Section 5.3

After viewing the lecture videos and reading the textbook, you should be able to answer the following questions:

1. The **definite integral**, $\int_a^b f(x)\,dx$, of a function f over a closed interval $[a, b]$ is defined to be

$$\int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(c_k) \cdot \Delta x_k$$

where P is a partition of $[a, b]$, provided the limit exists.

   a. What is the integral sign?
   b. What is the lower limit of integration?
   c. What is the upper limit of integration?
   d. What is the integrand?
   e. What does dx represent?

2. The properties of definite integrals are:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$$

$$\int_a^b \big(f(x) \pm g(x)\big)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\min f \cdot (b-a) \le \int_a^b f(x)\,dx \le \max f \cdot (b-a)$$

$$f(x) \ge g(x) \text{ on } [a,b] \Longrightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$$

$$f(x) \geq 0 \text{ on } [a,b] \Longrightarrow \int_a^b f(x)\,dx \geq 0$$

Suppose $\int_1^{13} f(x)\,dx = 3$, $\int_1^5 f(x)\,dx = 17$ and $\int_1^{13} g(x)\,dx = 5$. Find the values of:

a. $\int_{13}^1 f(x)\,dx$

b. $\int_{11}^{11} f(x)\,dx$

c. $\int_1^{13} 7 \cdot f(x)\,dx$

d. $\int_1^{13} \big(f(x) + g(x)\big)\,dx$

e. $\int_1^{13} \big(f(x) - g(x)\big)\,dx$

f. $\int_5^{13} f(x)\,dx$

3. You were given the following formulas to calculate some simple definite integrals:

$$\int_a^b c\,dx = c \cdot (b-a), \qquad c \text{ any constant}$$

$$\int_a^b x\,dx = \frac{b^2}{2} - \frac{a^2}{2}, \qquad a < b$$

$$\int_a^b x^2\,dx = \frac{b^3}{3} - \frac{a^3}{3}, \qquad a < b$$

Evaluate the following definite integrals:

a. $\int_5^{100} 2\,dx$

b. $\int_2^4 x\,dx$

c. $\int_3^9 x^2\,dx$

4. Which <u>one</u> of the following expressions is not the same as the others:

a. $\int_a^b 1\,dx$

b. $\int_a^b dx$

c. $b - a$

d. 1

5. What is the average value of a continuous function, $y = f(x)$, over a closed interval, $[a,b]$?