Chapter 9 Impulse and Momentum

Chapter Goal: To understand and apply the new concepts of impulse and momentum.

Chapter 9 Preview

Momentum

An object's **momentum** is the product of its mass and velocity: $\vec{p} = m\vec{v}$.

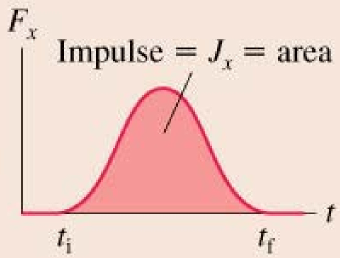
An object can have a large momentum by having a large mass or a large velocity.

Momentum is a vector. Paying attention to the *signs* of the components of momentum will be especially important.

You'll learn to write Newton's second law in terms of momentum.

Impulse

A force of short duration is an *impulsive force*. The **impulse** J_x is the area under the force-versus-time curve.



We say that the bat delivers an impulse to the ball.

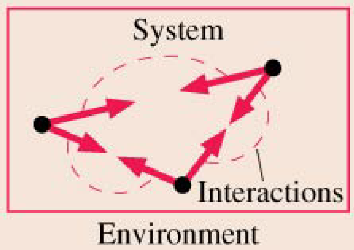
The **impulse-momentum theorem** says that an impulse changes a particle's momentum: $\Delta p_x = J_x$.

Chapter 9 Preview

Conservation Laws

Part I of this textbook was about how interactions cause things to change. Part II will explore how some things are *not* changed by the interactions. We say they are *conserved*.

The particles of an **isolated system** interact with each other—perhaps very intensely—but not with the external environment.



Chapter 9 Preview

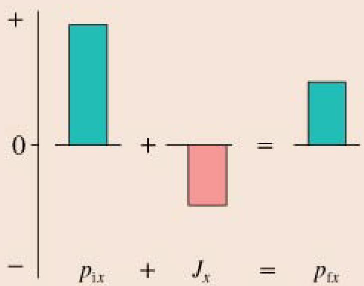
The mass, the momentum, and the energy of an isolated system are conserved. Conservation laws will be the basis of a new and powerful problem solving strategy:

$$\text{final value} = \text{initial value}$$

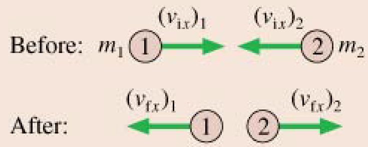
Conservation of momentum for an isolated system is a consequence of Newton's third law.

Representations

Conservation laws require new visual tools. You will learn to draw and use:



Momentum bar charts to show how impulse changes momentum.



Before-and-after pictorial representations to compare quantities before and after an interaction.

Chapter 9 Preview

Collisions and Explosions

You will learn to apply conservation of momentum to the analysis of *collisions* and *explosions*.

A **collision** is when two or more particles come together for a short but intense interaction.

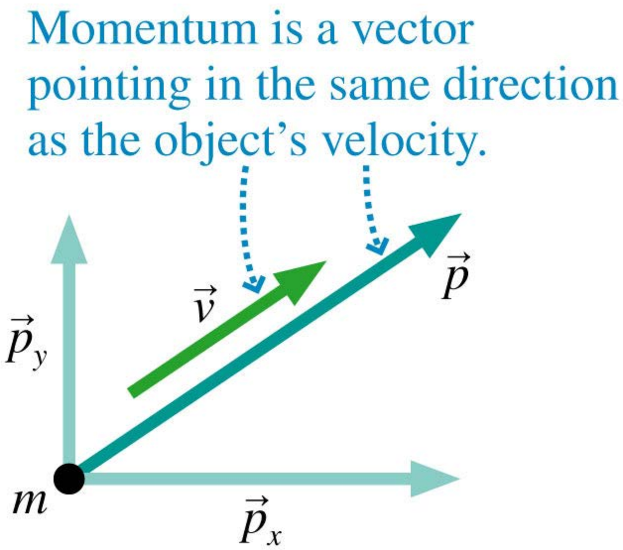
An **explosion** is when a short but intense interaction causes two or more particles to move apart.

Momentum

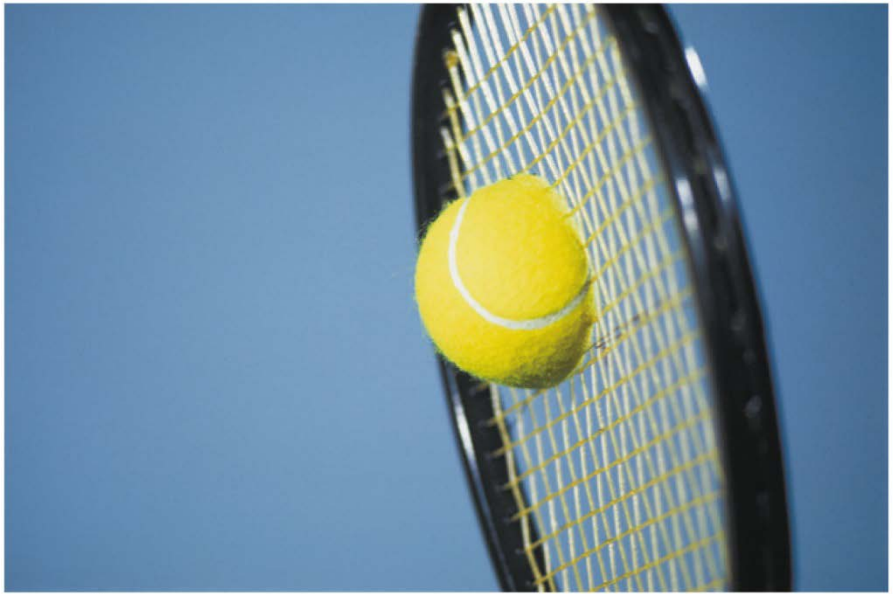
- The product of a particle's mass and velocity is called the *momentum* of the particle:

$$\textbf{momentum} = \vec{p} \equiv m\vec{v}$$

- Momentum is a vector, with units of kg m/s.
- A particle's momentum vector can be decomposed into *x*- and *y*-components.

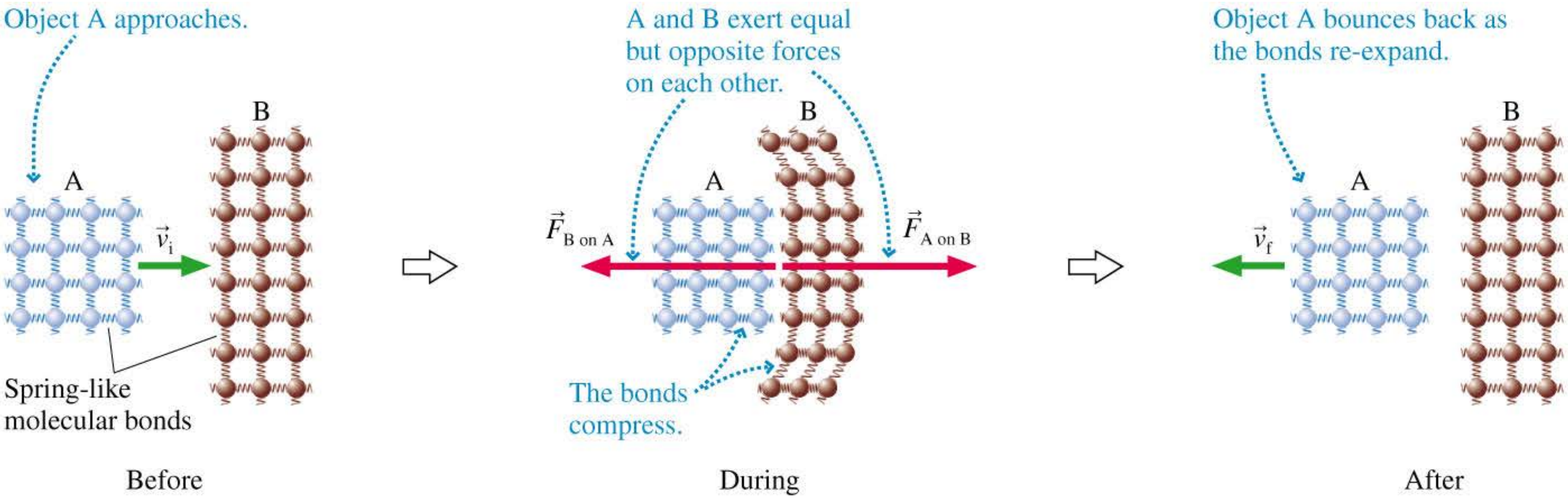


Collisions



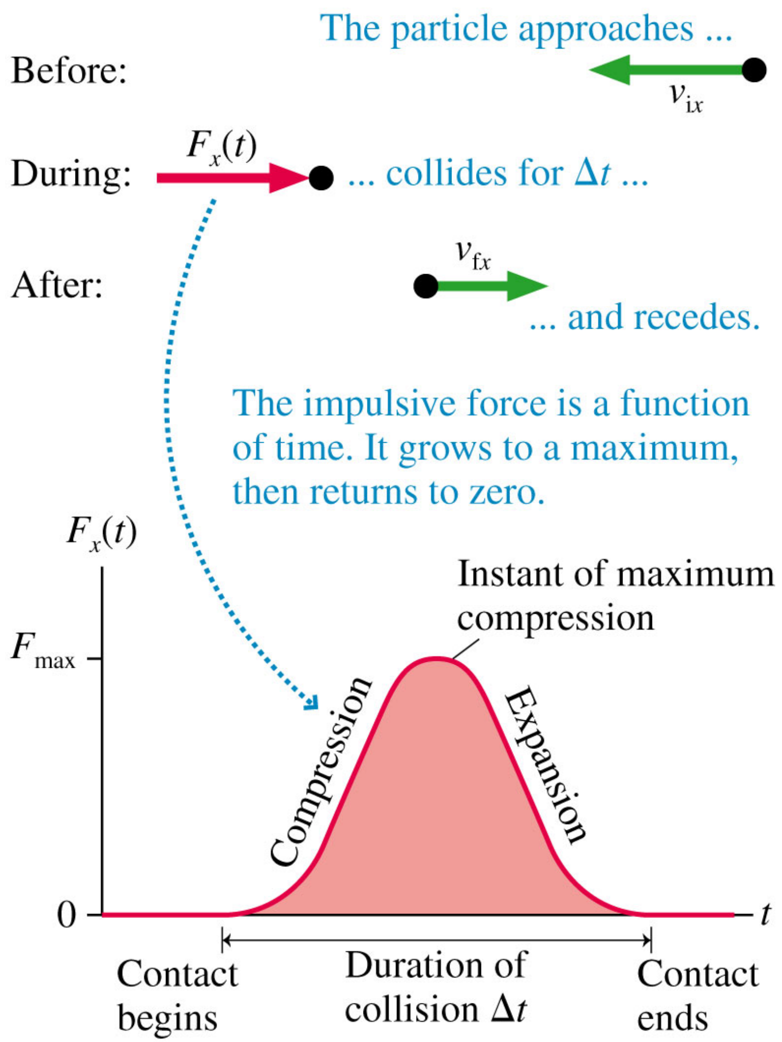
- A collision is a short-duration interaction between two objects.
- The collision between a tennis ball and a racket is quick, but it is *not* instantaneous.
- Notice that the right side of the ball is flattened.
- It takes time to compress the ball, and more time for the ball to re-expand as it leaves the racket.

Atomic Model of a Collision



Impulse During a Collision

- A large force exerted for a small interval of time is called an **impulsive force**.
- The figure shows a particle with initial velocity $\vec{v}_i$.
- The particle experiences an impulsive force of short duration Δt.
- The particle leaves with final velocity $\vec{v}_f$.



Impulse

- Newton's second law may be formulated in terms of momentum rather than acceleration:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

- Rearranging, and integrating over time, we have:

$$\Delta p_x = p_{\mathrm{f}x} - p_{\mathrm{i}x} = \int_{t_\mathrm{i}}^{t_\mathrm{f}} F_x(t)\, dt$$

- We define the right-hand side to be the *impulse.*

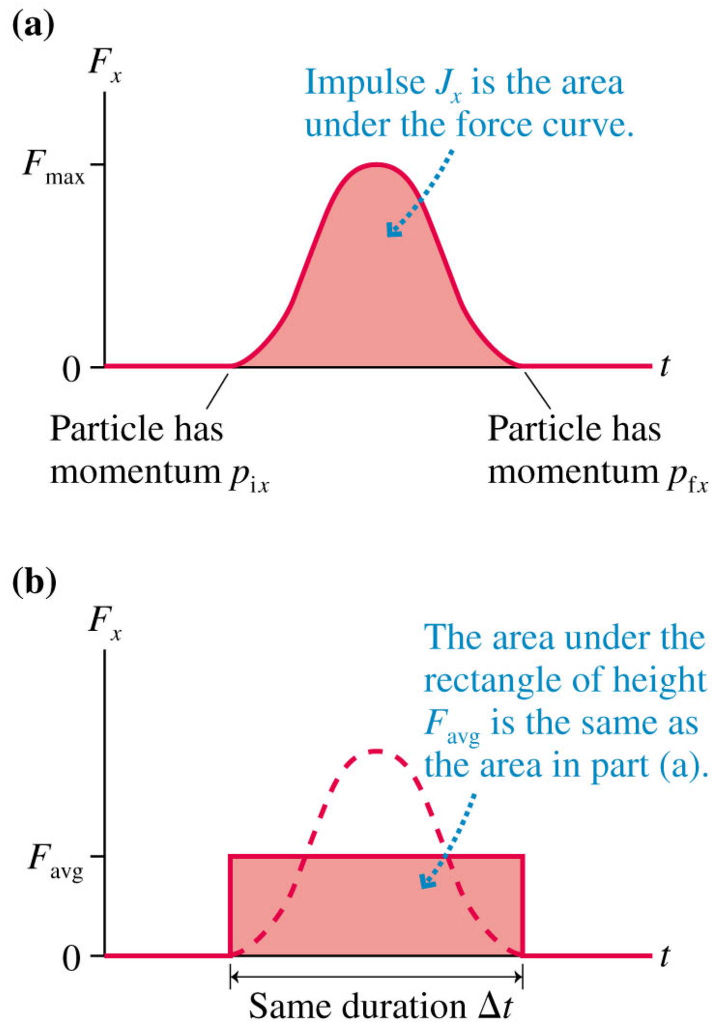
$$\textbf{impulse} = J_x \equiv \int_{t_\mathrm{i}}^{t_\mathrm{f}} F_x(t)\, dt$$
$$= \text{area under the } F_x(t) \text{ curve between } t_\mathrm{i} \text{ and } t_\mathrm{f}$$

- Impulse has units of N s, which are equivalent to kg m/s.

Impulse

- Figure (a) portrays the impulse graphically.
- Figure (b) shows that the average force F_{avg} is the height of a rectangle that has the same impulse as the real force curve.
- The impulse exerted during the collision is:

$$J_x = F_{avg}\,\Delta t$$



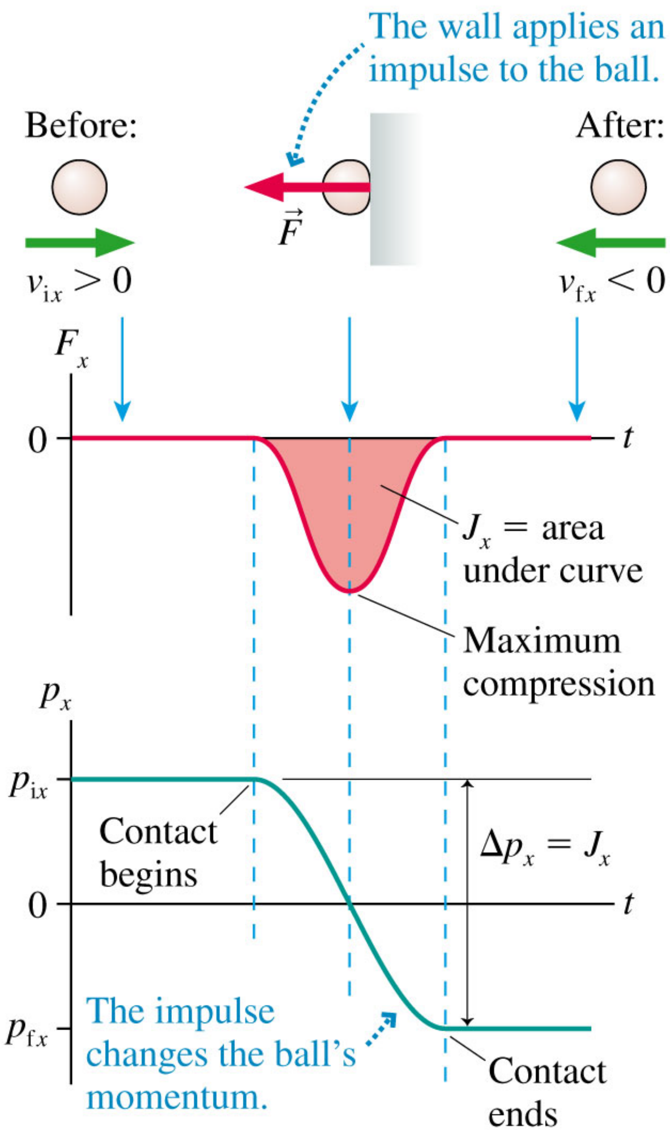
The Impulse-Momentum Theorem

- A particle experiences an impulsive force in the *x*-direction.
- The impulse delivered to the particle is equal to the change in the particle's momentum.

$$\Delta p_x = J_x \qquad \text{(impulse-momentum theorem)}$$

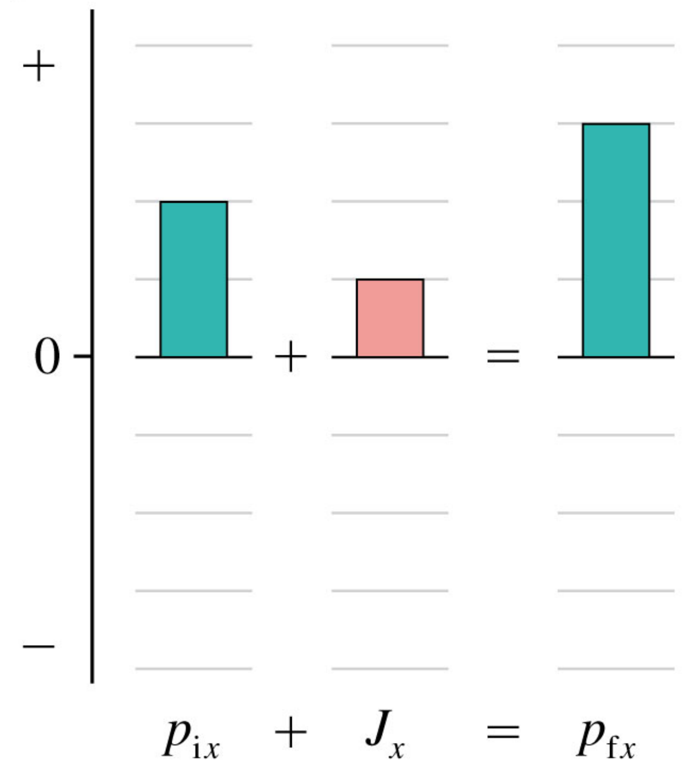
The Impulse-Momentum Theorem

- A rubber ball bounces off a wall.
- The ball is initially traveling toward the right, so v_{ix} and p_{ix} are positive.
- After the bounce, v_{fx} and p_{fx} are negative.
- The force *on the ball* is toward the left, so F_x is negative.
- In this example, the impulse, or area under the force curve, has a negative value.



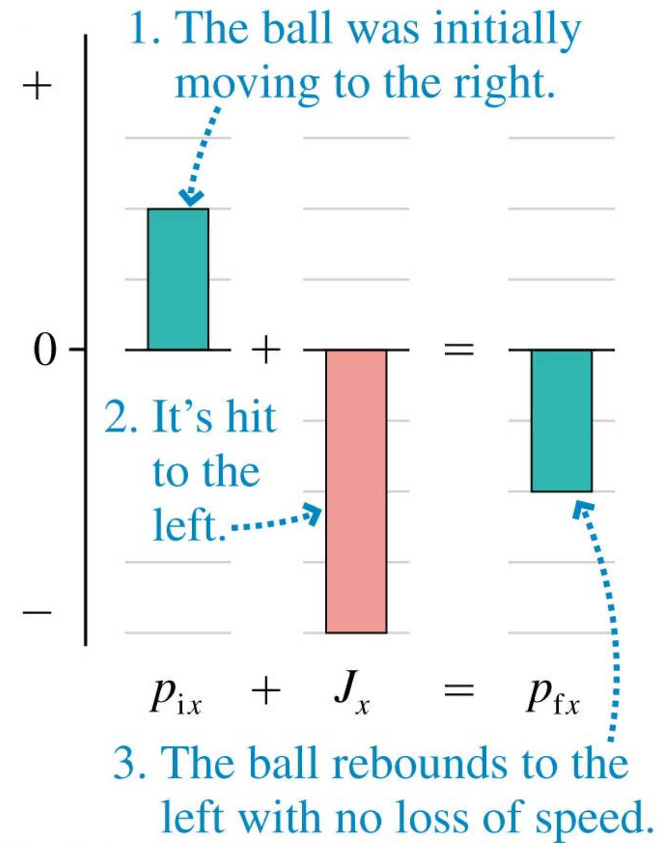
Momentum Bar Charts

- Impulse J_x transfers momentum to an object.
- If an object has an initial momentum of 2 kg m/s, a +1 kg m/s impulse exerted on the object increases its momentum to 3 kg m/s.
- $p_{fx} = p_{ix} + J_x$
- We can represent this "momentum accounting" with a **momentum bar chart**.
- The figure shows how one 1 unit of impulse adds to 2 units of initial momentum to give 3 units of final momentum.



Momentum Bar Charts

- A rubber ball is initially moving to the right with $p_{ix} = +2$ kg m/s.
- It collides with a wall which delivers an impulse of $J_x = -4$ N s.
- The figure shows the momentum bar chart to help analyze this collision.
- The final momentum is $p_{fx} = -2$ kg m/s.



Tactics: Drawing a Before-and-After Pictorial Representation

TACTICS BOX 9.1 **Drawing a before-and-after pictorial representation**

1. **Sketch the situation.** Use two drawings, labeled "Before" and "After," to show the objects *before* they interact and again *after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.

Exercises 17–19

Tactics: Drawing a Before-and-After Pictorial Representation

TACTICS BOX 9.1 **Drawing a before-and-after pictorial representation**

4. **List known information.** Give the values of quantities that are known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are simpler than the pictures for dynamics problems, so listing known information on the sketch is adequate.
5. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined in step 3.
6. If appropriate, **draw a momentum bar chart** to clarify the situation and establish appropriate signs.

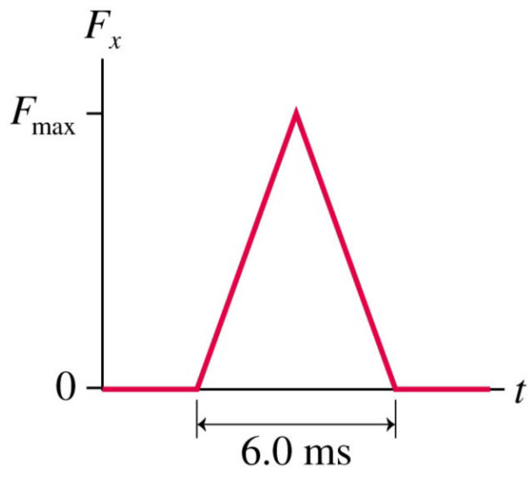
Exercises 17–19

Example 9.1 Hitting a Baseball

EXAMPLE 9.1 Hitting a baseball

A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The interaction force between the ball and the bat is shown in the figure below. What *maximum* force F_{max} does the bat exert on the ball? What is the *average* force of the bat on the ball?

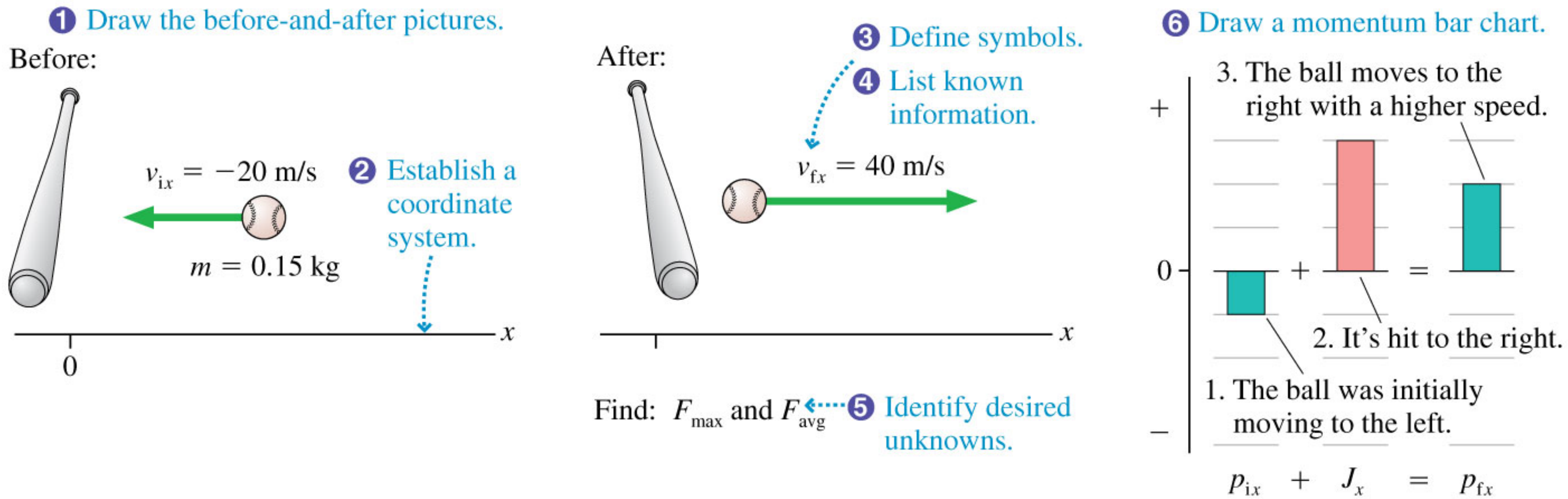
MODEL Model the baseball as a particle and the interaction as a collision.



Example 9.1 Hitting a Baseball

EXAMPLE 9.1 Hitting a baseball

VISUALIZE The figure below is a before-and-after pictorial representation. The steps from Tactics Box 9.1 are explicitly noted. Because F_x is positive (a force to the right), we know the ball was initially moving toward the left and is hit back toward the right. Thus we converted the statements about *speeds* into information about *velocities*, with v_{ix} negative.



Example 9.1 Hitting a Baseball

EXAMPLE 9.1 Hitting a baseball

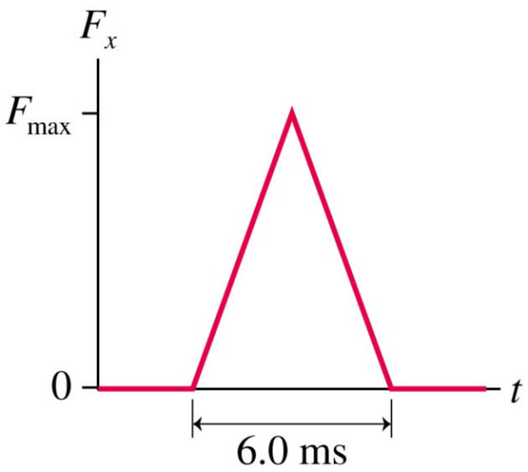
SOLVE Until now we've consistently started the mathematical representation with Newton's second law. Now we want to use the impulse-momentum theorem:

$$\Delta p_x = J_x = \text{area under the force curve}$$

We know the velocities before and after the collision, so we can calculate the ball's momenta:

$$p_{ix} = mv_{ix} = (0.15\text{ kg})(-20\text{ m/s}) = -3.0\text{ kg m/s}$$

$$p_{fx} = mv_{fx} = (0.15\text{ kg})(40\text{ m/s}) = 6.0\text{ kg m/s}$$



Example 9.1 Hitting a Baseball

EXAMPLE 9.1 Hitting a baseball

Thus the *change* in momentum is

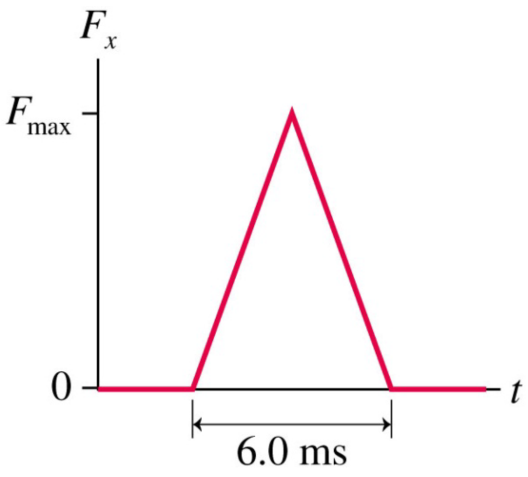
$$\Delta p_x = p_{fx} - p_{ix} = 9.0 \text{ kg m/s}$$

The force curve is a triangle with height F_{max} and width 6.0 ms. The area under the curve is

$$J_x = \text{area} = \frac{1}{2} \times F_{max} \times (0.0060 \text{ s}) = (F_{max})(0.0030 \text{ s})$$

According to the impulse-momentum theorem,

$$9.0 \text{ kg m/s} = (F_{max})(0.0030 \text{ s})$$



Example 9.1 Hitting a Baseball

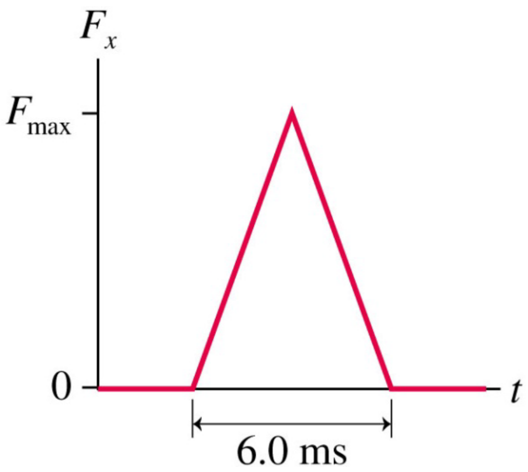
EXAMPLE 9.1 **Hitting a baseball**

Thus the *maximum* force is

$$F_{max} = \frac{9.0\ \text{kg m/s}}{0.0030\ \text{s}} = 3000\ \text{N}$$

The *average* force, which depends on the collision duration $\Delta t = 0.0060$ s, has the smaller value:

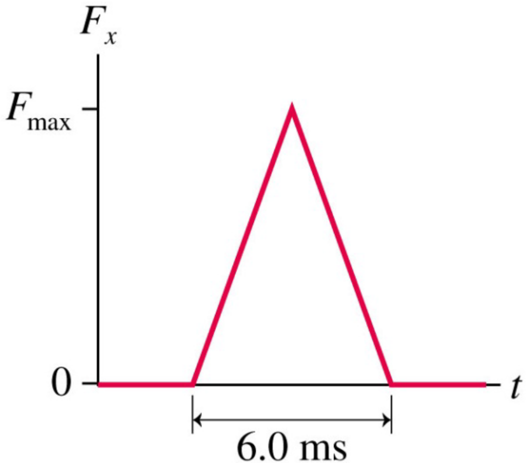
$$F_{avg} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0\ \text{kg m/s}}{0.0060\ \text{s}} = 1500\ \text{N}$$



Example 9.1 Hitting a Baseball

EXAMPLE 9.1 Hitting a baseball

ASSESS F_{max} is a large force, but quite typical of the impulsive forces during collisions. The main thing to focus on is our new perspective: An impulse changes the momentum of an object.



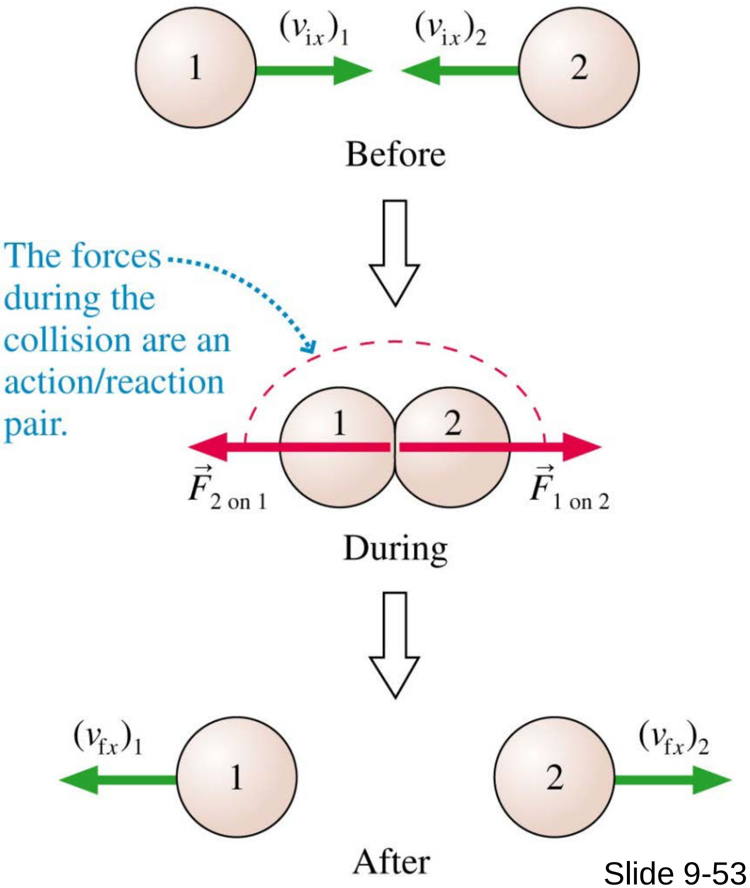
Conservation of Momentum

- Two objects collide, as shown.
- Neglect all outside forces on the objects.
- Due to the fact that the only forces on the objects are equal and opposite, the sum of their momenta is:

$$(p_x)_1 + (p_x)_2 = \text{constant}$$

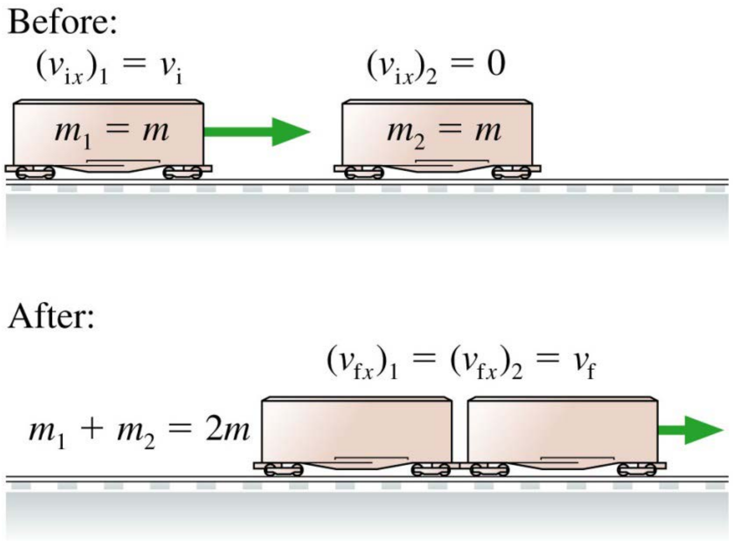
- This is a conservation law!
- The sum of the momenta *before* and *after* the collision are equal:

$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2$$



Conservation of Momentum: Quick Example

A train car moves to the right with initial speed v_i. It collides with a stationary train car of equal mass. After the collision the two cars are stuck together. What is the train cars' final velocity?



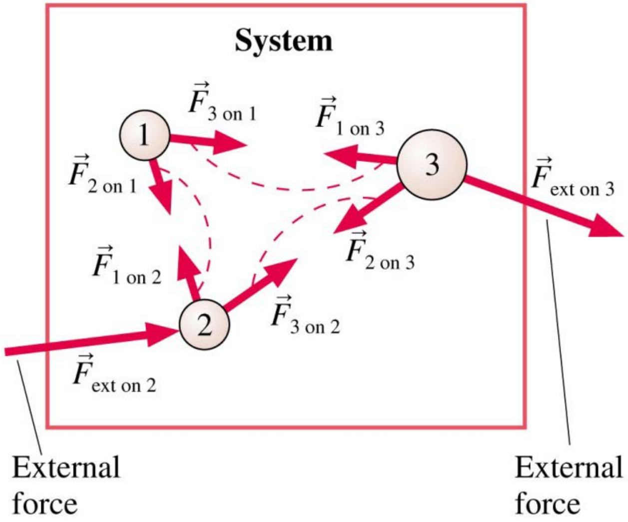
- According to conservation of momentum, before and after the collision:

$$m_1 (v_{fx})_1 + m_2 (v_{fx})_2 = m_1 (v_{ix})_1 + m_2 (v_{ix})_2$$

$$mv_f + mv_f = 2mv_f = mv_i + 0$$

- The mass cancels, and we find that the final velocity is $v_f = ½ v_i$.

Momentum of a System



- Consider a system of N interacting particles.
- The figure shows a simple case where $N = 3$.
- The system has a **total momentum**:

$$\vec{P} = \text{total momentum} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_N = \sum_{k=1}^{N} \vec{p}_k$$

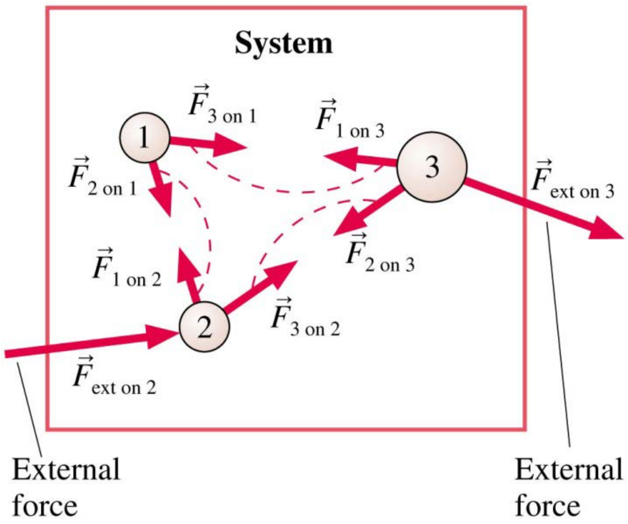
- Applying Newton's second law for each individual particle, we find the rate of change of the total momentum of the system is:

$$\frac{d\vec{P}}{dt} = \sum_k \sum_{j \neq k} \vec{F}_{j \text{ on } k} + \sum_k \vec{F}_{\text{ext on } k}$$

Momentum of a System

- The interaction forces come in action/reaction pairs, with $\vec{F}_{k \text{ on } j} = -\vec{F}_{j \text{ on } k}$.
- Consequently, the sum of all the interaction forces is zero.
- Therefore:

$$\frac{d\vec{P}}{dt} = \sum_k \vec{F}_{\text{ext on } k} = \vec{F}_{\text{net}}$$



- **The rate of change of the total momentum of the system is equal to the net force applied to the system.**
- This result *justifies* our particle model: Internal forces between atoms in an object do not affect the motion of the object as a whole.

Law of Conservation of Momentum

- An **isolated system** is a system for which the *net* external force is zero: $\vec{F}_{\text{net}} = \vec{0}$

- For an isolated system:

$$\frac{d\vec{P}}{dt} = \vec{0} \qquad \text{(isolated system)}$$

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

- Or, written mathematically:

$$\vec{P}_{\text{f}} = \vec{P}_{\text{i}}$$

Problem-Solving Strategy: Conservation of Momentum

PROBLEM-SOLVING STRATEGY 9.1 **Conservation of momentum**

MODEL Clearly define *the system.*

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapters 10 and 11, conservation of energy.

Problem-Solving Strategy: Conservation of Momentum

PROBLEM-SOLVING STRATEGY 9.1 **Conservation of momentum**

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 16

Example 9.4 Rolling Away

EXAMPLE 9.4 Rolling away

Bob sees a stationary cart 8.0 m in front of him. He decides to run to the cart as fast as he can, jump on, and roll down the street. Bob has a mass of 75 kg and the cart's mass is 25 kg. If Bob accelerates at a steady 1.0 m/s^2, what is the cart's speed just after Bob jumps on?

Example 9.4 Rolling Away

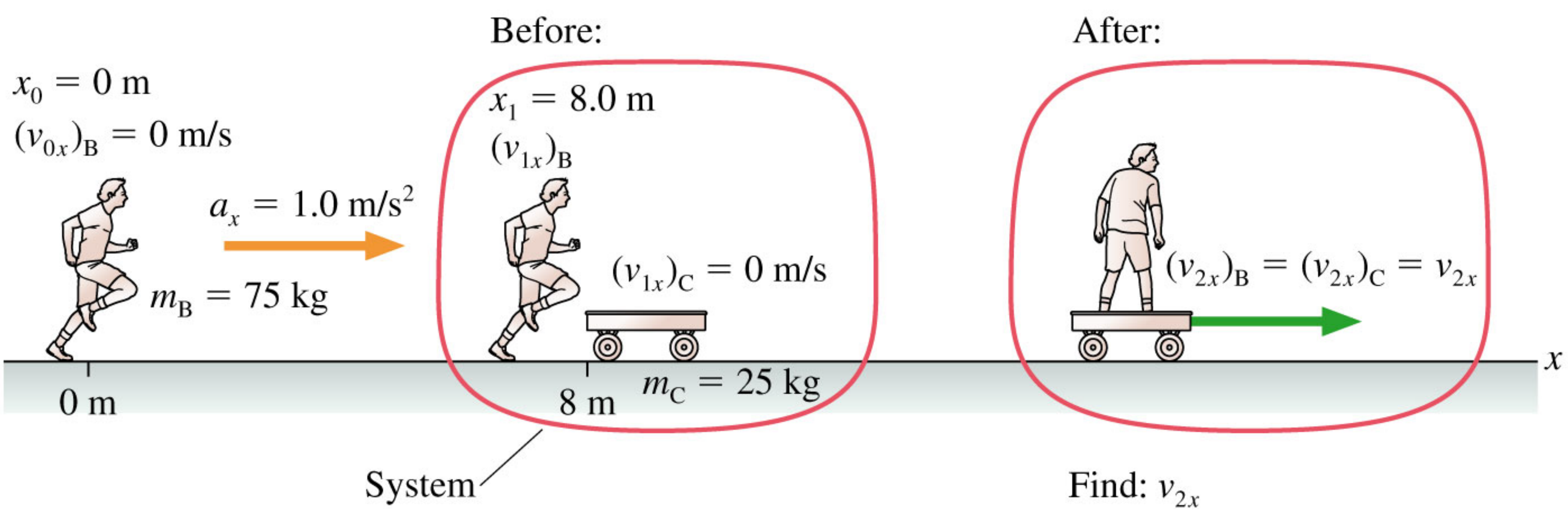
EXAMPLE 9.4 Rolling away

MODEL This is a two-part problem. First Bob accelerates across the ground. Then Bob lands on and sticks to the cart, a "collision" between Bob and the cart. The interaction forces between Bob and the cart (i.e., friction) act only over the fraction of a second it takes Bob's feet to become stuck to the cart. Using the impulse approximation allows the system Bob + cart to be treated as an isolated system during the brief interval of the "collision," and thus the total momentum of Bob + cart is conserved during this interaction. But the system Bob + cart is *not* an isolated system for the entire problem because Bob's initial acceleration has nothing to do with the cart.

Example 9.4 Rolling Away

EXAMPLE 9.4 Rolling away

VISUALIZE Our strategy is to divide the problem into an *acceleration* part, which we can analyze using kinematics, and a *collision* part, which we can analyze with momentum conservation. The pictorial representation below includes information about both parts. Notice that Bob's velocity $(v_{1x})_B$ at the end of his run is his "before" velocity for the collision.



Example 9.4 Rolling Away

EXAMPLE 9.4 Rolling away

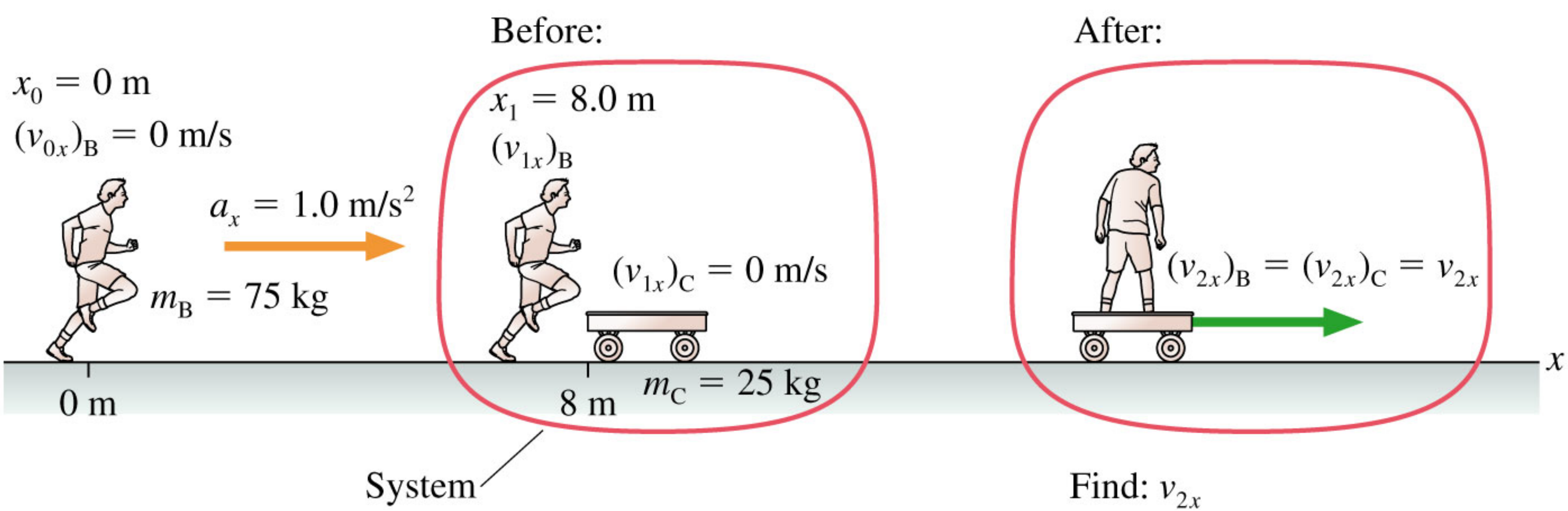
SOLVE The first part of the mathematical representation is kinematics. We don't know how long Bob accelerates, but we do know his acceleration and the distance. Thus

$$(v_{1x})_B^2 = (v_{0x})_B^2 + 2a_x\,\Delta x = 2a_x x_1$$

His velocity after accelerating for 8.0 m is

$$(v_{1x})_B = \sqrt{2a_x x_1} = 4.0 \text{ m/s}$$

The second part of the problem, the collision, uses conservation of momentum: $P_{2x} = P_{1x}$. Equation 9.21 is



Example 9.4 Rolling Away

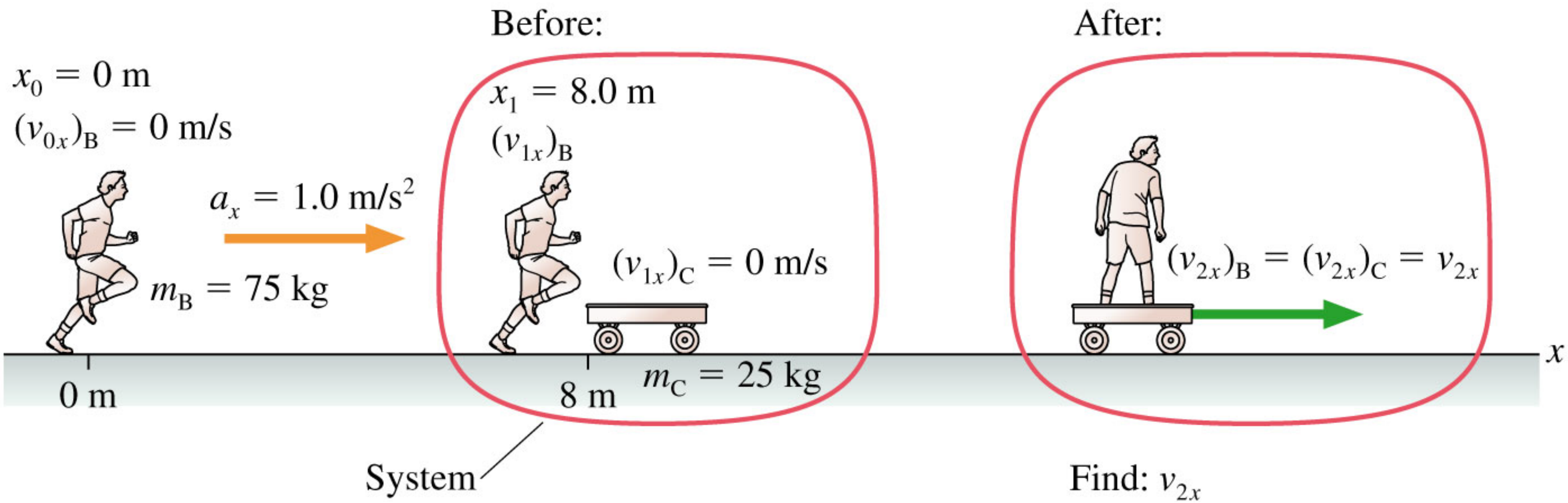
EXAMPLE 9.4 Rolling away

$$m_B(v_{2x})_B + m_C(v_{2x})_C = m_B(v_{1x})_B + m_C(v_{1x})_C = m_B(v_{1x})_B$$

where we've used $(v_{1x})_C = 0$ m/s because the cart starts at rest. In this problem, Bob and the cart move together at the end with a common velocity, so we can replace both $(v_{2x})_B$ and $(v_{2x})_C$ with simply v_{2x}. Solving for v_{2x}, we find

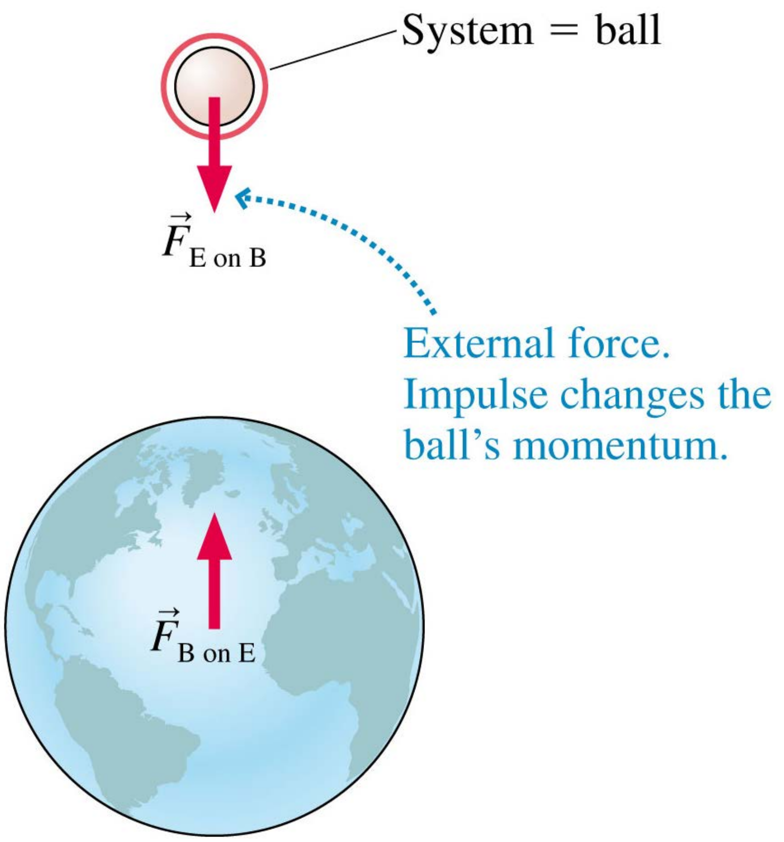
$$v_{2x} = \frac{m_B}{m_B + m_C}(v_{1x})_B = \frac{75\text{ kg}}{100\text{ kg}} \times 4.0\text{ m/s} = 3.0\text{ m/s}$$

The cart's speed is 3.0 m/s immediately after Bob jumps on.



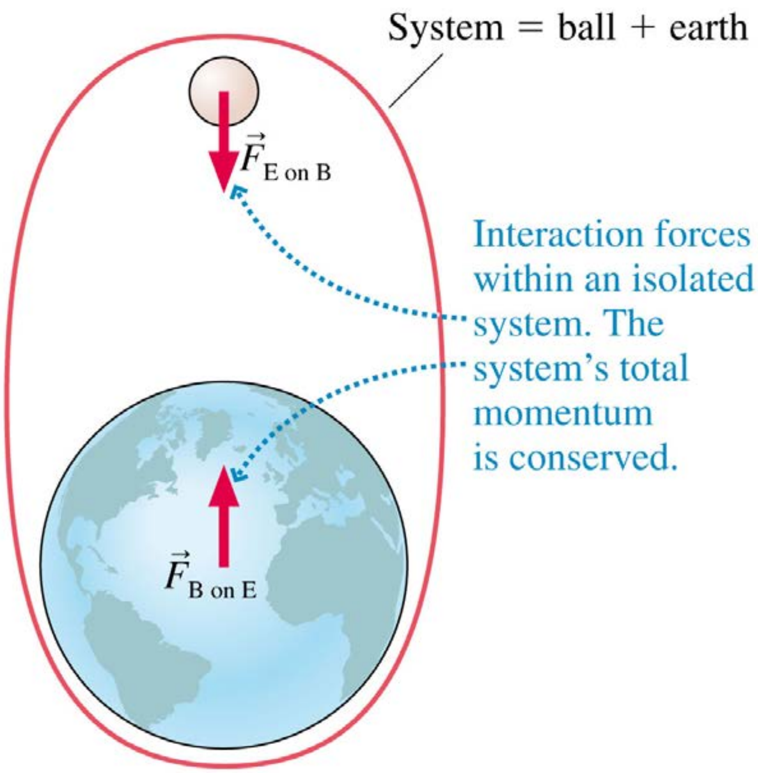
Conservation of Momentum Depends on the Choice of System

- A rubber ball is dropped, and falls toward earth.
- Define the ball as the System.
- The gravitational force of the earth is an external force.
- The momentum of the system is *not* conserved.



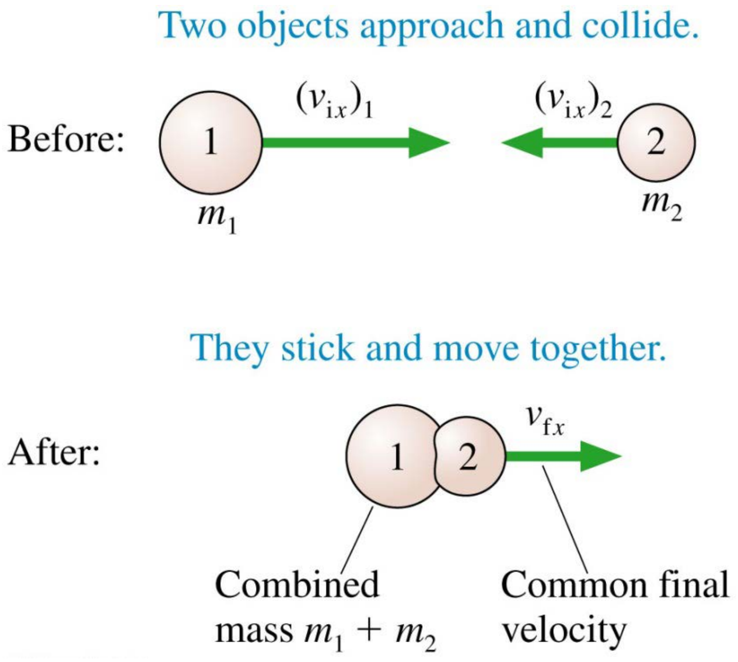
Conservation of Momentum Depends on the Choice of System

- A rubber ball is dropped, and falls toward earth.
- Define the ball + earth as the System.
- The gravitational forces are interactions *within* the system.
- This is an isolated system, so the total momentum $\vec{P} = \vec{p}_{\text{ball}} + \vec{p}_{\text{earth}}$ is conserved.



Inelastic Collisions

- A collision in which the two objects stick together and move with a common final velocity is called a perfectly inelastic collision.



- Examples of inelastic collisions:
  - A piece of clay hits the floor.
  - A bullet strikes a block of wood and embeds itself in the block.
  - Railroad cars coupling together upon impact.
  - A dart hitting a dart board.

Example 9.5 An Inelastic Glider Collision

EXAMPLE 9.5 An inelastic glider collision

In a laboratory experiment, a 200 g air-track glider and a 400 g air-track glider are pushed toward each other from opposite ends of the track. The gliders have Velcro tabs on the front and will stick together when they collide. The 200 g glider is pushed with an initial speed of 3.0 m/s. The collision causes it to reverse direction at 0.40 m/s. What was the initial speed of the 400 g glider?

Example 9.5 An Inelastic Glider Collision

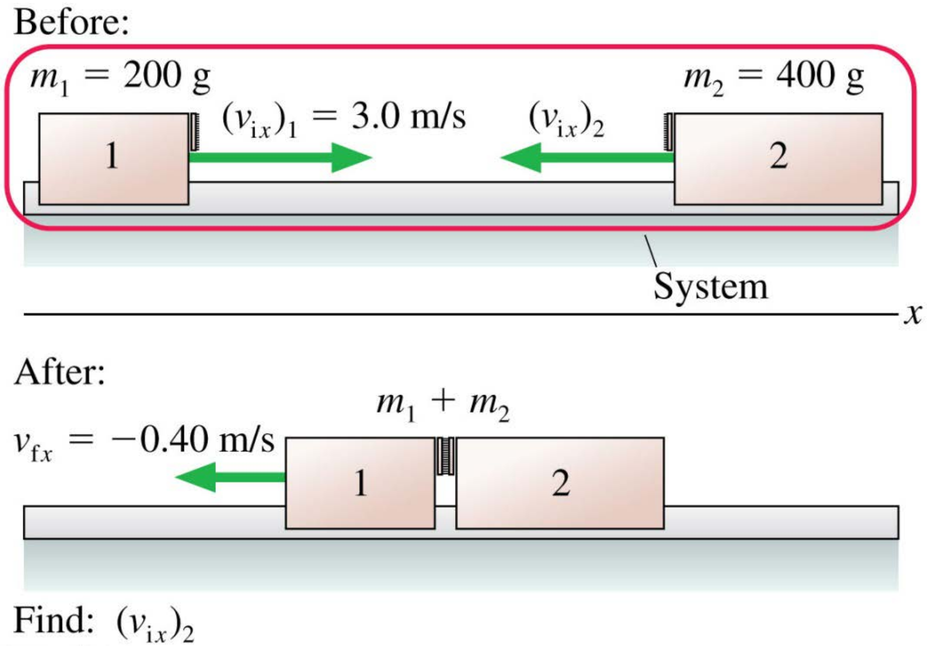
EXAMPLE 9.5 An inelastic glider collision

MODEL Model the gliders as particles. Define the two gliders together as the system. This is an isolated system, so its total momentum is conserved in the collision. The gliders stick together, so this is a perfectly inelastic collision.

Example 9.5 An Inelastic Glider Collision

EXAMPLE 9.5 An inelastic glider collision

VISUALIZE The figure below shows a pictorial representation. We've chosen to let the 200 g glider (glider 1) start out moving to the right, so $(v_{ix})_1$ is a positive 3.0 m/s. The gliders move to the left after the collision, so their common final velocity is $v_{fx} = -0.40$ m/s.



Example 9.5 An Inelastic Glider Collision

EXAMPLE 9.5 An inelastic glider collision

SOLVE The law of conservation of momentum, $P_{fx} = P_{ix}$, is

$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2$$

where we made use of the fact that the combined mass $m_1 + m_2$ moves together after the collision. We can easily solve for the initial velocity of the 400 g glider:

$$(v_{ix})_2 = \frac{(m_1 + m_2)v_{fx} - m_1(v_{ix})_1}{m_2}$$

$$= \frac{(0.60 \text{ kg})(-0.40 \text{ m/s}) - (0.20 \text{ kg})(3.0 \text{ m/s})}{0.40 \text{ kg}}$$

$$= -2.1 \text{ m/s}$$

The negative sign indicates that the 400 g glider started out moving to the left. The initial *speed* of the glider, which we were asked to find, is 2.1 m/s.

Explosions

- An **explosion** is the opposite of a collision.
- The particles first have a brief, intense interaction, then they move apart from each other.

- The explosive forces are *internal* forces.
- If the system is isolated, its total momentum during the explosion will be conserved.

Example 9.7 Recoil

EXAMPLE 9.7 Recoil

A 10 g bullet is fired from a 3.0 kg rifle with a speed of 500 m/s. What is the recoil speed of the rifle?

Example 9.7 Recoil

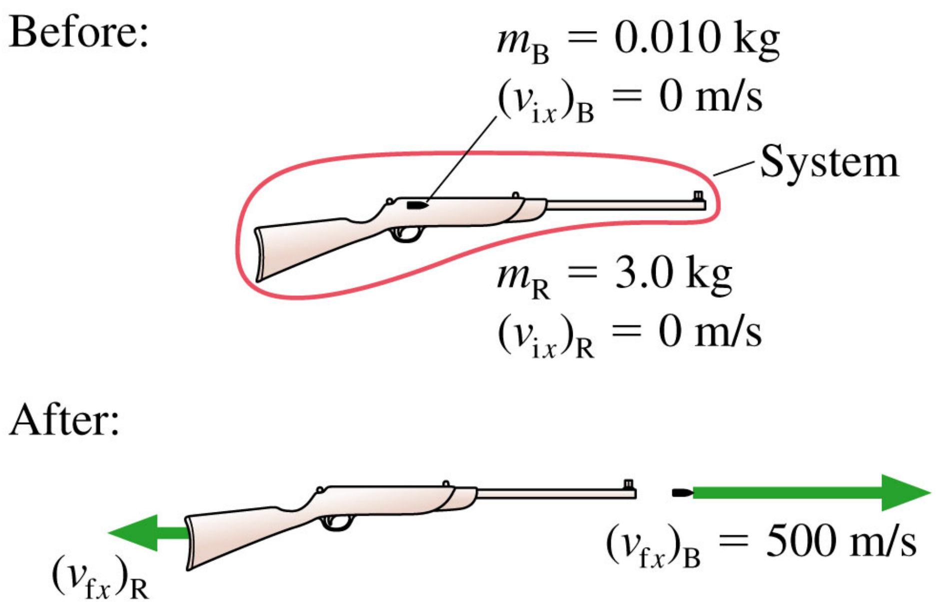
EXAMPLE 9.7 Recoil

MODEL The rifle causes a small mass of gunpowder to explode, and the expanding gas then exerts forces on *both* the bullet and the rifle. Let's define the system to be bullet + gas + rifle. The forces due to the expanding gas during the explosion are internal forces, within the system. Any friction forces between the bullet and the rifle as the bullet travels down the barrel are also internal forces. Gravity, the only external force, is balanced by the normal forces of the barrel on the bullet and the person holding the rifle, so $\vec{F}_{net} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.

Example 9.7 Recoil

EXAMPLE 9.7 Recoil

VISUALIZE The figure below shows a pictorial representation before and after the bullet is fired.



Find: $(v_{fx})_R$

Example 9.7 Recoil

EXAMPLE 9.7 Recoil

SOLVE The x-component of the total momentum is $P_x = (p_x)_B + (p_x)_R + (p_x)_{gas}$. Everything is at rest before the trigger is pulled, so the initial momentum is zero. The law of conservation of momentum is thus

$$P_{fx} = m_B(v_{fx})_B + m_R(v_{fx})_R = P_{ix} = 0$$

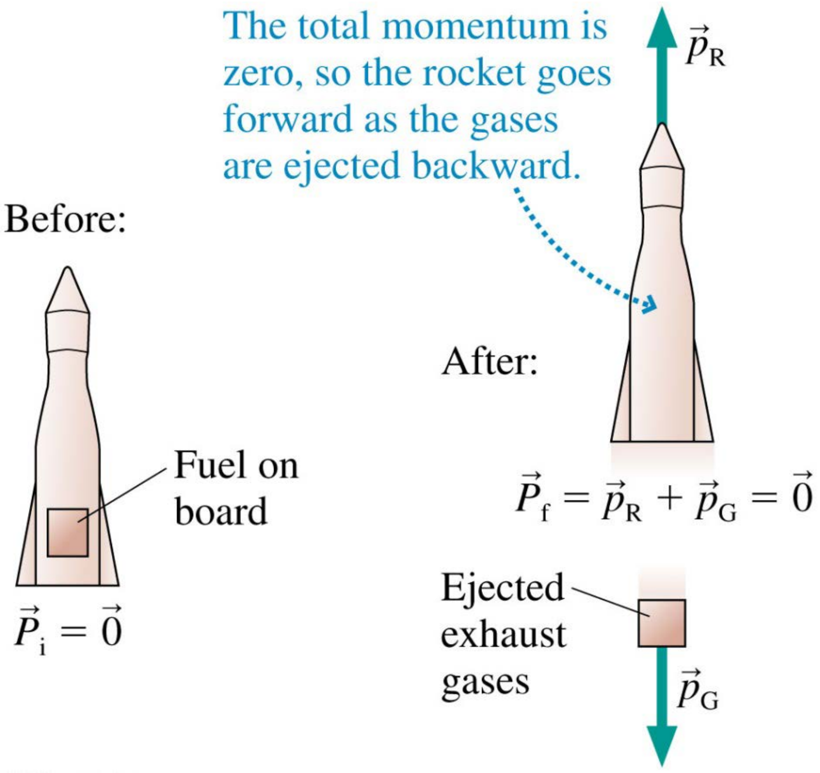
Solving for the rifle's velocity, we find

$$(v_{fx})_R = -\frac{m_B}{m_R}(v_{fx})_B = -\frac{0.010\text{ kg}}{3.0\text{ kg}} \times 500\text{ m/s} = -1.7\text{ m/s}$$

The minus sign indicates that the rifle's recoil is to the left. The recoil *speed* is 1.7 m/s.

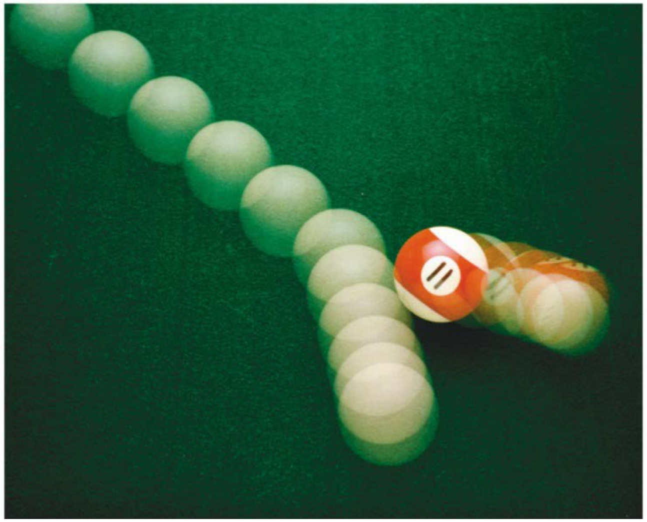
Rocket or Jet Propulsion

- The figure shows a rocket with a parcel of fuel on board.
- If we choose rocket + gases to be the system, the burning and expulsion are both internal forces.
- The exhaust gases gain backward momentum as they are shot out the back.
- The *total* momentum of the system remains zero.
- Therefore, the rocket gains forward momentum.



Momentum in Two Dimensions

- The total momentum $\vec{P}$ is a vector sum of the momenta $\vec{p} = m\vec{v}$ of the individual particles.



- Momentum is conserved only if each component of $\vec{P}$ is conserved:

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

Chapter 9 Summary Slides

General Principles

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

General Principles

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

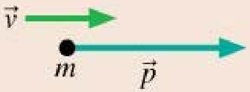
$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

Important Concepts

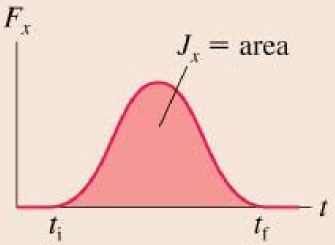
Momentum $\vec{p} = m\vec{v}$



Impulse $J_x = \int_{t_i}^{t_f} F_x(t)\,dt$ = area under force curve

Impulse and momentum are related by the **impulse-momentum theorem**

$$\Delta p_x = J_x$$

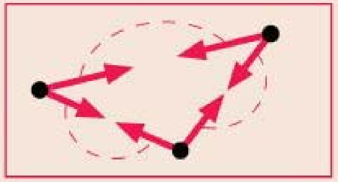


The impulse delivered to a particle causes the particle's momentum to change. This is an alternative statement of Newton's second law.

Important Concepts

System A group of interacting particles.

Isolated system A system on which there are no external forces or the net external force is zero.



Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.