 10\text{k}\Omega\) \(B = 15.9\text{k}\Omega\)

- \(Z = \sqrt{A^2 + B^2} = 18.8\text{k}\Omega\)

- \(\theta = \tan^{-1}\left(\frac{-15.9\text{k}\Omega}{10\text{k}\Omega}\right) = 57.8^\circ\)

- \(V_S = IZ = 0.2 \times 18.8 = 3.76\ V\)
Phasor diagrams that have reactance phasors can only be drawn for a single frequency because \( X_C = \frac{1}{\omega C} \) is a function of frequency.

As frequency changes, the impedance triangle for an RC circuit changes as illustrated here because \( X_C \) decreases with increasing f. This determines the frequency response of RC circuits.
Frequency response
Frequency response of the low-pass \( RC \) circuit
Cutoff Frequency

\[ f_c = \frac{1}{2\pi RC} \]
For a given frequency, a series RC circuit can be used to produce a phase lag by a specific amount between an input voltage and an output by taking the output across the capacitor. This circuit is also a basic low-pass filter, a circuit that passes low frequencies and rejects all others.
Reversing the components in the previous circuit produces a circuit that is a basic lead network. This circuit is also a basic **high-pass filter**, a circuit that passes high frequencies and rejects all others. This filter passes high frequencies down to a frequency called the **cutoff** frequency.
The current is 200 μA. Determine the source voltage.

\[ 2\pi fL = 2\pi (10 \text{ kHz})(100 \text{ mH}) = 6.28 \text{ kΩ} \]

The impedance is

\[ |Z| = \sqrt{(10 \text{ kΩ})^2 + (6.28 \text{ kΩ})^2} = 11.8 \text{ kΩ} \]

\[ \theta = \tan^{-1}\left(\frac{6.28 \text{ kΩ}}{10 \text{ kΩ}}\right) = 32.1^\circ \]

Applying Ohm’s law yields

\[ V_S = IZ = (200 \text{ μA})(11.8 \text{ kΩ}) = 2.36 \text{ V} \]
Example

- Determine the source voltage and the phase angle

- The source voltage is the phasor sum of $V_R$ and $V_L$.

- $V_S = \sqrt{V_R^2 + V_L^2} = \sqrt{50^2 + 35^2} = 61$ V

- The phase angle between the resistor voltage and the source voltage is

- $\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{35}{50}\right) = 35^\circ$
Phase Relationships of the Current and Voltages

\[ V_S = \sqrt{V_R^2 + V_L^2} \]
\[ \theta = \tan^{-1} \left( \frac{V_L}{V_R} \right) \]

I lags \( V_L \) by 90°.
\( V_R \) and I are in phase.
Amplitudes depend on the particular circuit.
Phasor diagrams that have reactance phasors can only be drawn for a single frequency because $X$ is a function of frequency.

As frequency changes, the impedance triangle for an $RL$ circuit changes as illustrated here because $j\omega L$ increases with increasing $f$.

This determines the *frequency response* of $RL$ circuits.