

# Indefinite Integrals and the Substitution Method

Part 2: Important Examples

# Example 1

Evaluate

$$\int \frac{dx}{e^x + e^{-x}}$$

Solution:

Sometimes you need to re-write the integrand – here we will multiply it by  $1 = e^x/e^x$ :

# Example 1 (continued)

$$\begin{aligned}\int \frac{dx}{e^x + e^{-x}} &= \int \frac{dx}{e^x + e^{-x}} \cdot \frac{e^x}{e^x} \\&= \int \frac{e^x dx}{e^{2x} + 1} \\&\quad \boxed{\begin{aligned}u &= e^x \Rightarrow u^2 = e^{2x} \\du &= e^x dx\end{aligned}} \\&= \int \frac{du}{u^2 + 1} = \tan^{-1}(u) + C \\&= \tan^{-1}(e^x) + C\end{aligned}$$

## Example 2

Evaluate

$$\int \sec(x) dx$$

Solution:

$$\begin{aligned}\int \sec(x) dx &= \int \sec(x) \cdot \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} dx \\ &= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x) + \tan(x)} dx\end{aligned}$$

## Example 2 (continued)

$$\int \sec(x) dx = \int \frac{\sec^2(x) + \sec(x) \tan(x)}{\sec(x) + \tan(x)} dx$$

$$\begin{aligned} u &= \sec(x) + \tan(x) \\ du &= (\sec(x) \tan(x) + \sec^2(x))dx \end{aligned}$$

$$\begin{aligned} &= \int \frac{du}{u} = \ln|u| + C \\ &= \ln|\sec(x) + \tan(x)| + C \end{aligned}$$

# Example 3

Evaluate

$$\int \sin^2(x) dx$$

Solution:

Here we will use a trigonometric identity:

## Example 3 (continued)

$$\begin{aligned}\int \sin^2(x) dx &= \int \frac{1 - \cos(2x)}{2} dx \\&= \frac{1}{2} \int (1 - \cos(2x)) dx \\&= \frac{1}{2} \left( x - \frac{1}{2} \sin(2x) \right) + C \\&= \frac{1}{2} x - \frac{1}{4} \sin(2x) + C\end{aligned}$$

# Example 4

Evaluate

$$\int \cos^2(x) dx$$

Solution:

As in Example 3, we will use a trigonometric identity:

$$\begin{aligned}\int \cos^2(x) dx &= \int \frac{1 + \cos(2x)}{2} dx \\ &= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C\end{aligned}$$

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HANG UP  
AND DERIVE!

$$y\left(\frac{x^2-y^2}{x^2+1} + \frac{z^2}{x^2+1}\right) =$$

$$D^2 \approx 10^{-57} \times$$

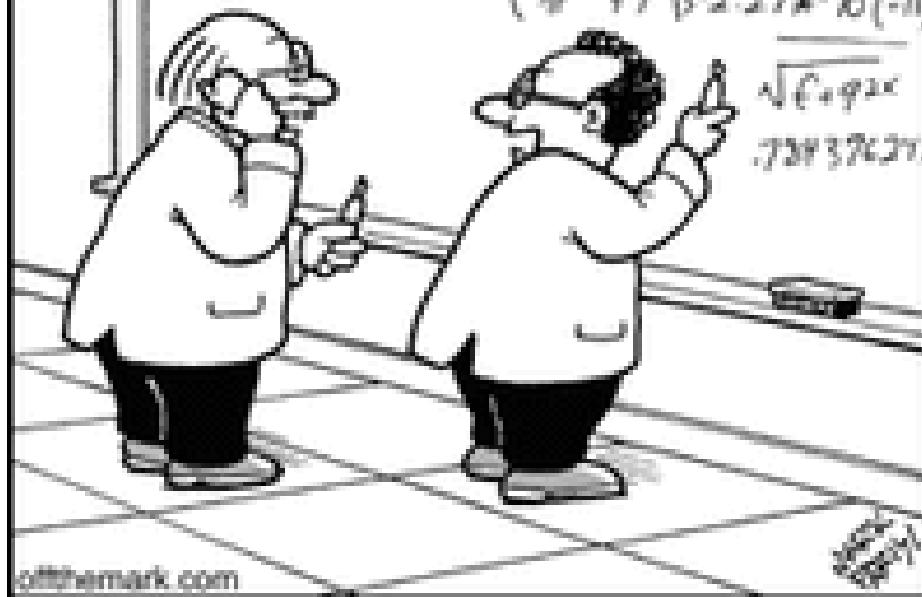
$$\Delta x \approx 1.87/(x)$$

$$(xx+1) = \omega \approx$$

$$\left(\frac{\pi^2}{4} + \frac{\pi^2}{4}\right) \beta \cdot 2.27 \times 10(-1)$$

$$\sqrt{6.92 \times}$$

$$7.27376271$$



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