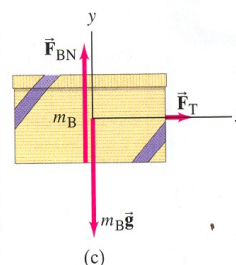
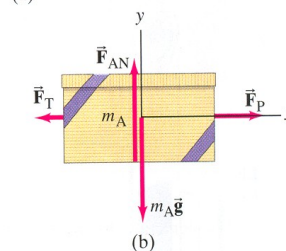
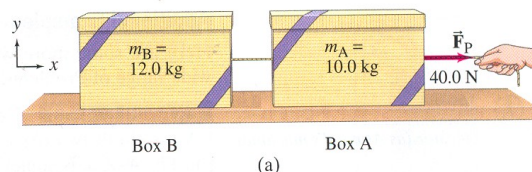


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FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0\text{ N}$. (b) Free-body diagram for box A. (c) Free-body diagram for box B.



Our next Example involves two boxes connected by a cord. We can refer to this group of objects as a system. A *system* is any group of one or more objects we choose to consider and study.

EXAMPLE 4–12 Two boxes connected by a cord. Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg . A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (a) the acceleration of each box, and (b) the tension in the cord connecting the boxes.

APPROACH We streamline our approach by not listing each step. We have two boxes so we need to draw a free-body diagram for each box. To draw them correctly, we must consider the forces on *each* box by itself, so that Newton's second law can be applied to each. The person exerts a force F_P on box A. Box A exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box A (Newton's third law). These two horizontal forces on box A are shown in Fig. 4–22b, along with the force of gravity $m_A \vec{g}$ downward and the normal force $\vec{F}_{AN}$ exerted upward by the table. The cord is light, so we neglect its mass. The tension at each end of the cord is thus the same. Hence the cord exerts a force F_T on the second box. Figure 4–22c shows the forces on box B, which are $\vec{F}_T$, $m_B \vec{g}$, and the normal force $\vec{F}_{BN}$. There will be only horizontal motion. We take the positive x axis to the right.

SOLUTION (a) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \quad [\text{box A}]$$

For box B, the only horizontal force is F_T , so

$$\Sigma F_x = F_T = m_B a_B. \quad [\text{box B}]$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a . Thus $a_A = a_B = a$. We are given $m_A = 10.0\text{ kg}$ and $m_B = 12.0\text{ kg}$. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\text{ N}}{22.0\text{ kg}} = 1.82\text{ m/s}^2.$$

This is what we sought.

Alternate Solution We would have obtained the same result had we considered a single system, of mass $m_A + m_B$, acted on by a net horizontal force equal to F_P . (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

(b) From the equation above for box B ($F_T = m_B a_B$), the tension in the cord is

$$F_T = m_B a = (12.0\text{ kg})(1.82\text{ m/s}^2) = 21.8\text{ N}.$$

Thus, F_T is less than $F_P (= 40.0\text{ N})$, as we expect, since F_T acts to accelerate only m_B .

NOTE It might be tempting to say that the force the person exerts, F_P , acts not only on box A but also on box B. It doesn't. F_P acts only on box A. It affects box B via the tension in the cord, F_T , which acts on box B and accelerates it.

PROBLEM SOLVING
An alternate analysis

CAUTION
For any object, use only the forces on that object in calculating $\Sigma F = ma$

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To evaluate the tension in the cord between the boxes we use the acceleration we have calculated to write

$$T = m a$$

$$T = 12.0 \text{ kg} * 1.82 \frac{\text{m}}{\text{s}^2} = 21.8 \text{ N}$$

In the above “alternative solution” we consider the two boxes as one, with the cord supplying an *internal* force.

PHYSICS APPLIED
Elevator (as Atwood's machine)

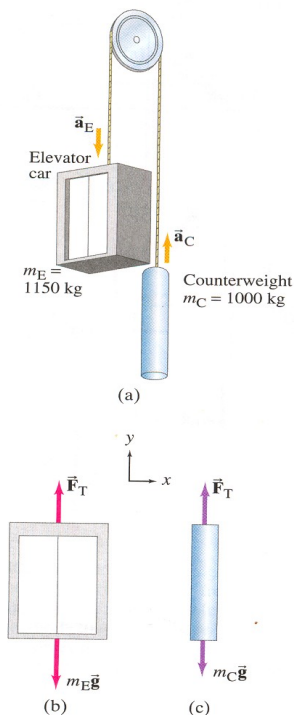


FIGURE 4-23 Example 4-13.
(a) Atwood's machine in the form of an elevator-counterweight system. (b) and (c) Free-body diagrams for the two objects.

PROBLEM SOLVING
Check your result by seeing if it works in situations where the answer is easily guessed

Additional Examples

Here are some more worked-out Examples to give you practice in solving a wide range of Problems.

EXAMPLE 4-13 Elevator and counterweight (Atwood's machine).

A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4-23a, is sometimes referred to as an *Atwood's machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000$ kg. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150$ kg. For the latter case ($m_E = 1150$ kg), calculate (a) the acceleration of the elevator and (b) the tension in the cable.

APPROACH Again we have two objects, and we will need to apply Newton's second law to each of them separately. Each mass has two forces acting on it: gravity downward and the cable tension pulling upward, $\vec{F}_T$. Figures 4-23b and c show the free-body diagrams for the elevator (m_E) and for the counterweight (m_C). The elevator, being the heavier, will accelerate downward, whereas the counterweight will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable doesn't stretch). For the counterweight, $m_C g = (1000 \text{ kg})(9.80 \text{ m/s}^2) = 9800 \text{ N}$, so F_T must be greater than 9800 N (in order that m_C will accelerate upward). For the elevator, $m_E g = (1150 \text{ kg})(9.80 \text{ m/s}^2) = 11,300 \text{ N}$, which must have greater magnitude than F_T so that m_E accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N.

SOLUTION (a) To find F_T as well as the acceleration a , we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$\begin{aligned} F_T - m_E g &= m_E a_E = -m_E a \\ F_T - m_C g &= m_C a_C = +m_C a. \end{aligned}$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a :

$$a = \frac{m_E - m_C}{m_E + m_C} g = \frac{1150 \text{ kg} - 1000 \text{ kg}}{1150 \text{ kg} + 1000 \text{ kg}} g = 0.070 g = 0.68 \text{ m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070 g = 0.68 \text{ m/s}^2$.

(b) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations, setting $a = 0.070 g = 0.68 \text{ m/s}^2$:

$$\begin{aligned} F_T - m_E g &= m_E a = m_E (g - a) \\ &= 1150 \text{ kg} (9.80 \text{ m/s}^2 - 0.68 \text{ m/s}^2) = 10,500 \text{ N}, \\ F_T &= m_C g + m_C a = m_C (g + a) \\ &= 1000 \text{ kg} (9.80 \text{ m/s}^2 + 0.68 \text{ m/s}^2) = 10,500 \text{ N}, \end{aligned}$$

which are consistent. As predicted, our result lies between 9800 N and 11,300 N.

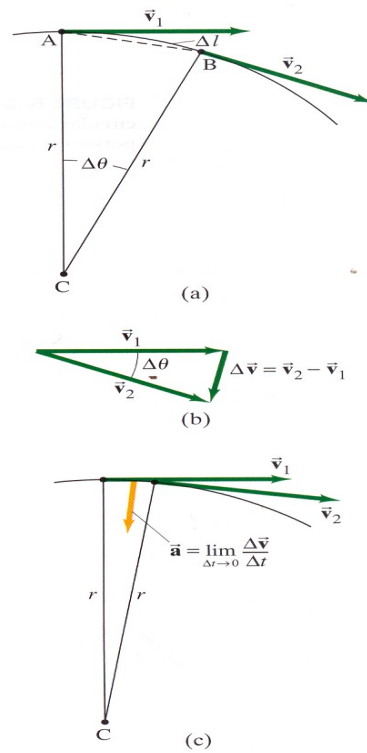
NOTE We can check our equation for the acceleration a in this Example by noting that if the masses were equal ($m_E = m_C$), then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_C = 0$), then the other mass ($m_E \neq 0$) would be predicted by our equation to accelerate at $a = g$, again as expected.

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Chapter 5 Circular Motion and Gravity

In circular motion the direction of the acceleration is toward the center of the circle

FIGURE 5-2 Determining the change in velocity, $\Delta \vec{v}$, for a particle moving in a circle. The length Δl is the distance along the arc, from A to B.



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Deducing the magnitude of the centripetal acceleration.

1. The magnitude of the centripetal acceleration *increases* as the curve gets sharper, I. e. as the radius decreases.

2. The magnitude of the centripetal acceleration *increases* as the speed *increases*

First try at a formula

$$a_c = \frac{v}{r}$$

Dimensional check $\frac{\frac{m}{s}}{r} = \frac{1}{s}$ which is wrong

Use clue that the centripetal acceleration does NOT depend on direction of the rotation. So $-v$ should be the same as v

$$a_c = \frac{v^2}{r}$$

Dimension check

$$\frac{\left[\frac{m}{s}\right]^2}{m} = \frac{m}{s^2} \text{ which is correct.}$$

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There is no real "centrifugal force"

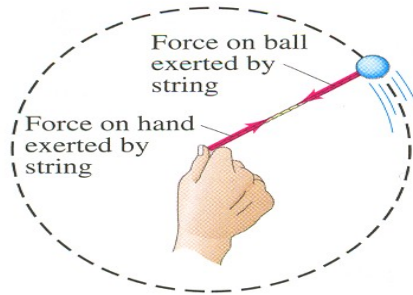
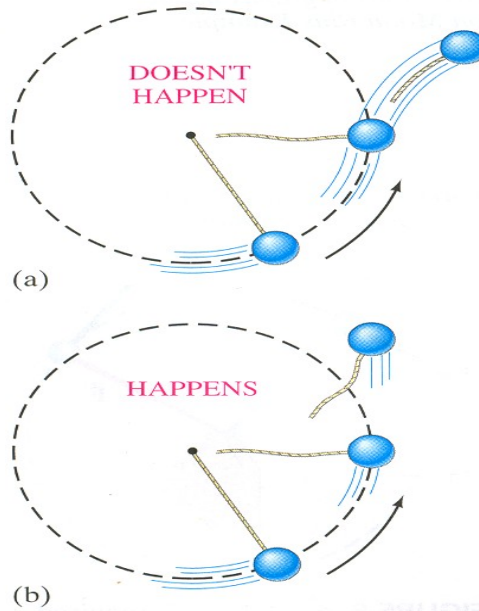


FIGURE 5-5 Swinging a ball on the end of a string.

FIGURE 5-6 If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b). For example, in (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.



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EXAMPLE 5-4 Revolving ball (vertical circle). A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. (a) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (b) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (a).

APPROACH The ball moves in a vertical circle and is *not* undergoing uniform circular motion. The radius is assumed constant, but the speed v changes because of gravity. Nonetheless, Eq. 5-1 is valid at each point along the circle, and we use it at points 1 and 2. The free-body diagram is shown in Fig. 5-8 for both positions 1 and 2.

SOLUTION (a) At the top (point 1), two forces act on the ball: $m\vec{g}$, the force of gravity, and $\vec{F}_{T1}$, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R . We apply Newton's second law, for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m \frac{v_1^2}{r} \quad [\text{at top}]$$

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$, for which we have

$$mg = m \frac{v_1^2}{r} \quad [\text{minimum speed at top}]$$

We solve for v_1 :

$$v_1 = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(1.10 \text{ m})} = 3.28 \text{ m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(b) When the ball is at the bottom of the circle (point 2 in Fig. 5-8), the cord exerts its tension force F_{T2} upward, whereas the force of gravity, $m\vec{g}$, still acts downward. So we apply Newton's second law, this time choosing *upward* as positive since the acceleration is upward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m \frac{v_2^2}{r} \quad [\text{at bottom}]$$

The speed v_2 is given as twice that in (a), namely 6.56 m/s. We solve for F_{T2} :

$$F_{T2} = m \frac{v_2^2}{r} + mg$$

$$= (0.150 \text{ kg}) \frac{(6.56 \text{ m/s})^2}{(1.10 \text{ m})} + (0.150 \text{ kg})(9.80 \text{ m/s}^2) = 7.34 \text{ N}.$$

EXERCISE B In a tumble dryer, the speed of the drum should be just large enough so that the clothes are carried nearly to the top of the drum and then fall away, rather than being pressed against the drum for the whole revolution. Determine whether this speed will be different for heavier wet clothes than for lighter dry clothes.

EXERCISE C A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5-9). Is the normal force that the seat exerts on the rider at the top of the wheel (a) less than, (b) more than, or (c) the same as, the force the seat exerts at the bottom of the wheel?

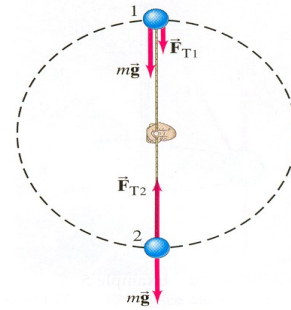


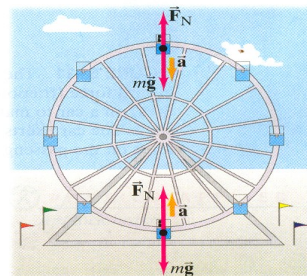
FIGURE 5-8 Example 5-4. Free-body diagrams for positions 1 and 2.

Cord tension and gravity together provide centripetal acceleration

Gravity provides centripetal acceleration

String tension and gravity acting in opposite directions provide centripetal acceleration

FIGURE 5-9 Exercise C.



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The Law of Gravity

Up to this point we have talked about forces without any fundamental definition of what a force is.

Now we are going to define the first of the 4 forces which exist in the universe.

The hypothesis that that same force which causes objects to fall on the surface of the Earth is the same as the force as that which causes the planets to stay in their orbits.

The clue from the Moon.

The distance from the Earth to the moon could be fairly accurately measured.

By knowing the distance the centripetal acceleration of the moon can be calculated.

It results in that fact that the acceleration of the moon is $\frac{1}{3600}$ of the acceleration of objects on the Earth.

Since the distance to the Moon is 60 times the radius of the Earth, the force of gravity is

$$\frac{1}{60^2} = \frac{1}{3600}$$

The *inverse square* relationship --- the force of gravity is inversely proportional to the distance between the objects.

The dependence on mass

The fact that objects of different masses fall with the same speed means that the force of gravity must be proportional to the mass.

$$F_G = m g$$

To have a force we need *two* masses.

By Newton's 3rd law the force must be dependent on *both* masses.

If we put these 2 fact together we get that

$$F_G \propto \frac{m_1 m_2}{r_{12}^2}$$

The Universal Law of Gravitation

To make this proportionality into an equation we use the **universal gravitational constant G**

$$F_G = G \frac{m_1 m_2}{r_{12}^2}$$