Chapter 6:
Series-Parallel Circuits

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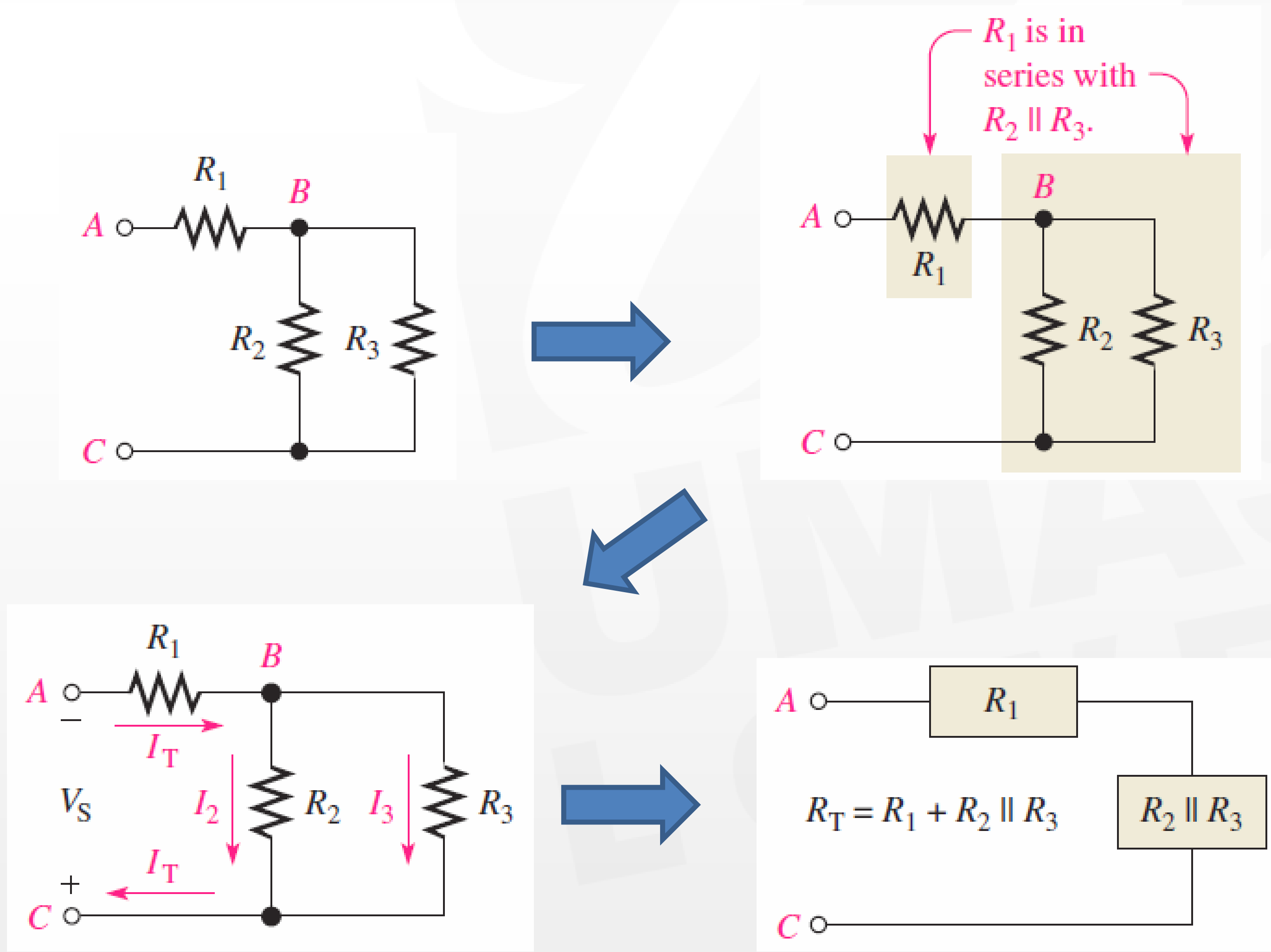
Identifying series-parallel relationships

- Most practical circuits have combinations of series and parallel components.
- Components that are connected in series will share a common path.
- Components that are connected in parallel will be connected across the same two nodes.

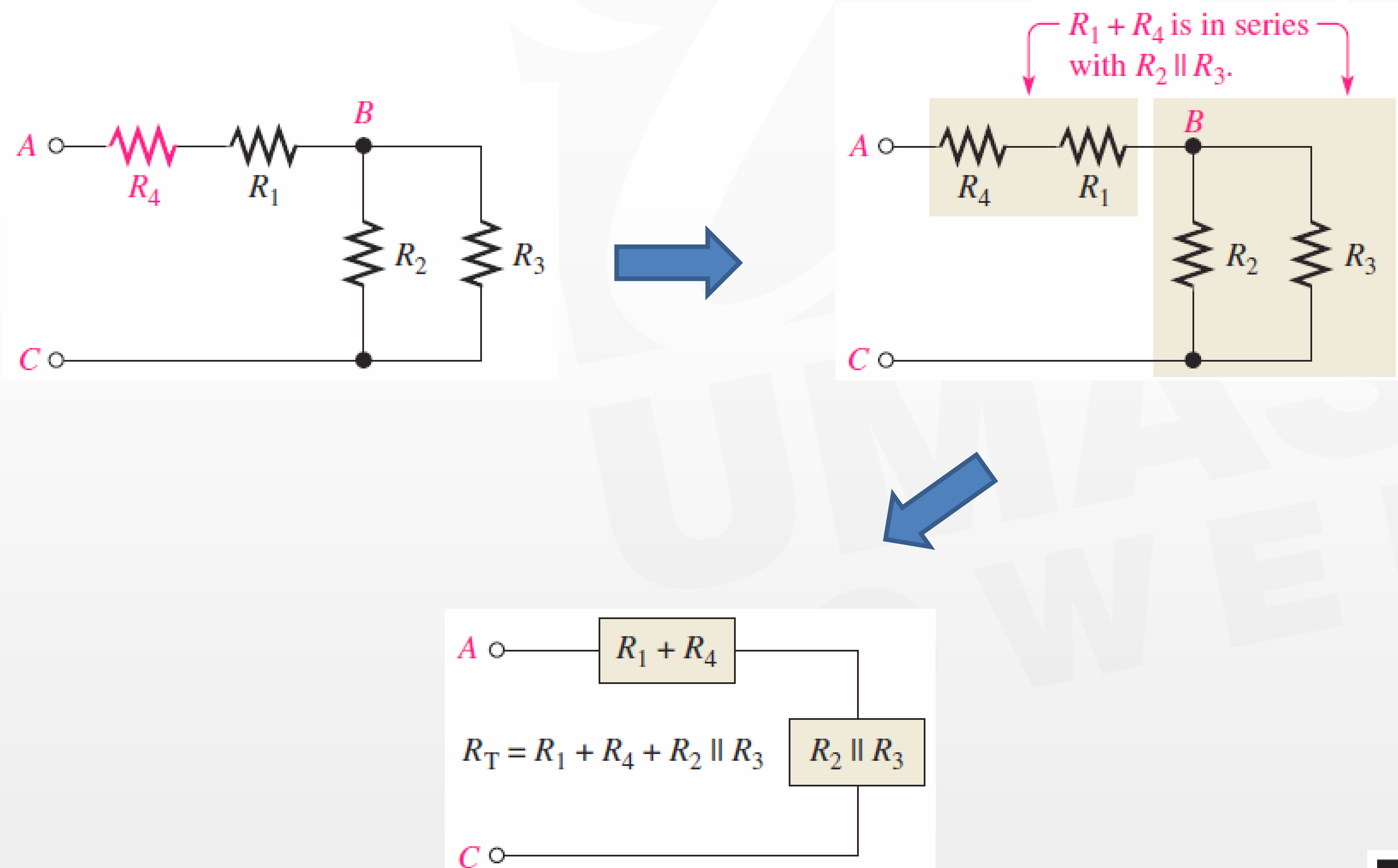
Combination circuits

- Most practical circuits have various combinations of series and parallel components. You can frequently simplify analysis by combining series and parallel components.
- An important analysis method is to form an equivalent circuit.
- An equivalent circuit is one that has characteristics that are electrically the same as another circuit but is generally simpler.

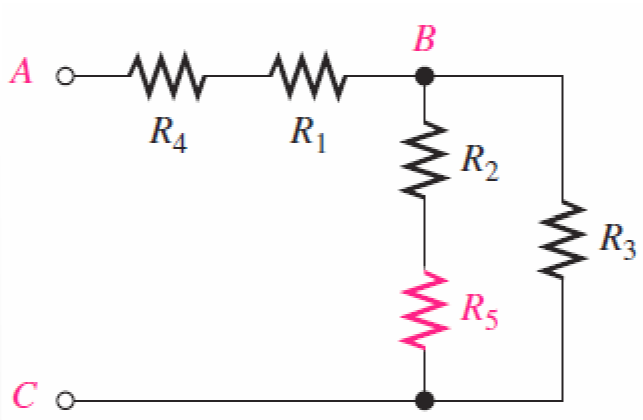
Identifying series-parallel relationships

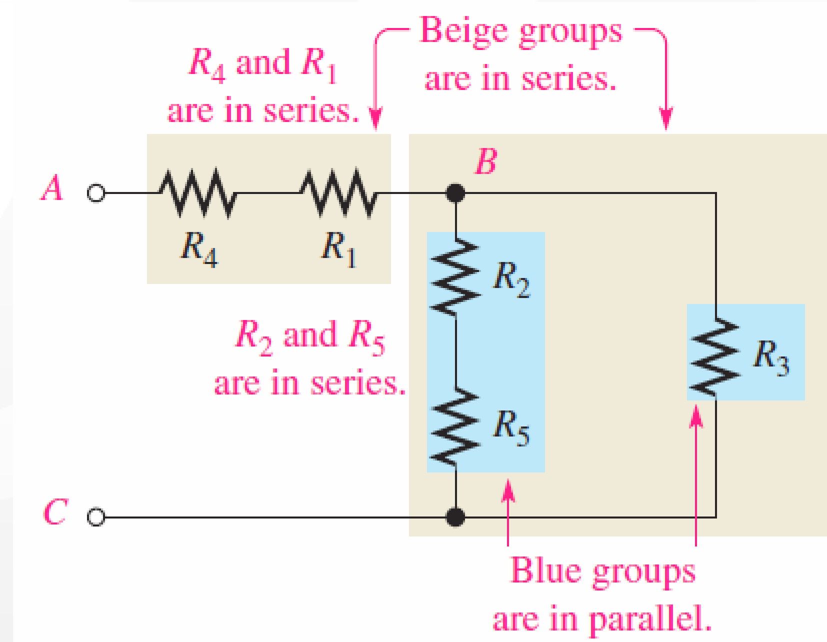


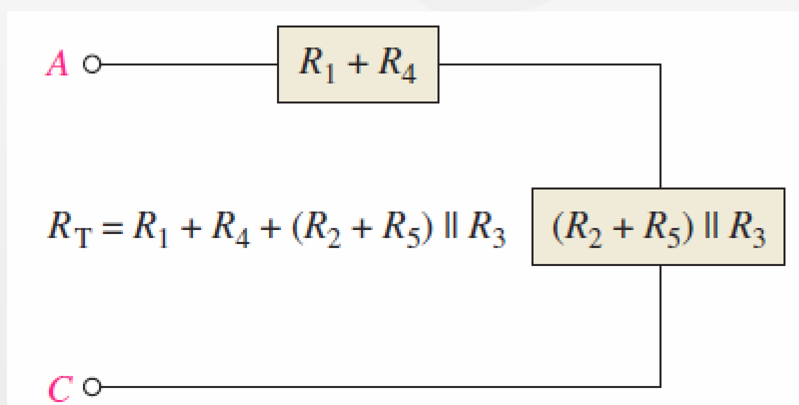
Identifying series-parallel relationships



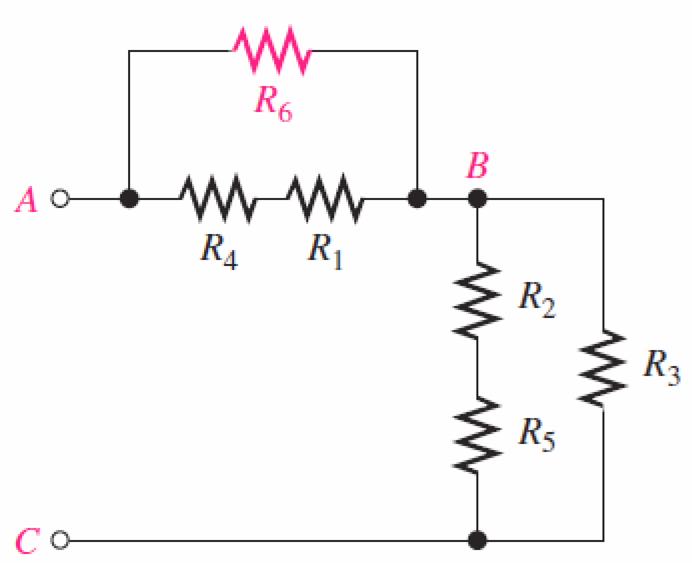
Identifying series-parallel relationships

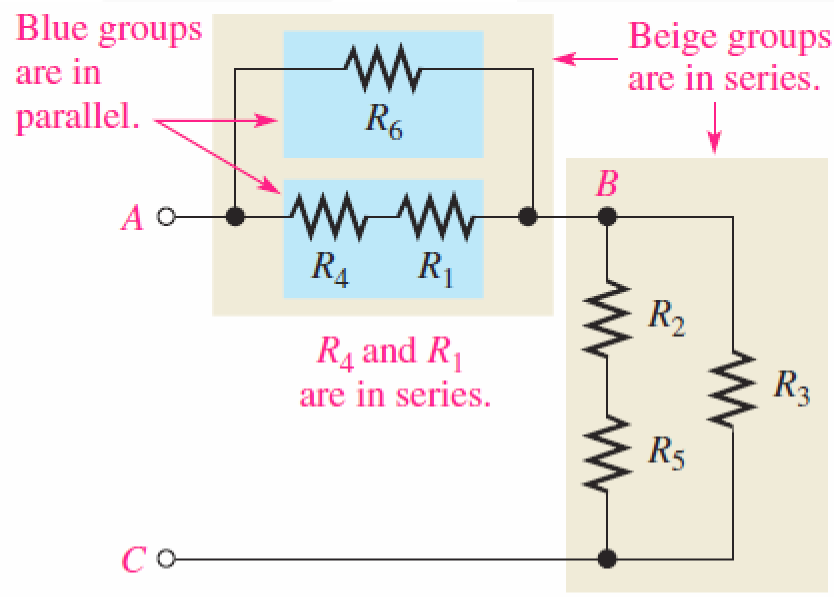


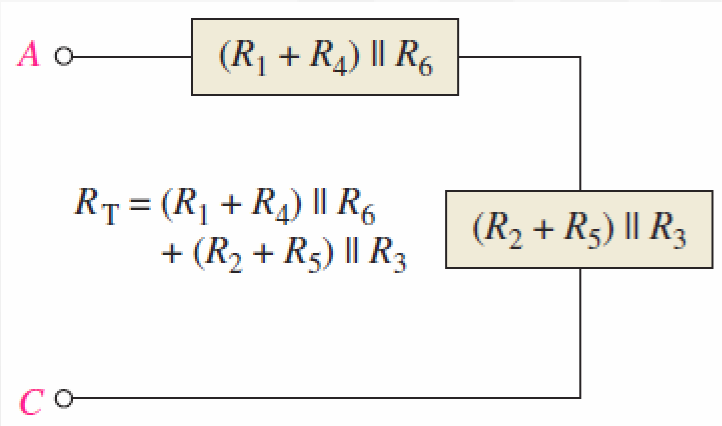




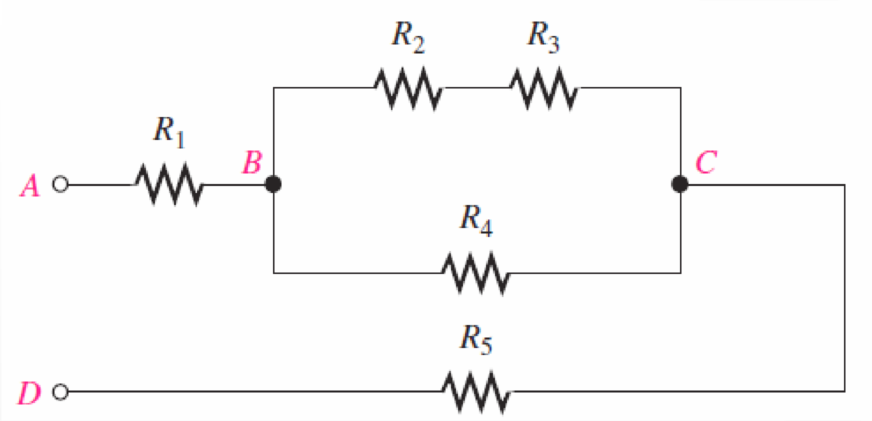
Identifying series-parallel relationships

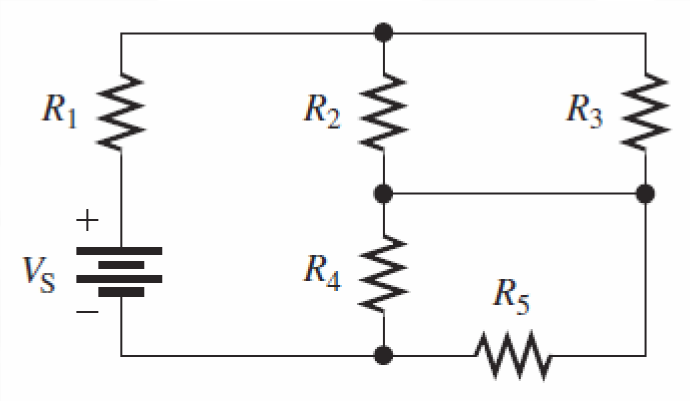






Examples

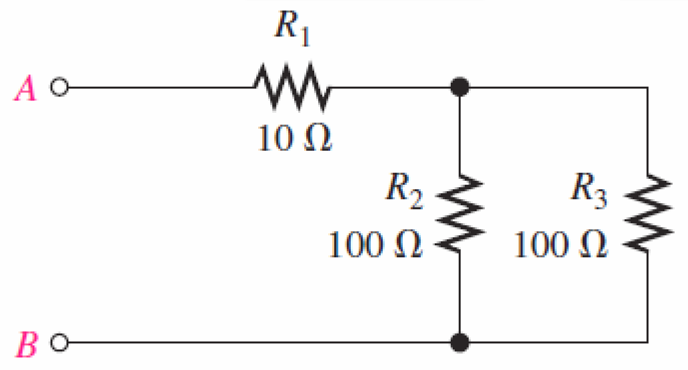




$R_T = R_1 + R_5 + R_4 \,||\, (R_2 + R_3)$

$R_T = R_1 + R_2 \,||\, R_3 + R_4 \,||\, R_5$

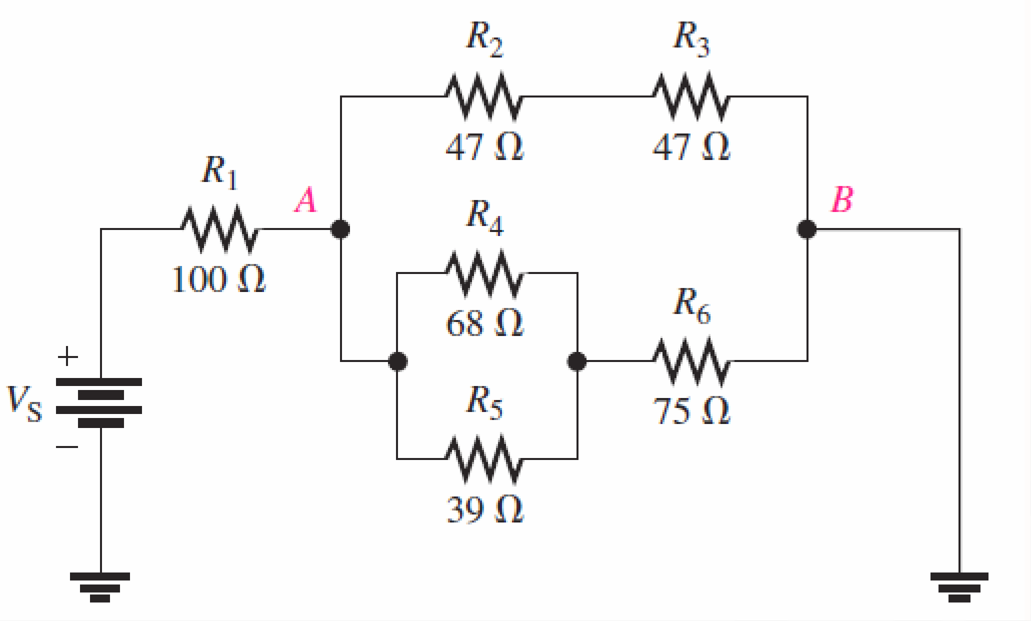
Total resistance



- Identifying the components

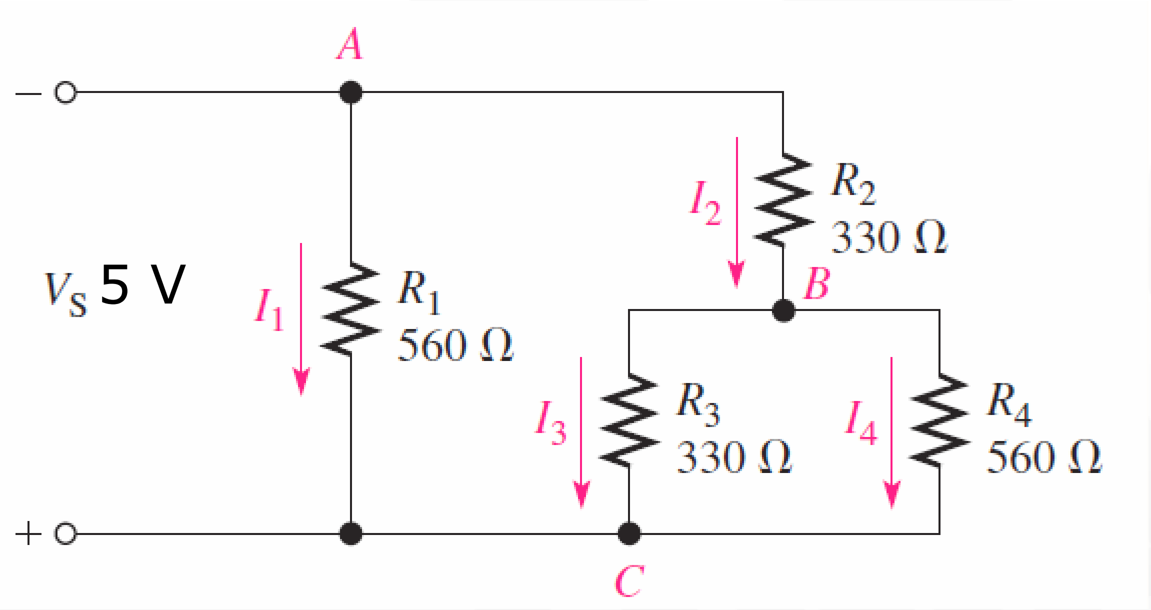
- Total resistance R_T= $R_1 + R_2 \,||\, R_3 = 10 + 50 = 60\ \Omega$

Example



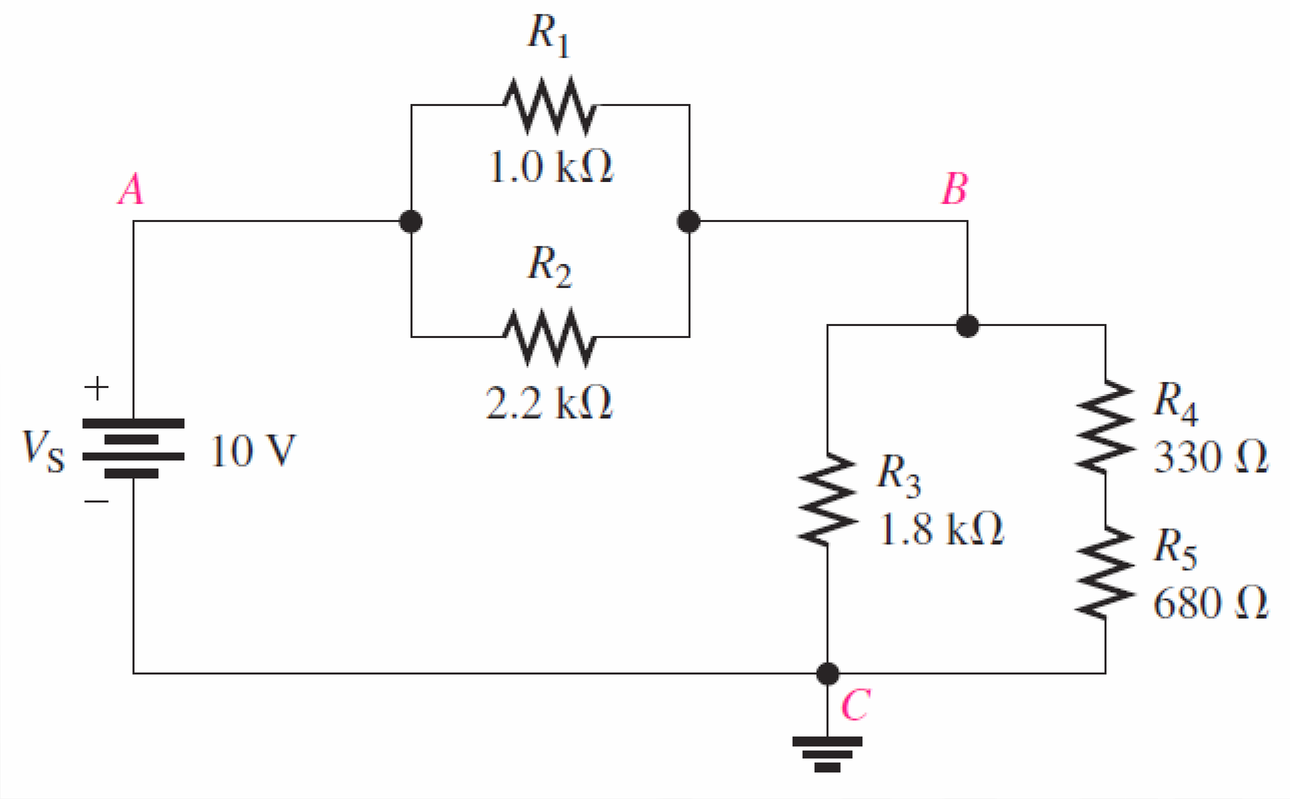
- Find R_T= 148 Ω

Total Current



- Ohm's law: $I_T = \dfrac{V_S}{R_T}$
- Find I_4 = 3.45 mA (${I_2}$ = 9.29 mA)

Voltage Relationships



- Find R_{AB}, R_{BC} & R_T
- Find V_1, V_2, V_3, V_4, and V_5

$V_1 = V_2 = 5.15$ V

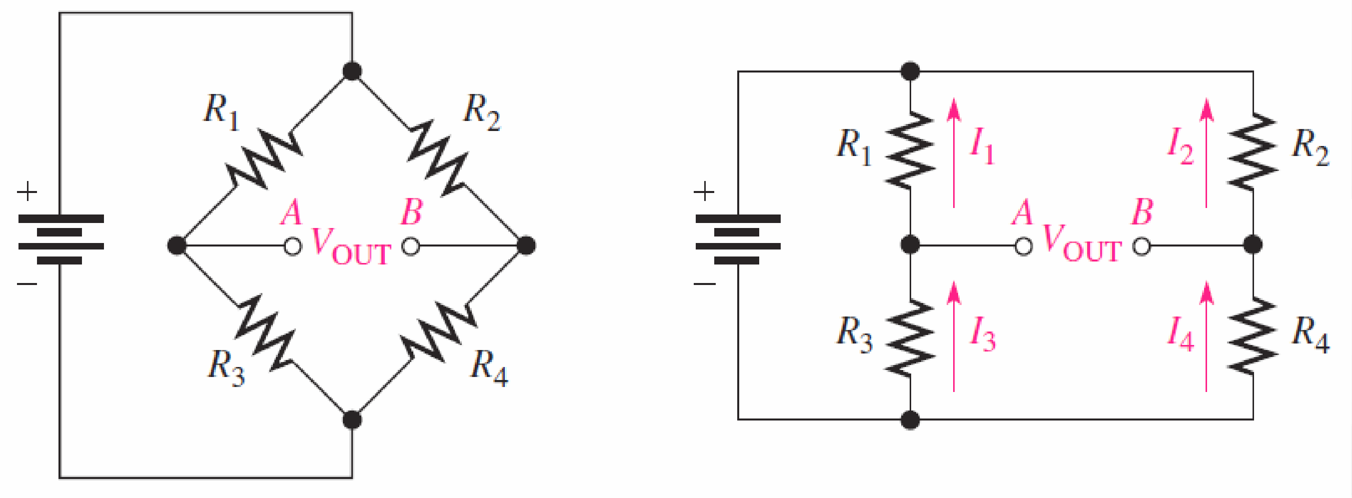
$V_3 = 4.85$ V

$V_4 = 1.58$ V

$V_5 = 3.27$ V

THE WHEATSTONE BRIDGE

Sir Charles Wheatstone, 1802-1875

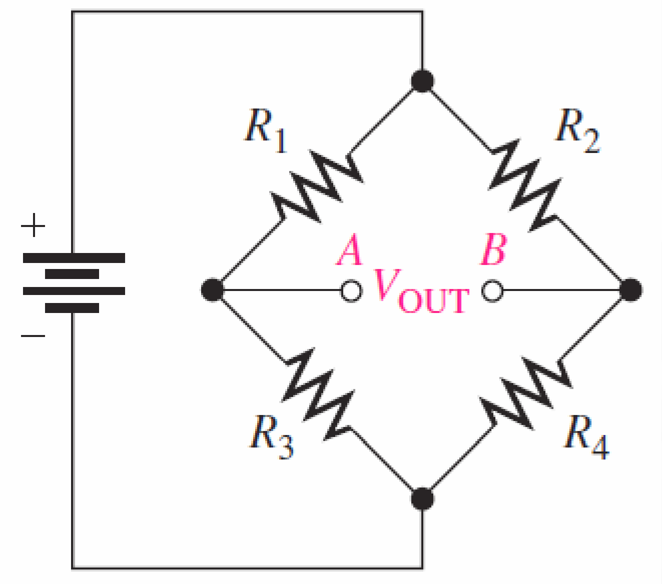


- Used to precisely measure resistance
- Used also in conjunction with transducers
- **Transducer = sensor**.
  Change physical parameter in resistance, for example.

THE WHEATSTONE BRIDGE

The balanced bridge

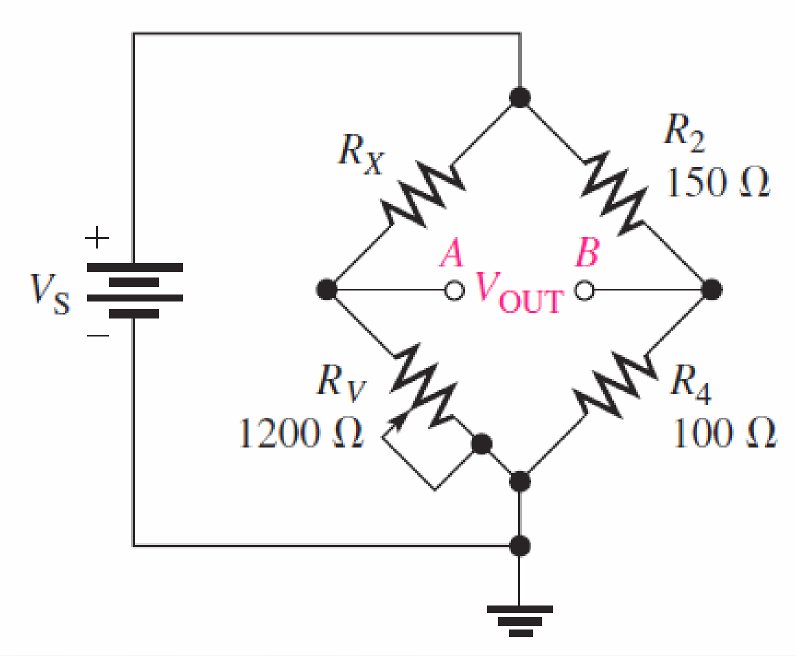
- Balanced bridge when the output
  $V_{OUT} = 0V$
- We have then $V_1 = V_2$ and $V_3 = V_4$
- Therefore $\frac{V_1}{V_3} = \frac{V_2}{V_4}$ and $\frac{I_1 R_1}{I_3 R_3} = \frac{I_2 R_2}{I_4 R_4}$
- Since $I_1 = I_3$ and $I_2 = I_4$
- We got $\frac{R_1}{R_3} = \frac{R_2}{R_4}$ and finally $R_1 = R_3 \frac{R_2}{R_4}$
- **Bridge to find an unknown resistance**



THE WHEATSTONE BRIDGE

Example

Determine the value of R_X.
The bridge is balanced (V_{OUT} = 0 V)
when R_V is set at 1200 Ω



$$R_X = R_V \frac{R_2}{R_4} = 1200 * \frac{150}{100} = 1800\ \Omega$$

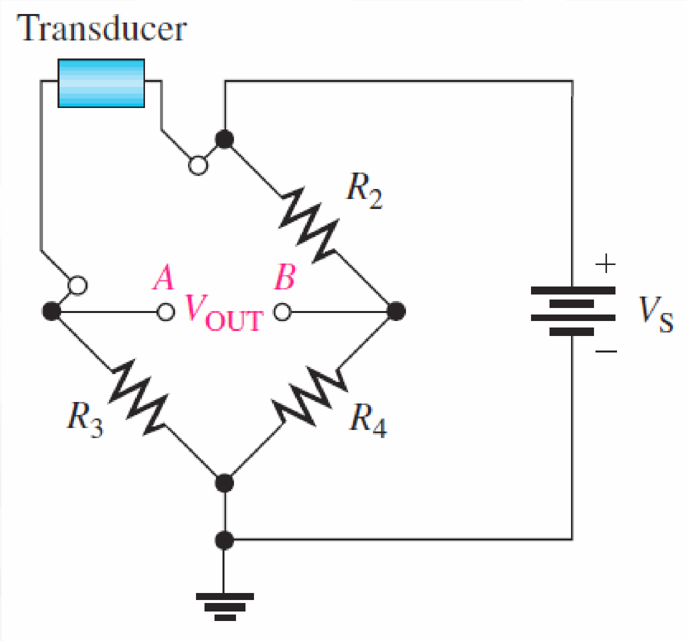
THE WHEATSTONE BRIDGE

The unbalanced bridge

- Unbalanced bridge when the output

  $V_{OUT} \neq 0V$

- Used to measure several types of physical quantities such as mechanical strain, temperature, or pressure.
- Connecting the transducer in one leg of the bridge.

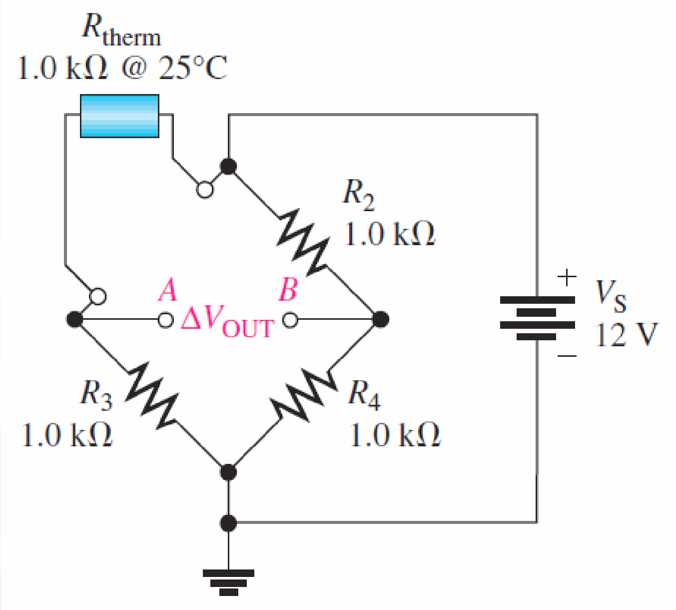


- The resistance of the transducer changes proportionally to the changes in the parameter that it is measuring.
- If the bridge is balanced at a known point, then the amount of deviation indicates the amount of change in the parameter being measured.
- Therefore, the value of the parameter being measured can be determined by the amount that the bridge is unbalanced

THE WHEATSTONE BRIDGE

Example

- Determine the output voltage of the temperature-measuring bridge circuit if the thermistor is exposed to a temperature of 50ºC and its resistance at 25ºC is 1.0 kΩ.
- Assume the resistance of the thermistor decreases to 900 Ω at 50°C.



- At 25ºC the bridge is balanced.
- At 50ºC the bridge is unbalanced. We can apply the voltage-divider formula to the left and right sides.
- $V_A = \left(\frac{R_3}{R_3+R_{therm}}\right) V_S = \left(\frac{1k\Omega}{1k\Omega+900\Omega}\right) 12\ V = 6.32\ V$
- $V_B = \left(\frac{R_4}{R_2+R_4}\right) V_S = \left(\frac{1k\Omega}{2k\Omega}\right) 12\ V = 6.\ V$
- $V_{OUT} = V_A - V_B = 6.32 - 6 = 0.32$ V at 50ºC

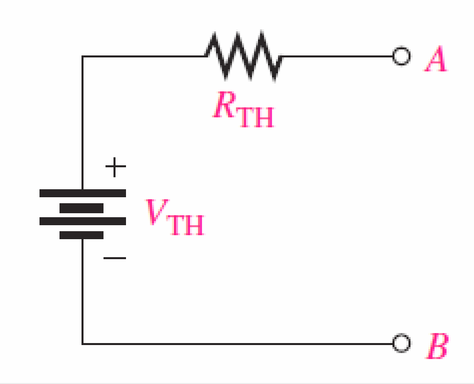
THEVENIN's Theorem

Léon Charles Thévenin, French Engineer, 1857-1926

- What for ?
- To simplify Electric Engineer's Life !!!

- Simplify a complicate series-parallel circuit into an equivalent circuit

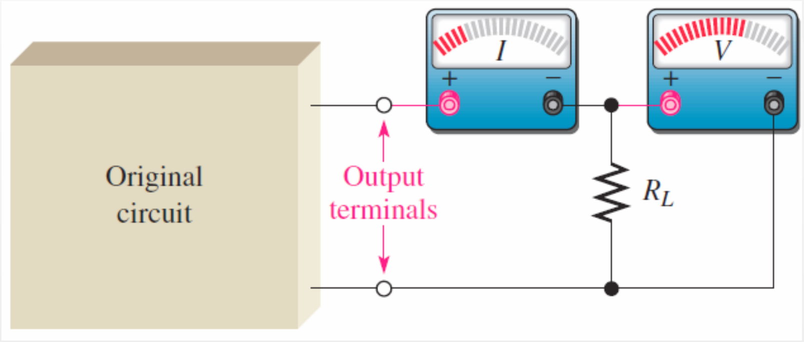
- Consists of an equivalent voltage source V_{TH}
- And an equivalent resistance R_{TH}

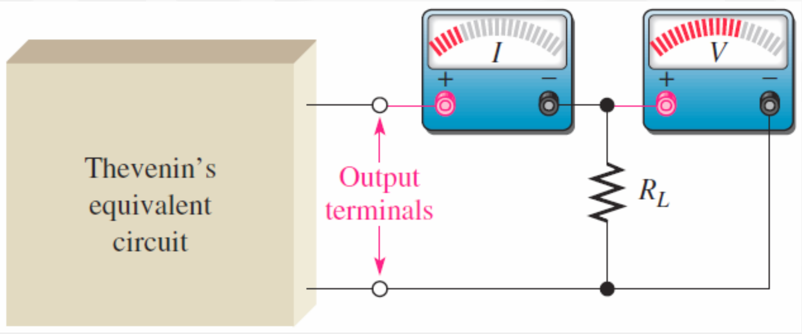


THEVENIN’s Theorem

- The Thevenin equivalent voltage V_{TH} is the open circuit (no-load) voltage between two specified output terminals in a circuit.

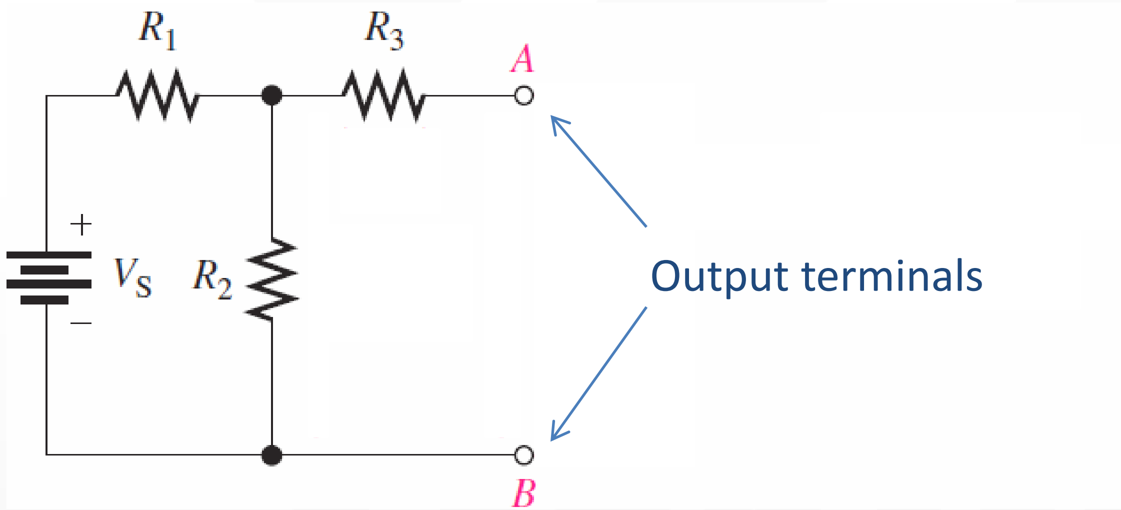
- The Thevenin equivalent resistance R_{TH} is the total resistance appearing between two specified output terminals in a circuit with all sources replaced by their internal resistances (which for an ideal voltage source is zero).





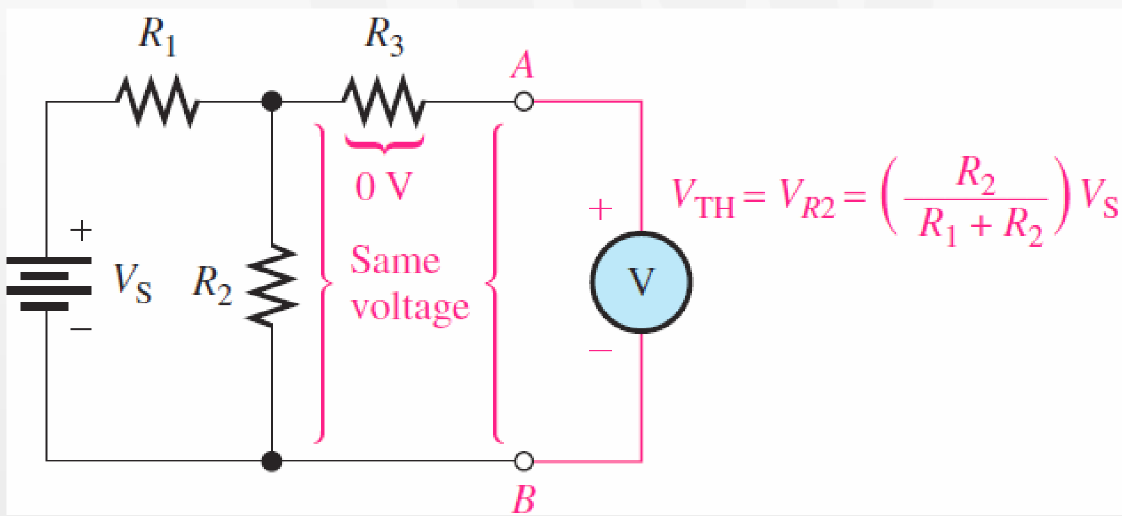
Three steps: #1

Circuit "complicate"



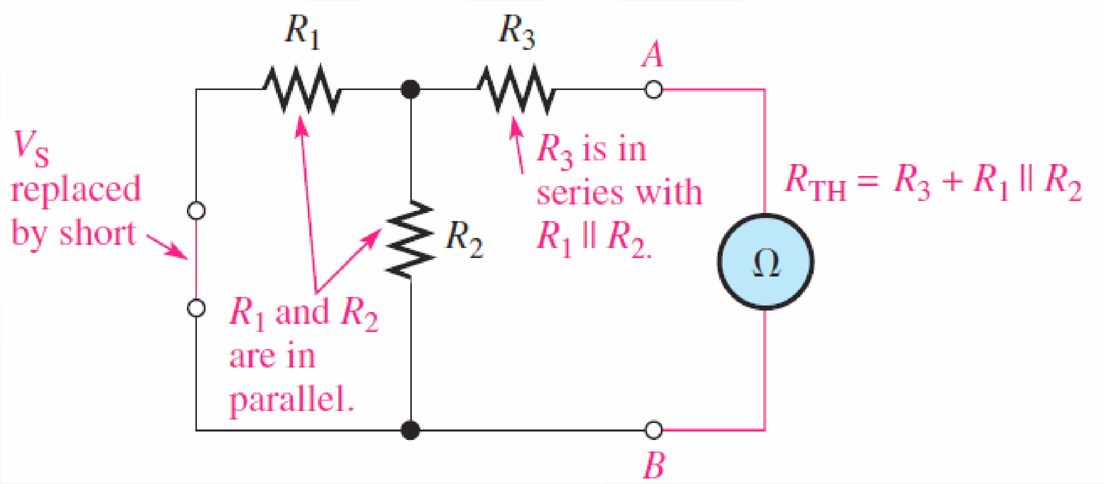
Output terminals

- Step 1: Find V_{TH} = Find the voltage between A and B



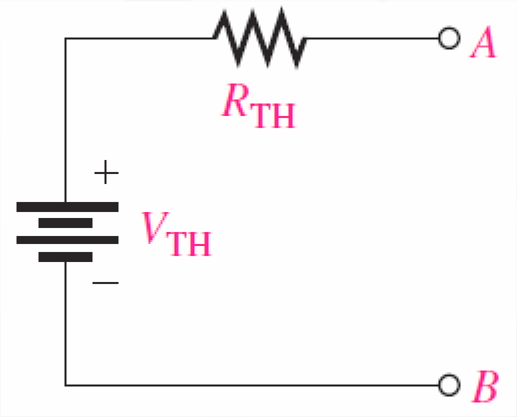
Three steps: #2

- Step 2: Find R_{TH}
- Short-circuiting the battery
- Find the resistance between A and B

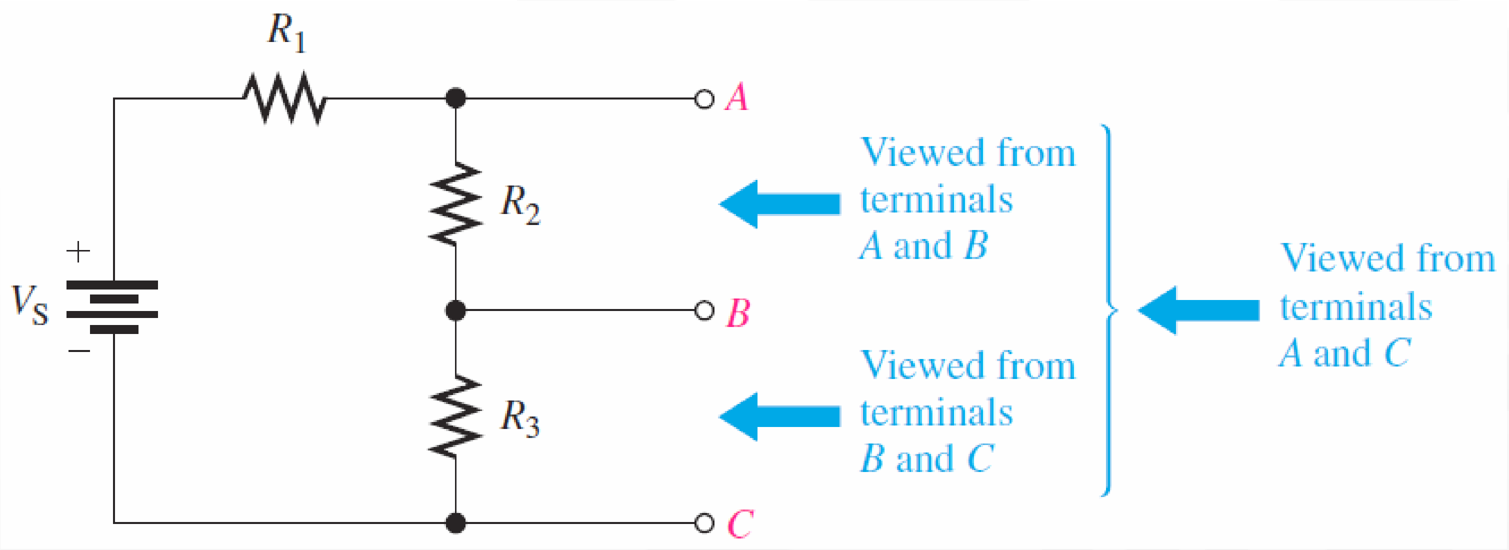


Three steps: #3

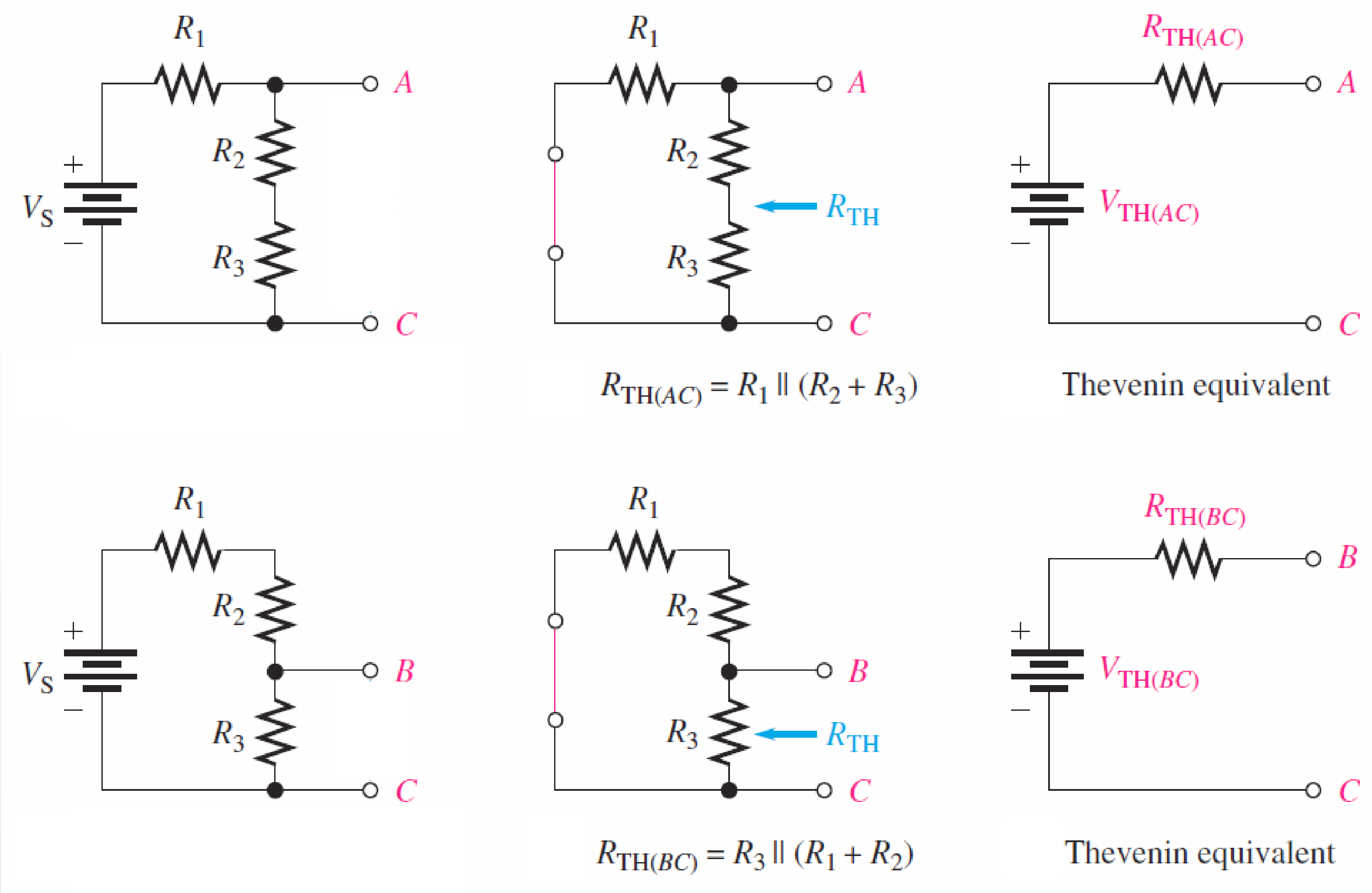
- Step 3: Combining both V_{TH} and R_{TH}
- Thevenin equivalent circuit



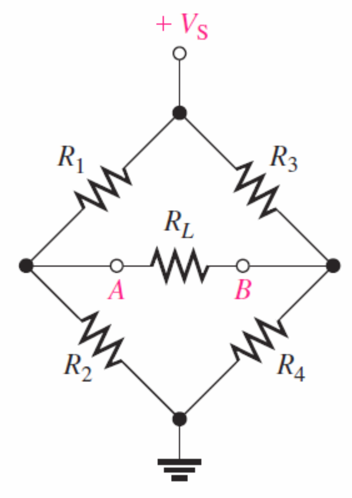
Depends on the viewpoint

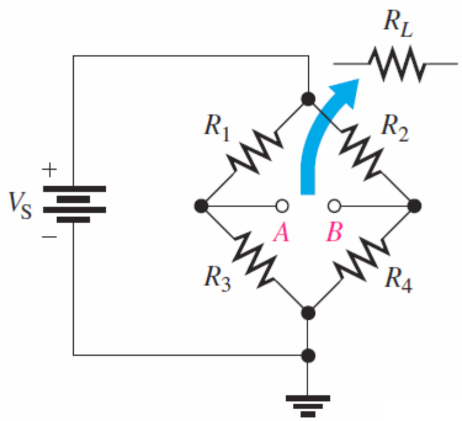


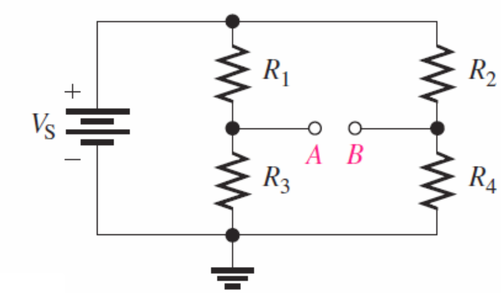
Depends on the viewpoint

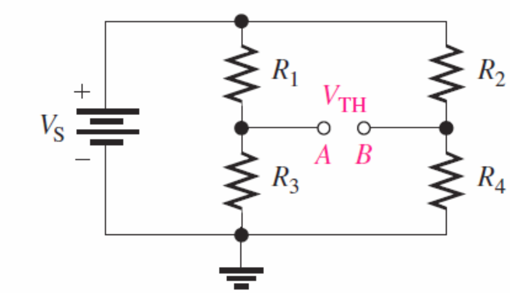


Thevenizing a Bridge Circuit

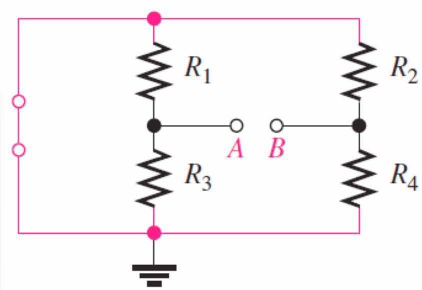


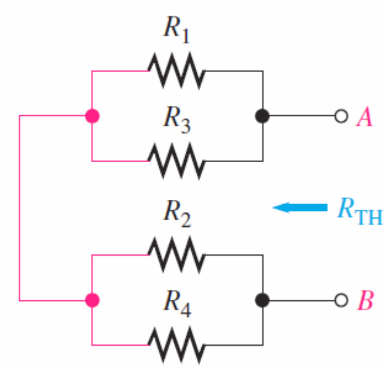


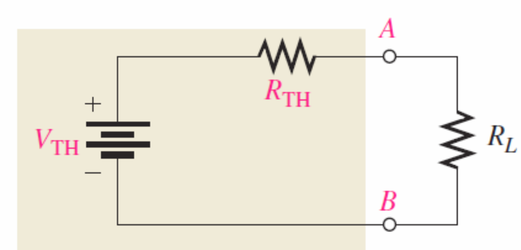




$$V_{TH} = V_A - V_B = \left(\frac{R_3}{R_1 + R_3}\right)V_S - \left(\frac{R_4}{R_2 + R_4}\right)V_S$$



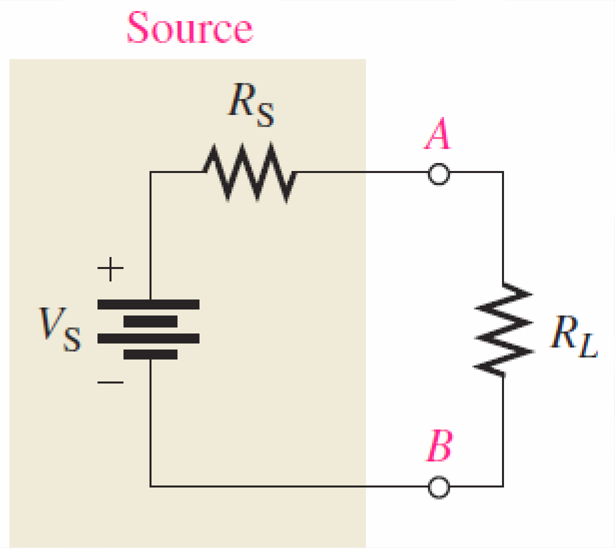




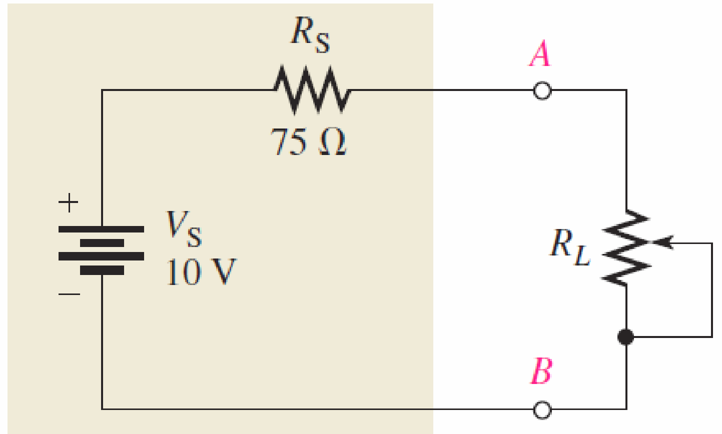
Maximum Power Transfer Theorem

- For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.

- Maximum power is transferred to the load when $R_L = R_S$.



Maximum Power Transfer Theorem

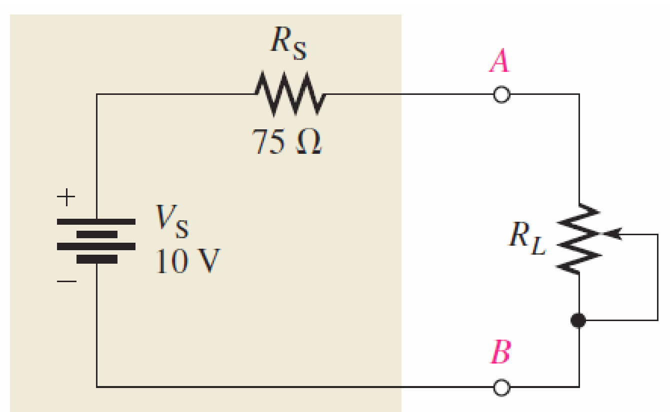


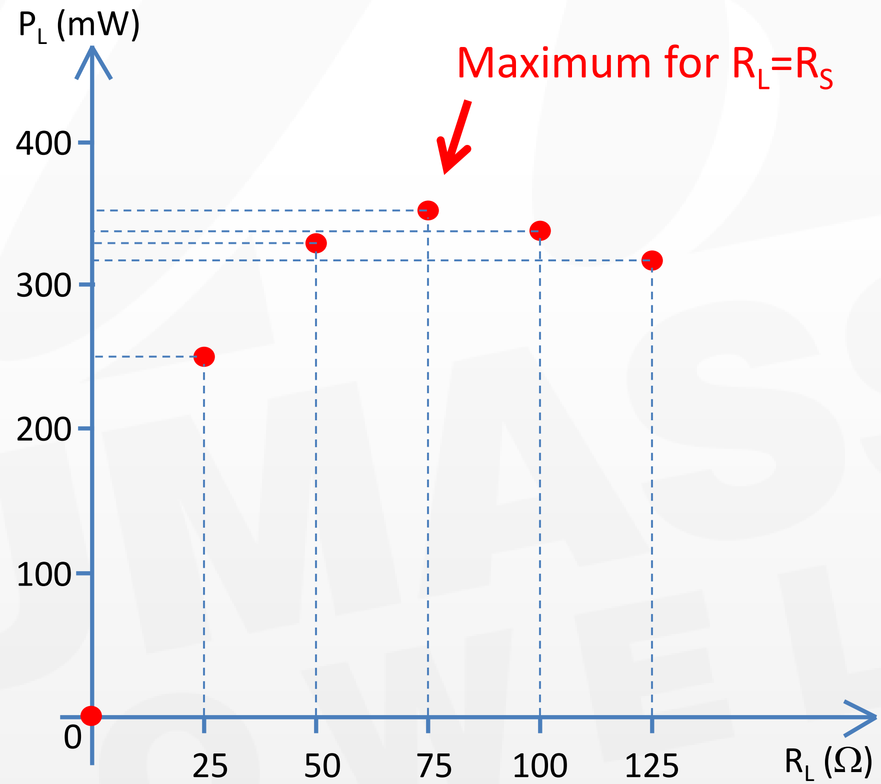
- Example: Determine the load power for different values of the variable load resistance [0:125] Ω

- Solution: Using Ohm's law and Power Formula
- For R_L = 0 Ω

$$I = \frac{V_S}{R_S + R_L} = \frac{10}{75 + 0} = 133\,mA$$
$$P_L = I^2 R_L = 133^2 * 0 = 0\,W$$

Maximum Power Transfer Theorem





For $R_L = 0\ \Omega : P_L = 0$ W

For $R_L = 25\ \Omega : P_L = 250$ mW

For $R_L = 50\ \Omega : P_L = 320$ mW

For $R_L = 75\ \Omega : P_L = 334$ mW

For $R_L = 100\ \Omega : P_L = 326$ mW

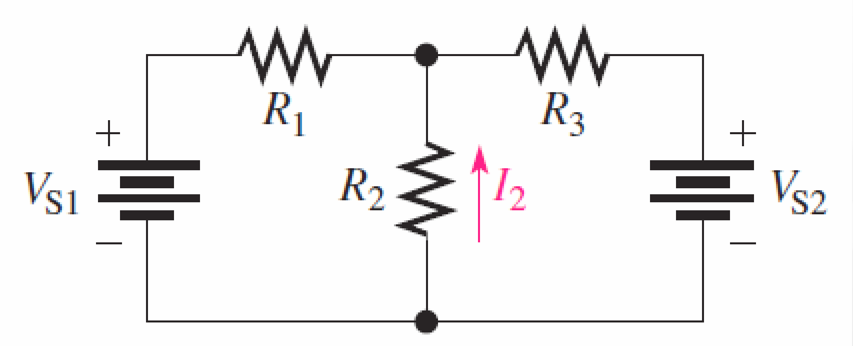
For $R_L = 125\ \Omega : P_L = 313$ mW

- Note that $R_S = R_{TH}$ if we are using the Thevenin's Theorem

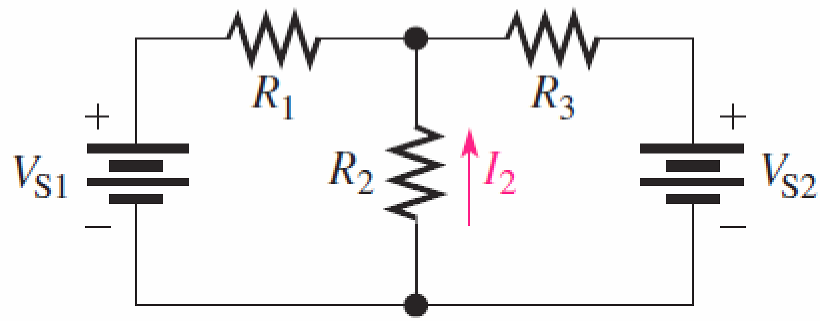
Superposition Theorem

What is happening when there are two or more voltage sources ???

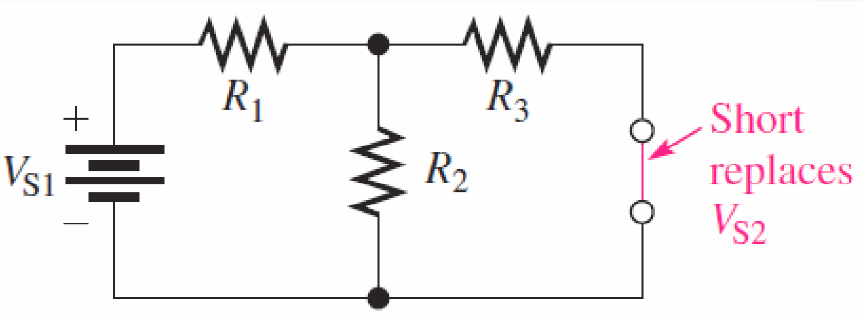
How do we calculate I_2 ???



Superposition Theorem

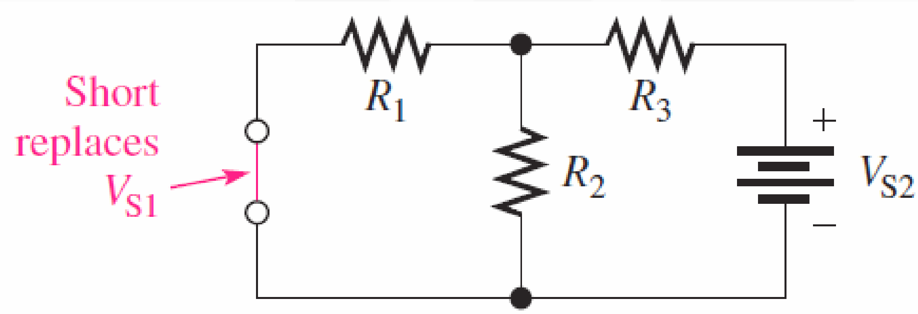


Problem: Find I_2



Determine R_{T1} and I_{T1}

Then $I_{2(S1)}$



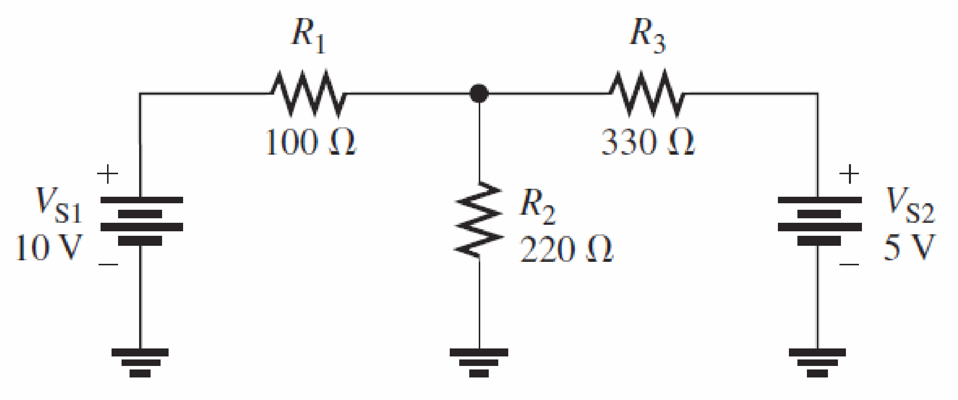
Determine R_{T2} and I_{T2}

Then $I_{2(S2)}$

$$I_2 = I_{2(S1)} + I_{2(S2)}$$

Superposition Theorem

Example



- Find I_2, the current through R_2

- 1_ short replace V_{S2}, find R_{T1}, then I_{T1} and finally $I_{2(S1)}$
- R_{T1}=232 W // I_{T1} = 43.1 mA // $I_{2(S1)}$=25.9 mA
- 2_ short replace V_{S1}, find R_{T2}, then I_{T2} and finally $I_{2(S2)}$
- R_{T1}=399 W // I_{T1} = 12.5 mA // $I_{2(S1)}$=3.9 mA
- $I_2 = I_{1(S1)} + I_{2(S2)}$ =25.9 + 3.9 = 29.8 mA