Chapter 16:
Mutual Inductance

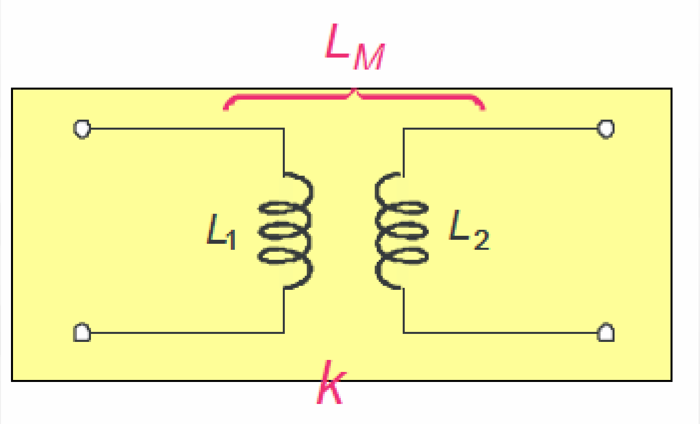
Instructor: Jean-François MILLITHALER

http://faculty.uml.edu/JeanFrancois_Millithaler/FunElec/Spring2017

Mutual Inductance

- When two coils are placed close to each other, a changing flux in one coil will cause an induced voltage in the second coil. The coils are said to have **mutual inductance** (L_M), which can either add or subtract from the total inductance depending on if the fields are aiding or opposing.

- The coefficient of coupling is a measure of how well the coils are linked; it is a number between 0 and 1.



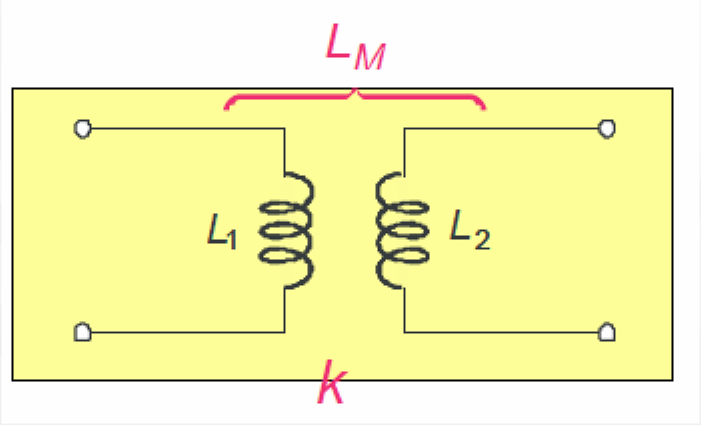
Mutual Inductance

- The formula for mutual inductance is

$$L_M = k\sqrt{L_1 L_2}$$

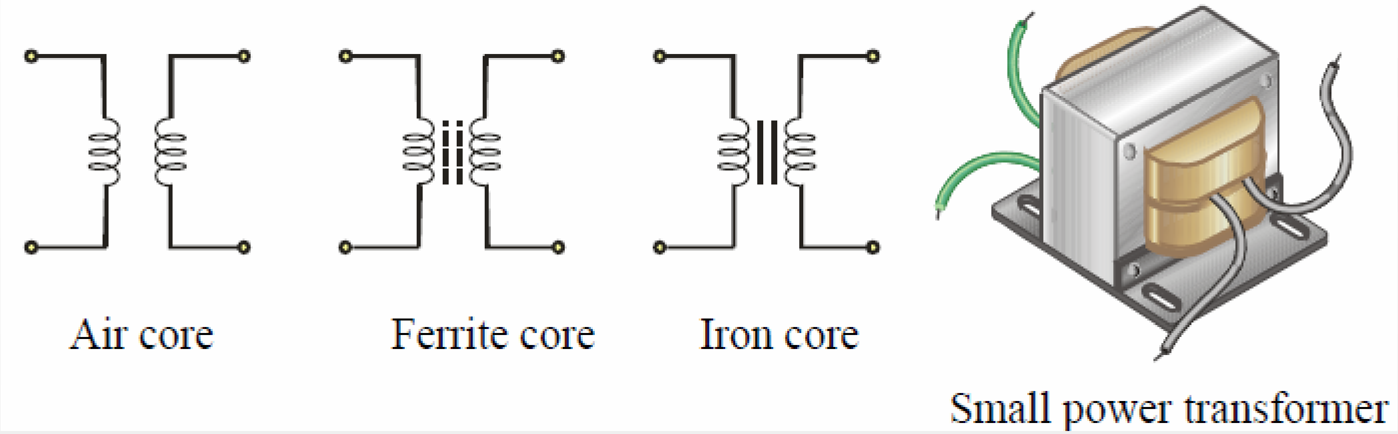
- k = the coefficient of coupling (dimensionless)
- L_1, L_2 = inductance of each coil (H)

- The coefficient of coupling depends on factors such as the orientation of the coils to each other, their proximity, and if they are on a common core.



Basic Transfomer

- The basic transformer is formed from two coils that are usually wound on a common core to provide a path for the magnetic field lines. Schematic symbols indicate the type of core



Turn ratio

- A useful parameter for ideal transformers is the turns ratio defined as

$$n = \frac{N_{sec}}{N_{pri}}$$

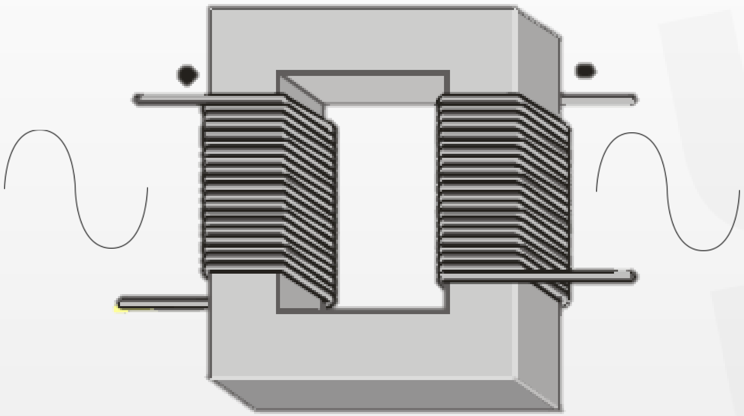
- N_{sec} = number of secondary windings
- N_{pri} = number of primary windings

- Most transformers are not marked with turns ratio, however it is a useful parameter for understanding transformer operation.

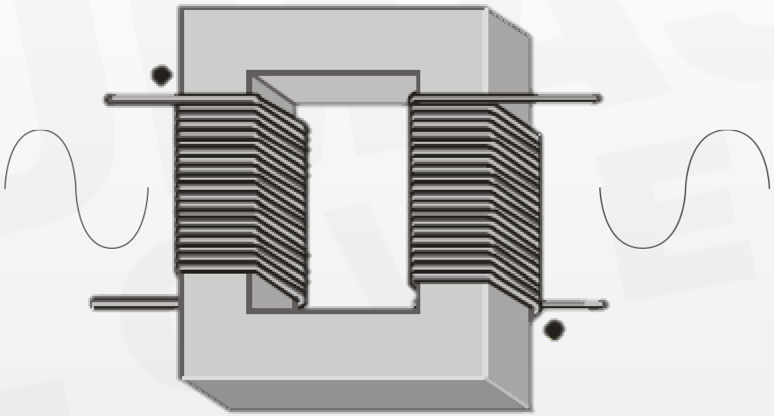
- <u>Example:</u> A transformer has 800 turns on the primary and a turns ratio of 0.25. How many turns are on the secondary?

Direction of windings

- The direction of the windings determines the polarity of the voltage across the secondary winding with respect to the voltage across the primary. Phase dots are sometimes used to indicate polarities.



In Phase



Out of Phase

Step-up and step-down transformers

- In a **step-up transformer**, the secondary voltage is greater than the primary voltage and *n* > 1.
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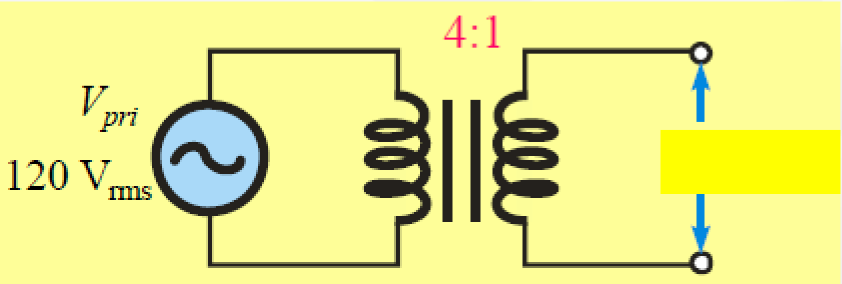
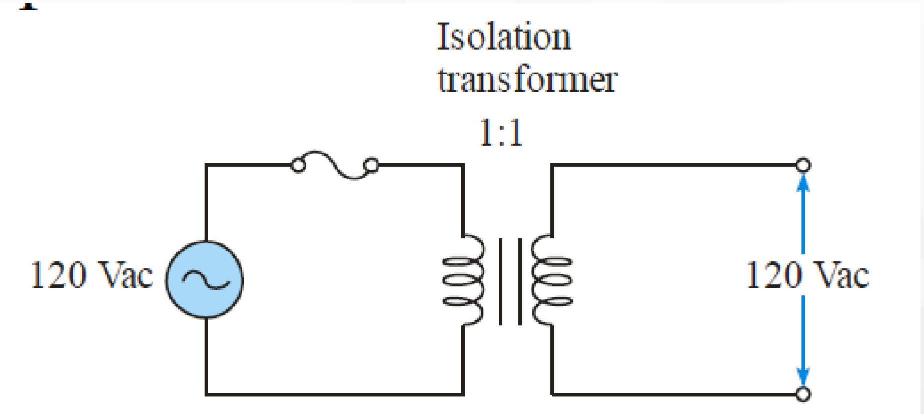
- In a **step-down transformer**, the secondary voltage is less than the primary voltage and *n* < 1.

- Example: What is the secondary voltage?



Isolation Transformers

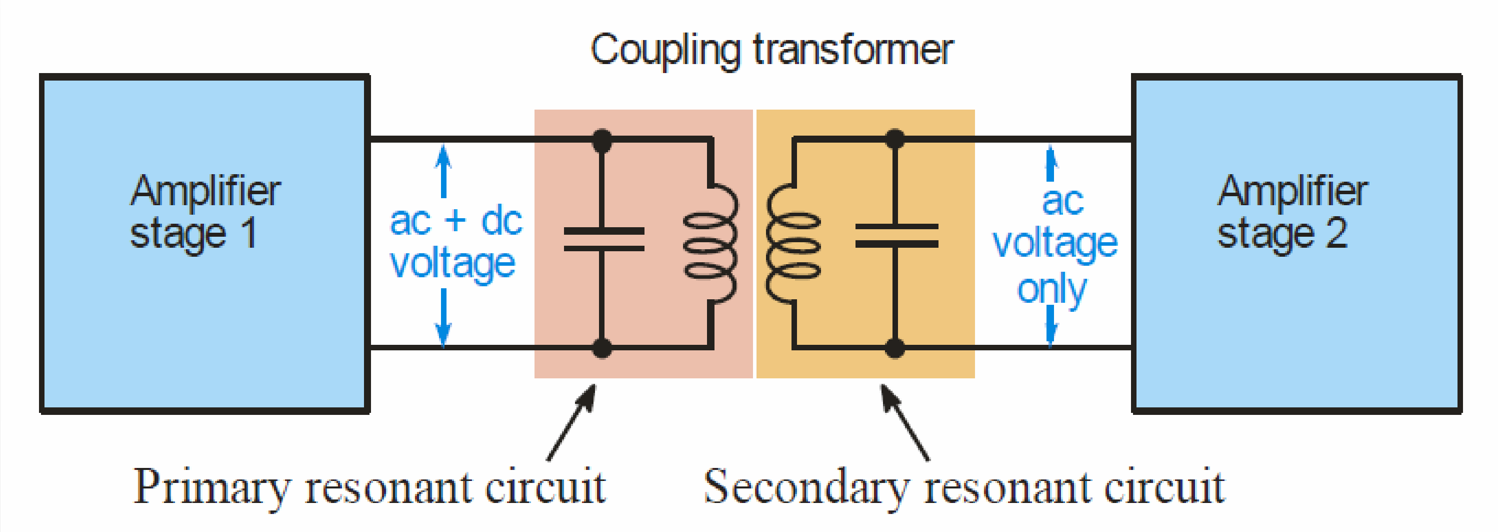
- A special transformer with a turns ratio of 1 is called an **isolation transformer**. Because the turns ratio is 1, the secondary voltage is the same as the primary voltage, hence ac is passed from one circuit to another.



- The isolation transformer breaks the dc path between two circuits while maintaining the ac path. The dc is blocked by the transformer, because magnetic flux does not change with dc.

Coupling Transformers

- Coupling transformers are used to pass a higher frequency signal from one stage to another. Because they are high frequency transformers, they typically are configured with a resonant circuit on the primary and the secondary. Some specialty isolation amplifiers use transformer coupling to isolate power.



Current

- Transformers cannot increase the applied power. If the secondary voltage is higher than the primary voltage, then the secondary current must be lower than the primary current and vice-versa.

- The ideal transformer turns ratio equation for current is

$$n = \frac{I_{pri}}{I_{sec}}$$

- Notice that the primary current is in the numerator

Power

- The *ideal* transformer does not dissipate power. Power delivered from the source is passed on to the load by the transformer. This important idea can be summarized as

$$P_{pri} = P_{sec}$$

$$V_{pri} I_{pri} = V_{sec} I_{sec}$$

$$\frac{V_{sec}}{V_{pri}} = \frac{I_{pri}}{I_{sec}}$$

- These last ratios are, of course, the turns ratio, n.

Reflected Resistance

- A transformer changes both the voltage and current on the primary side to different values on the secondary side. This makes a load resistance appear to have a different value on the primary side.
- From Ohm's law, $R_{pri} = \frac{V_{pri}}{I_{pri}}$ and $R_L = \frac{V_{sec}}{I_{sec}}$
- Taking the ratio of R_{pri} to R_L,

$$\frac{R_{pri}}{R_L} = \left(\frac{V_{pri}}{V_{sec}}\right)\left(\frac{I_{sec}}{I_{pri}}\right) = \left(\frac{1}{\text{n}}\right)\left(\frac{1}{\text{n}}\right) = \frac{1}{n^2}$$

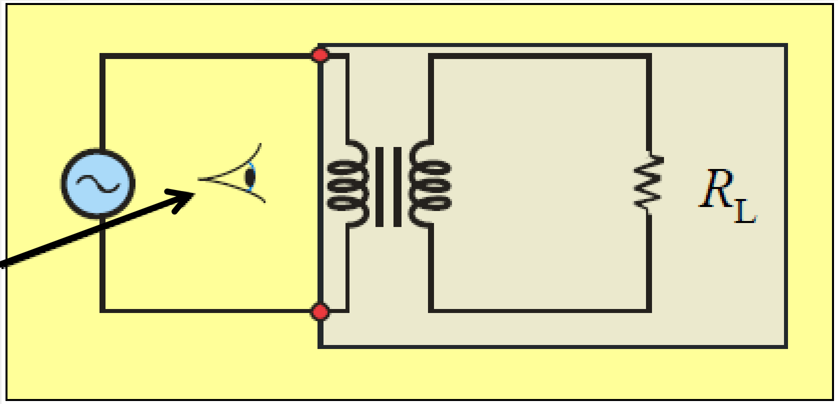
Reflected Resistance

- The resistance "seen" on the primary side is called the **reflected resistance**.

$$R_{pri} = \left(\frac{1}{n}\right)^2 R_L$$

- If you "look" into the primary side of the circuit, you see an effective load that is changed by the reciprocal of the turns ratio squared.

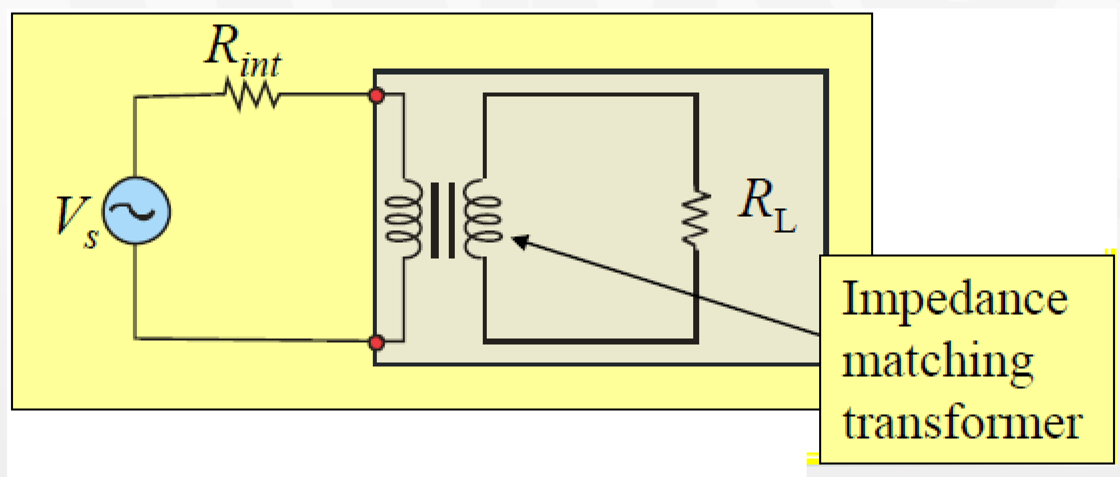
- You see the primary side resistance, so the load resistance is effectively changed.



Impedance matching

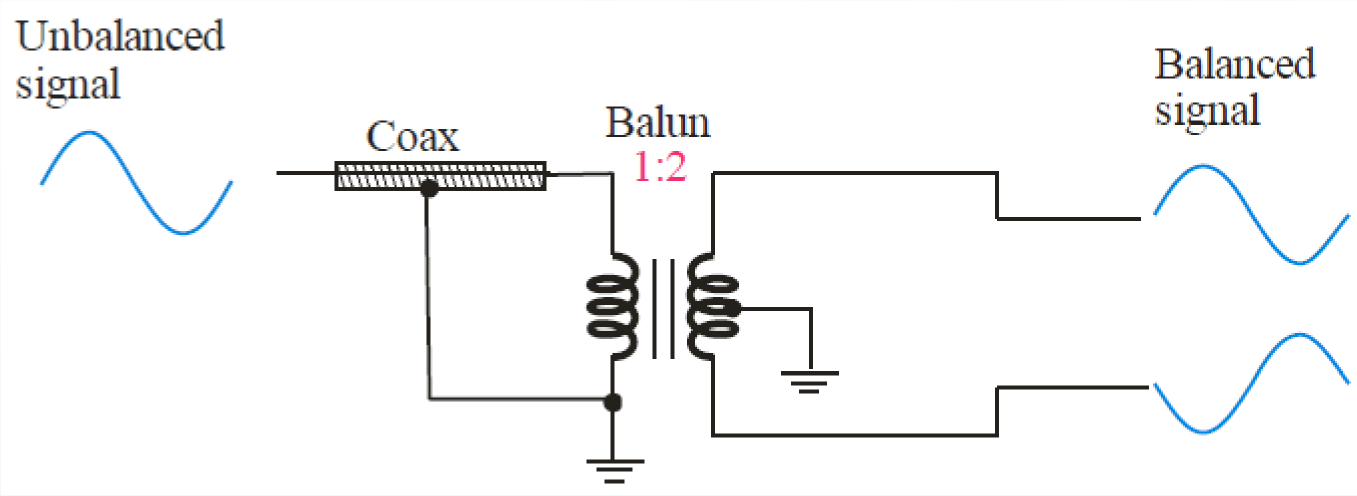
- The word *impedance* is used in ac work to take into account resistance and reactance effects. To match a load resistance to the internal source resistance (and hence transfer maximum power to the load), a special impedance matching transformer is used.

- Impedance matching transformers are designed for a wider range of frequencies than power transformers, hence tend to be not ideal.



Impedance matching

- The balun is a specialized transformer to match a balanced line to an unbalanced line and vice-versa (hence the name *balun*). A *balanced signal* is composed of two equal-amplitude signals that are 180º out-of-phase with each other. An *unbalanced signal* is one that is referenced to ground. In the illustration, an unbalanced signal is converted to a balanced signal by the balun transformer.



Non-ideal transformers

- An ideal transformer has no power loss; all power applied to the primary is all delivered to the load. Actual transformers depart from this ideal model. Some loss mechanisms are:

- **Winding resistance** (causing power to be dissipated in the windings.)
- **Hysteresis loss** (due to the continuous reversal of the magnetic field.)
- **Core losses** due to circulating current in the core (eddy currents).
- **Flux leakage** flux from the primary that does not link to the secondary
- **Winding capacitance** that has a bypassing effect for the windings.

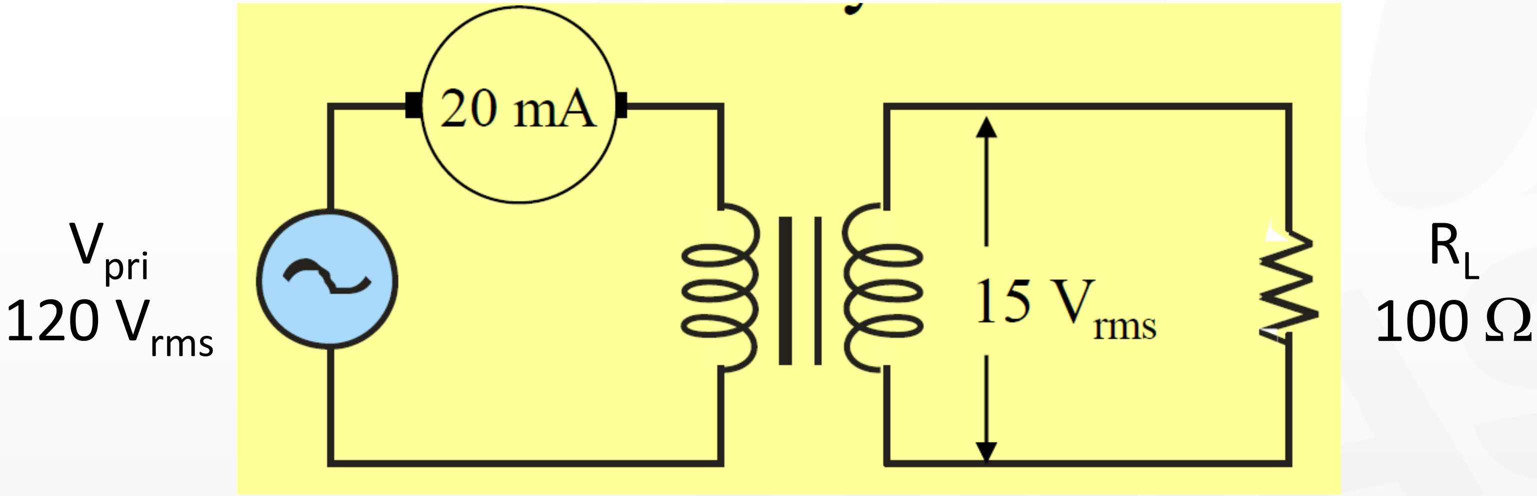
Transformer efficiency

- The efficiency of a transformer is the ratio of power delivered to the load (P_{out}) to the power delivered to the primary (P_{in}). Than is

$$\eta = \left(\frac{P_{out}}{P_{in}}\right) 100\%$$

Transformer efficiency

- What is the efficiency of the transformer?

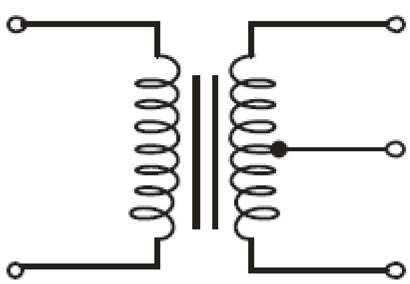


$$\eta = \left(\frac{P_{out}}{P_{in}}\right) 100\% =$$

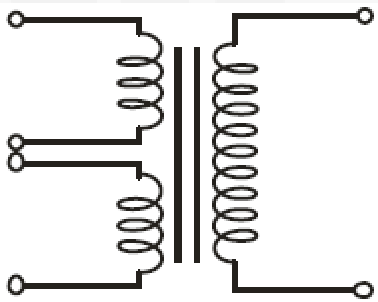
$$\left(\frac{\frac{{V_L}^2}{R_L}}{(V_{pri})(I_{pri})}\right) 100\% = \left(\frac{\frac{15^2}{100}}{120 * 0.02}\right) 100\% = 94\%$$

Tapped and multiple-winding transformers

- Frequently, it is useful to tap a transformer to allow for a different reference or to achieve different voltage ratings, either on the primary side or the secondary side.
- Multiple windings can be on either the primary or secondary side. One application for multiple windings is to connect to either 120 V or 240 V operation.

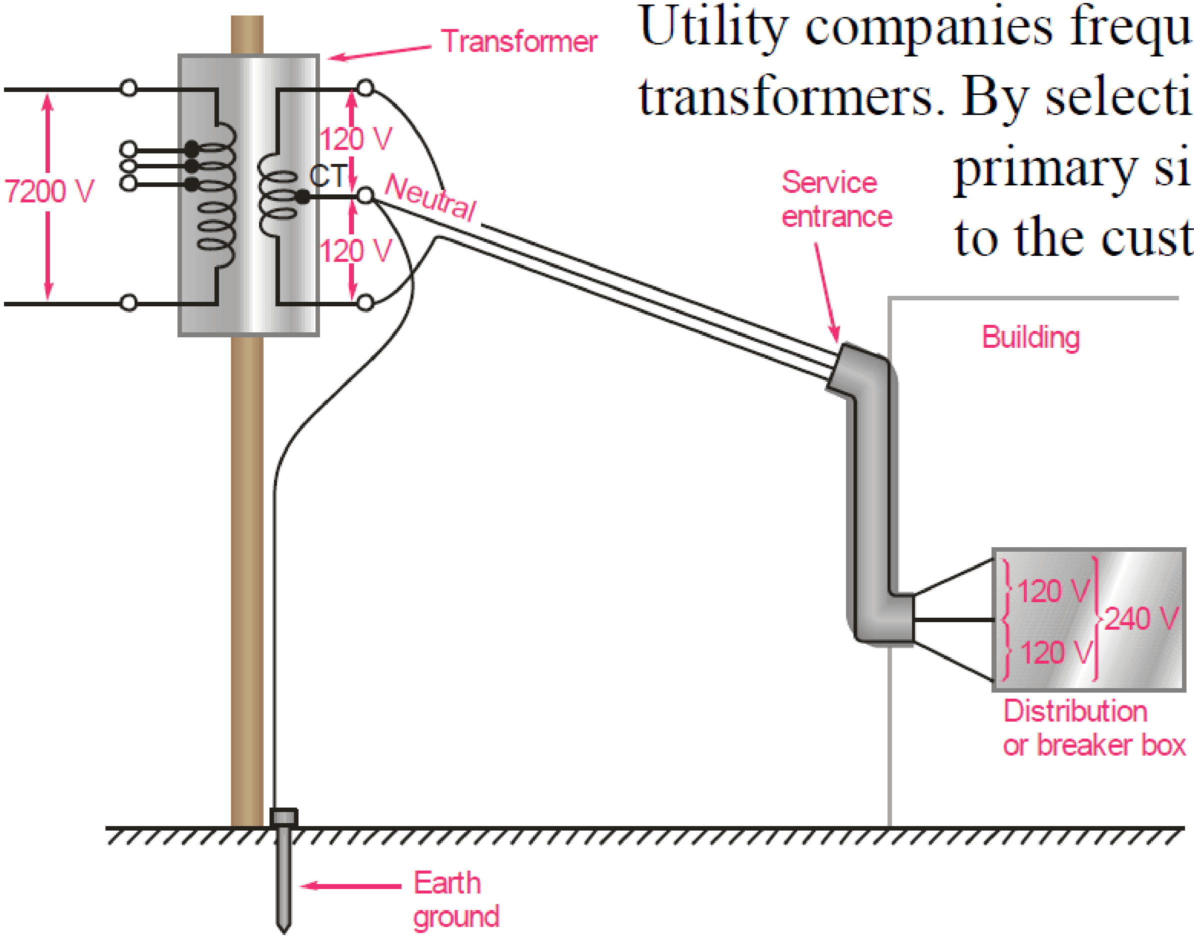


Secondary with center-tap



Primary with multiple-windings

Tapped and multiple-winding transformers



Utility companies frequently use multiple-tapped transformers. By selecting different taps, on the primary side, the voltage delivered to the customer can be adjusted.

The center-tapped secondary allows household wiring to select either 120 V or 240 V, depending on the circuit.