Indefinite Integrals and the Substitution Method

Part 2: Important Examples

Example 1

Evaluate

$$\int \frac{dx}{e^x + e^{-x}}$$

Solution:

Sometimes you need to re-write the integrand – here we will multiply it by $1 = e^x/e^x$:

Example 1 (continued)

$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + e^{-x}} \cdot \frac{e^x}{e^x}$$

$$= \int \frac{e^x dx}{e^{2x} + 1}$$

$$u = e^x \Rightarrow u^2 = e^{2x}$$
$$du = e^x dx$$

$$= \int \frac{du}{u^2 + 1} = \tan^{-1}(u) + C$$

$$= \tan^{-1}(e^x) + C$$

Example 2

Evaluate

$$\int \sec(x)\,dx$$

<u>Solution</u>:

$$\int \sec(x)\,dx = \int \sec(x) \cdot \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)}\,dx$$

$$= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x) + \tan(x)}\,dx$$

Example 2 (continued)

$$\int \sec(x)\,dx = \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x) + \tan(x)}\,dx$$

$$u = \sec(x) + \tan(x)$$
$$du = (\sec(x)\tan(x) + \sec^2(x))dx$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sec(x) + \tan(x)| + C$$

Example 3

Evaluate

$$\int \sin^2(x)\,dx$$

Solution:

Here we will use a trigonometric identity:

Example 3 (continued)

$$\int \sin^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx$$

$$= \frac{1}{2}\int (1-\cos(2x))\,dx$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C$$

$$= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

Example 4

Evaluate

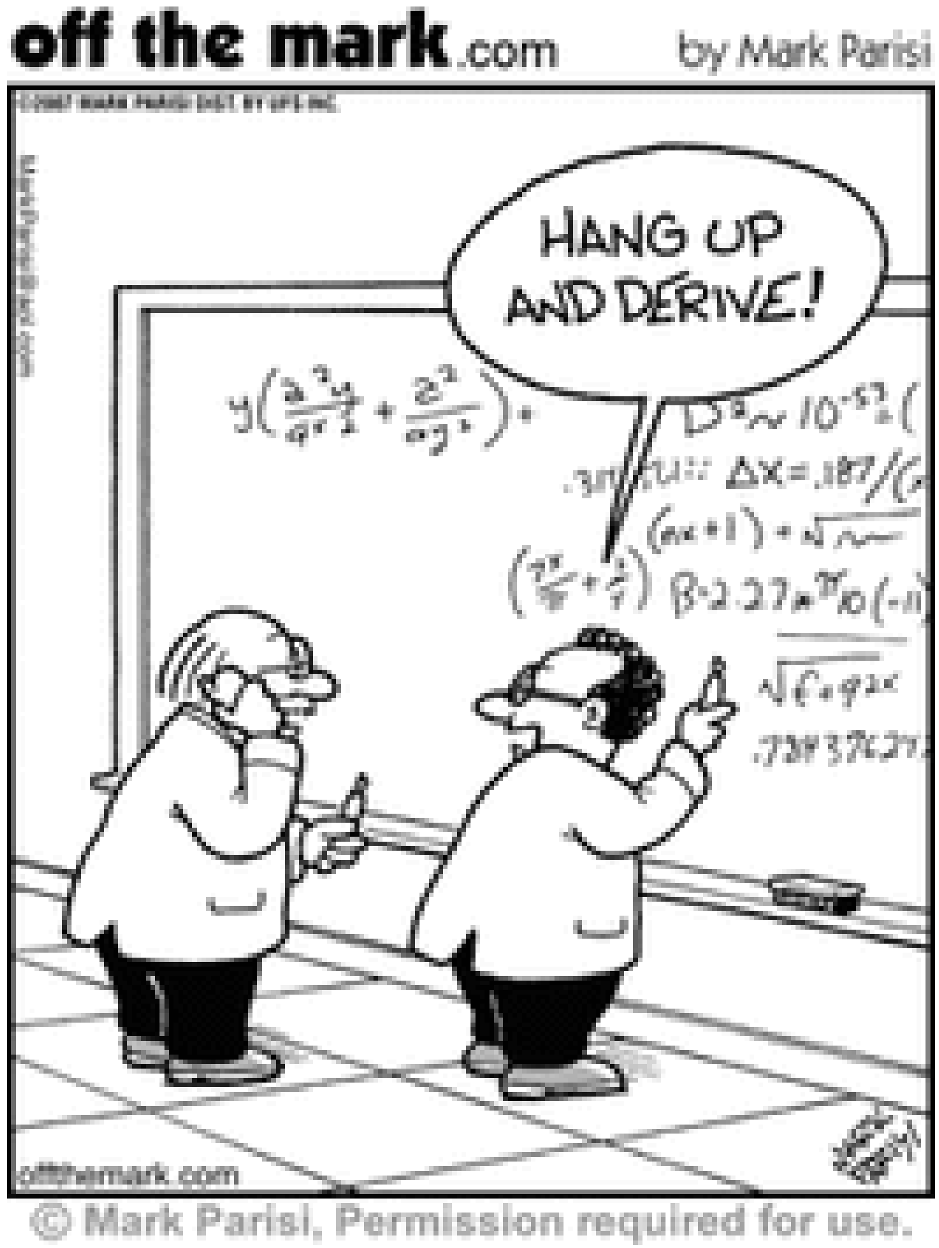
$$\int \cos^2(x)\, dx$$

Solution:

As in Example 3, we will use a trigonometric identity:

$$\int \cos^2(x)\, dx = \int \frac{1+\cos(2x)}{2}\, dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$$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