# Area and Estimating with Finite Sums

Part 2

Finite Sums with
Distance Traveled and Displacement

# Finite Sums and Distance Traveled

- $v(t) =$ velocity of an object moving in one direction along a straight line
- $s(t) =$ position of the object
- $v = \frac{ds}{dt} \Leftrightarrow s = \int v \, dt$

# Finite Sums and Distance Traveled

- If $F(t)$ is any antiderivative of $v(t)$, then $s(t) = F(t) + C$
- The **total distance traveled** by the object over the time interval $[a, b]$ is given by

$$s(b) - s(a) = [F(b) + C] - [F(a) + C]$$
$$= F(b) - F(a)$$

# Finite Sums and Distance Traveled

We don't always know a formula for $v(t)$

To approximate the total distance traveled:

- Subdivide $[a, b]$ into $n$ subintervals of equal width of $\Delta t$

- Let $t_k$ be any point in the $k$-th subinterval

- If $\Delta t$ is small enough, the total distance traveled on the $k$-th subinterval is approximately $v(t_k)\Delta t$

$$s(b) - s(a) \approx v(t_1)\Delta t + v(t_2)\Delta t + v(t_3)\Delta t + \cdots + v(t_n)\Delta t$$

# Finite Sums and Distance Traveled

$$\text{Total distance traveled } = s(b) - s(a)$$
$$\approx v(t_1)\Delta t + v(t_2)\Delta t + v(t_3)\Delta t + \cdots + v(t_n)\Delta t$$

This is the approximation for the area under the curve $y = v(t)$ over the interval $[a, b]$!

# Example 1

You are sitting on a bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour, with the results in the accompanying table. About how far upstream did the bottle travel during that hour? Find an estimate using 12 subintervals of length 5 with

a. Left endpoint values

b. Right endpoint values

| **Time (min)** | **0** | **5** | **10** | **15** | **20** | **25** | **30** | **35** | **40** | **45** | **50** | **55** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Velocity (m/sec) | 1 | 1.2 | 1.7 | 2 | 1.8 | 1.6 | 1.4 | 1.2 | 1 | 1.8 | 1.5 | 1.2 | 0 |

# Example 1 (continued)

Solution (a):

$$[a, b] = [0, 60] \text{ minutes}$$

$$n = 12$$

$$\Delta t = \frac{\text{width of interval}}{\text{number of subintervals}}$$

$$= \frac{60 - 0}{12} = 5 \text{ minutes} = 300 \text{ seconds}$$

(We change time to seconds to match velocity!)

# Example 1 (continued)

| Time (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Velocity (m/sec) | 1 | 1.2 | 1.7 | 2 | 1.8 | 1.6 | 1.4 | 1.2 | 1 | 1.8 | 1.5 | 1.2 | 0 |

- Interval 1 is [0,5]; Interval 2 is [5,10]; Interval 3 is [15,20];…;Interval 12 is [55,60]
- $t_k =$ Left endpoint value of $k$-th interval
- $t_1 = 0;\ t_2 = 5; t_3 = 10; \dots; t_{12} = 55$

# Example 1 (continued)

| Time (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Velocity (m/sec) | 1 | 1.2 | 1.7 | 2 | 1.8 | 1.6 | 1.4 | 1.2 | 1 | 1.8 | 1.5 | 1.2 | 0 |

$$\begin{aligned}\text{distance traveled} &\approx v(t_1)\Delta t + v(t_2)\Delta t + v(t_3)\Delta t + \cdots + v(t_{12})\Delta t \\ &= v(0)\Delta t + v(5)\Delta t + v(10)\Delta t + \cdots + v(55)\Delta t \\ &= (1)300 + (1.2)300 + (1.7)300 + (2)300 + (1.8)300 \\ &+ (1.6)300 + (1.4)300 + (1.2)300 + (1)300 + (1.8)\,300 \\ &+ (1.5)300 + (1.2)300 = 5220\text{ m}\end{aligned}$$

Answer (a): The bottle traveled about 5220 m upstream during that hour.

# Example 1 (continued)

Solution (b):

The only change is that we are using the right endpoint values.

$$\Delta t = 300 \text{ seconds}$$

| **Time (min)** | **0** | **5** | **10** | **15** | **20** | **25** | **30** | **35** | **40** | **45** | **50** | **55** | **60** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Velocity (m/sec) | 1 | 1.2 | 1.7 | 2 | 1.8 | 1.6 | 1.4 | 1.2 | 1 | 1.8 | 1.5 | 1.2 | 0 |

- $t_k$ = Right endpoint value of $k$-th interval
- $t_1 = 5;\ t_2 = 10; t_3 = 15; \ldots; t_{12} = 60$

# Example 1 (continued)

| Time (min) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Velocity (m/sec) | 1 | 1.2 | 1.7 | 2 | 1.8 | 1.6 | 1.4 | 1.2 | 1 | 1.8 | 1.5 | 1.2 | 0 |

$$\begin{aligned}\text{distance traveled} &\approx v(t_1)\Delta t + v(t_2)\Delta t + v(t_3)\Delta t + \cdots + v(t_{12})\Delta t \\ &= v(5)\Delta t + v(10)\Delta t + v(15)\Delta t + \cdots + v(60)\Delta t \\ &= (1.2)300 + (1.7)300 + (2)300 + (1.8)300 + (1.6)300 \\ &+ (1.4)300 + (1.2)300 + (1)300 + (1.8)\,300 + (1.5)300 \\ &+ (1.2)300 + (0)300 = 4920 \text{ m}\end{aligned}$$

Answer (b): The bottle traveled about 4920 m upstream during that hour.

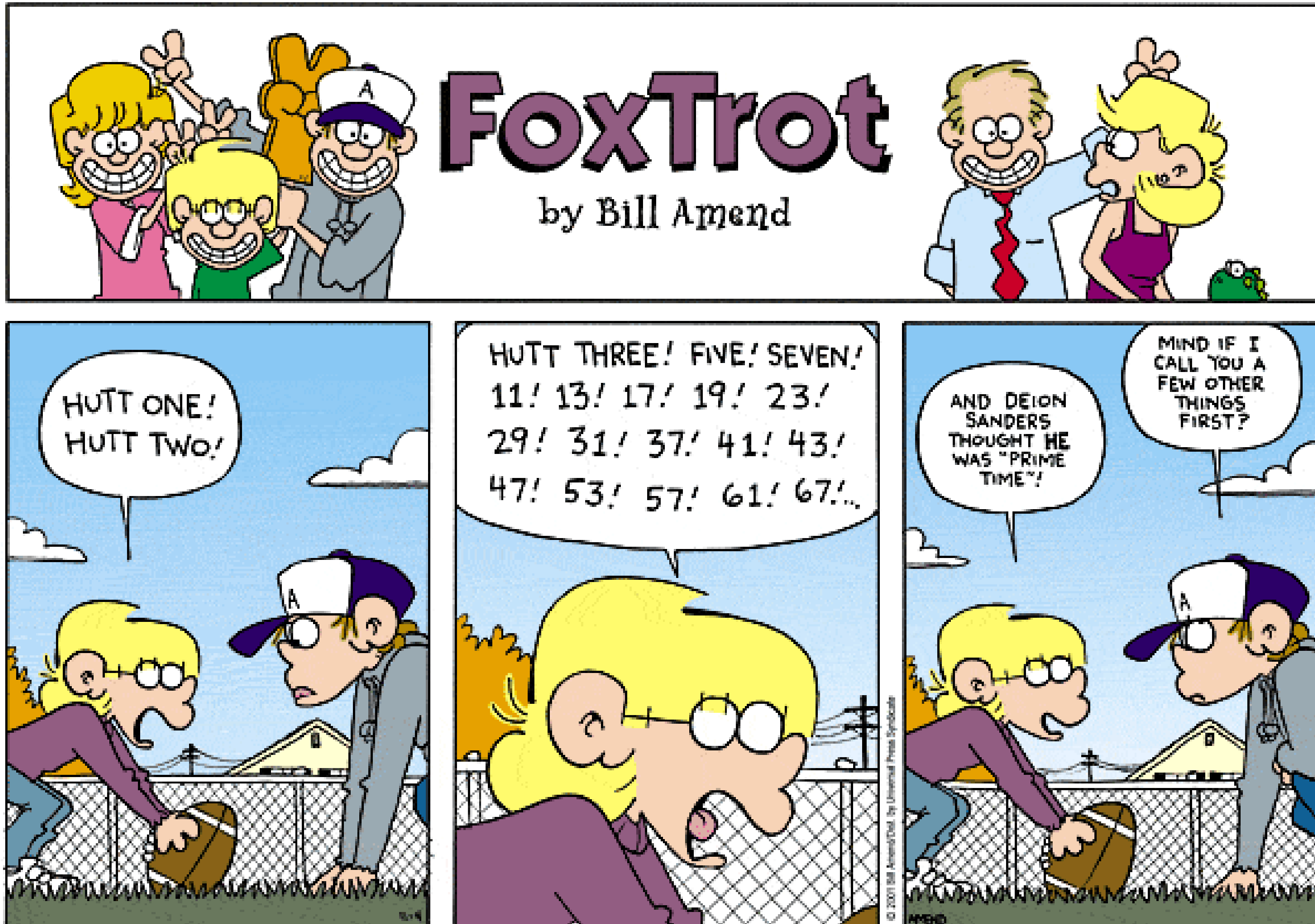
# Finite Sums and Distance Traveled

- $v(t) =$ velocity of an object moving in <u>both</u> directions along a straight line
- $s(t) =$ position of the object
- $v = \frac{ds}{dt} \Leftrightarrow s = \int v\,dt$
- $s(b) - s(a)$ *no longer represents the total distance traveled*
- $s(b) - s(a)$ is the **displacement** of the object (the difference between its initial and final positions)

# Finite Sums and Distance Traveled

The **total distance traveled** is approximately:

$$|v(t_1)|\Delta t + |v(t_2)|\Delta t + |v(t_3)|\Delta t + \cdots + |v(t_n)|\Delta t$$


FoxTrot
by Bill Amend
HUTT ONE! HUTT TWO!
HUTT THREE! FIVE! SEVEN! 11! 13! 17! 19! 23! 29! 31! 37! 41! 43! 47! 53! 57! 61! 67!...
AND DEION SANDERS THOUGHT HE WAS "PRIME TIME"!
MIND IF I CALL YOU A FEW OTHER THINGS FIRST?