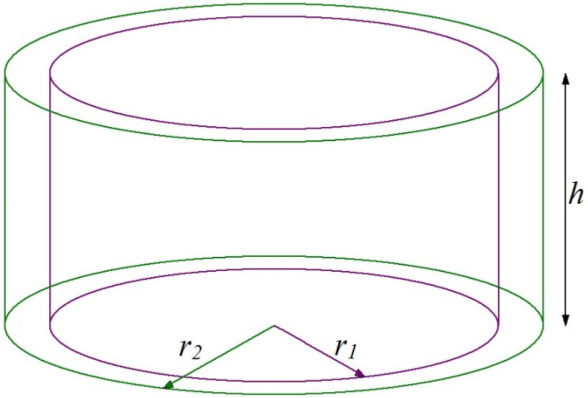
Volumes Using Cylindrical Shells

Volume of a Shell



Volume of outer cylinder − Volume of inner cylinder

$$= \pi r_2^2 h - \pi r_1^2 h$$

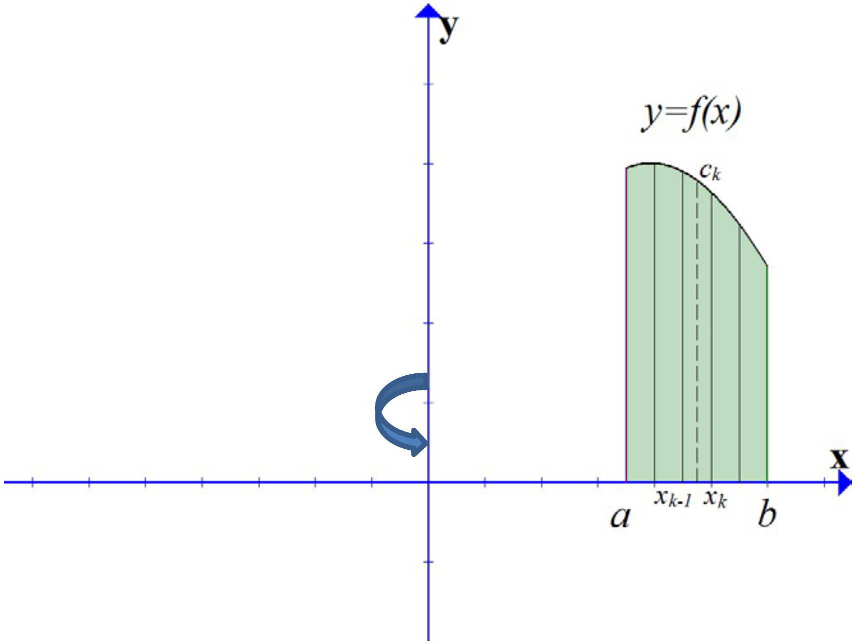
$$= \pi h(r_2^2 - r_1^2)$$

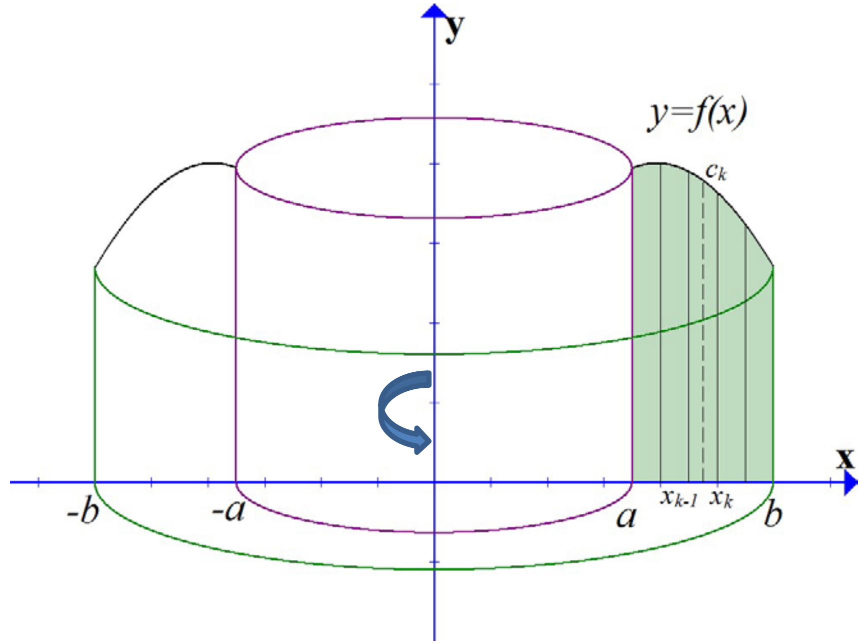
$$= \pi h(r_2 + r_1)(r_2 - r_1)$$

$$= 2\pi \left(\frac{r_2 + r_1}{2}\right) h(r_2 - r_1)$$

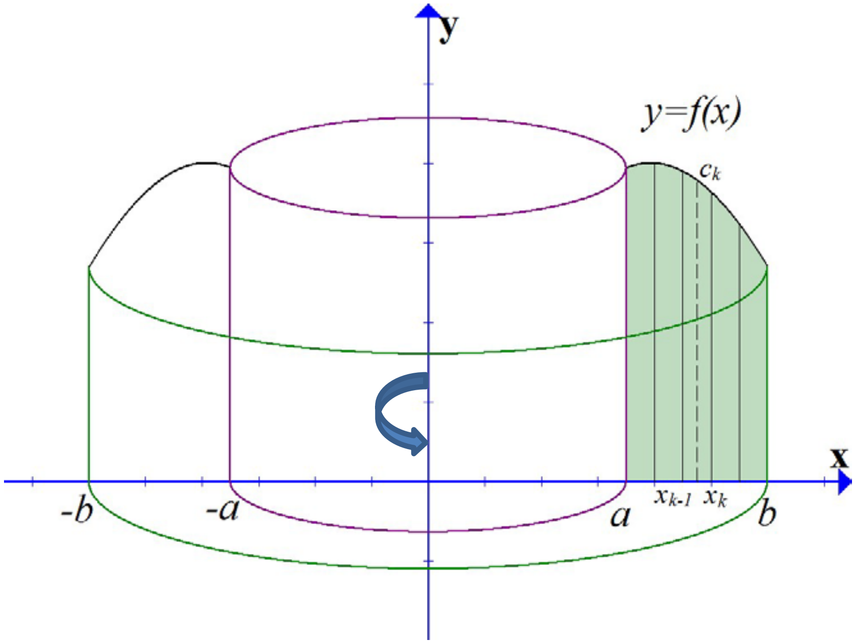
$= 2\pi$(average radius)(height)(thickness)

Revolve $y = f(x)$ about the y-axis





Revolve $y = f(x)$ about the y-axis



On each slice:

average radius

$$= \frac{x_k + x_{k-1}}{2} = c_k$$

$$\text{height} = f(c_k)$$

$$\text{thickness} = \Delta x_k$$

$$V_k \approx 2\pi c_k f(c_k) \Delta x_k$$

Method of Cylindrical Shells

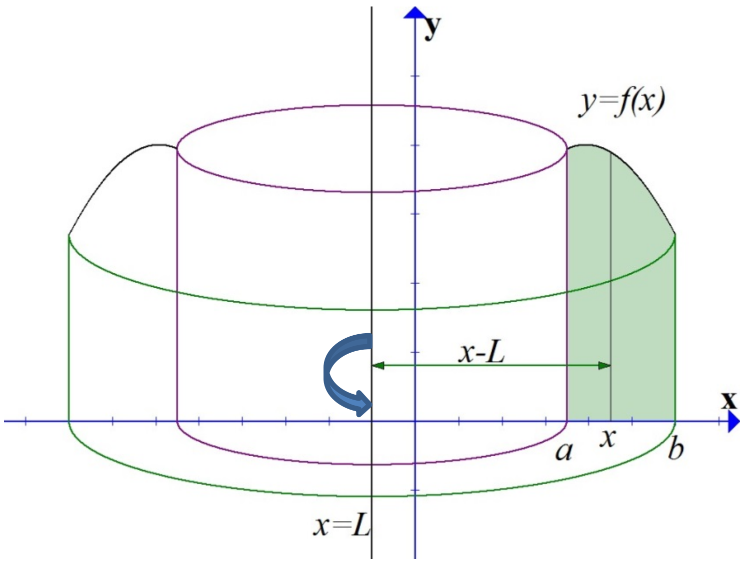
$$V = \sum_{k=1}^{n} V_k \approx \sum_{k=1}^{n} 2\pi c_k f(c_k) \Delta x_k$$

$$V = \int_a^b 2\pi x f(x)\, dx$$

$$V = \int_a^b \underbrace{2\pi(\text{radius})(\text{height})}_{\text{Surface area of a cylinder}}\, dx$$

Surface area of a cylinder

Revolve $y = f(x)$ about $x = L$

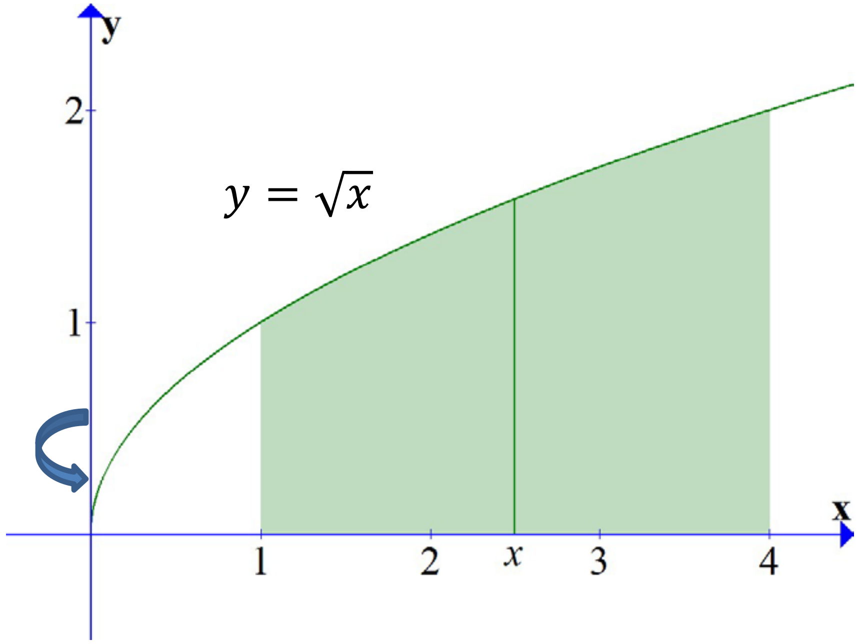


$$V = \int_a^b 2\pi(\text{radius})(\text{height})\,dx$$

$$= \int_a^b 2\pi|x - L|f(x)\,dx$$

Example 1

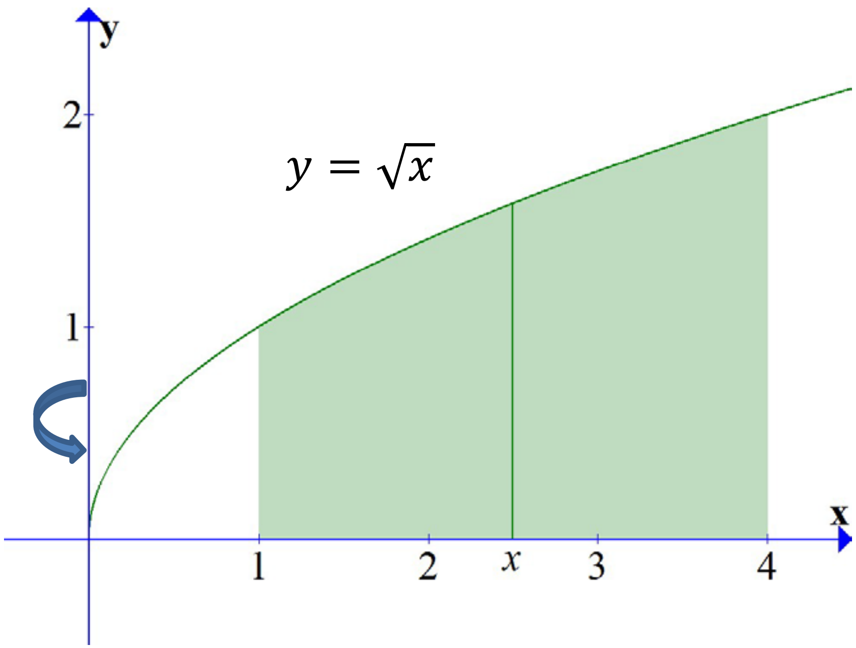
Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.



Solution:

Sketch the curve.

Example 1 (continued)



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, dx$$

$$= \int_1^4 2\pi x\sqrt{x}\, dx$$

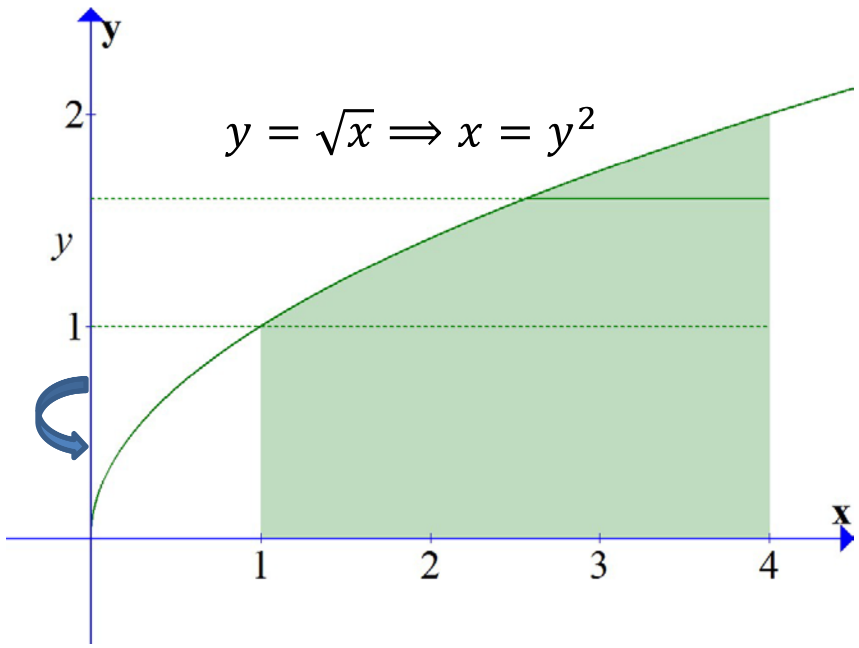
$$= \int_1^4 2\pi x^{3/2}\, dx$$

$$= \cdots = \frac{124\pi}{5}$$

Example 2

Redo Example 1 using the Washer Method.

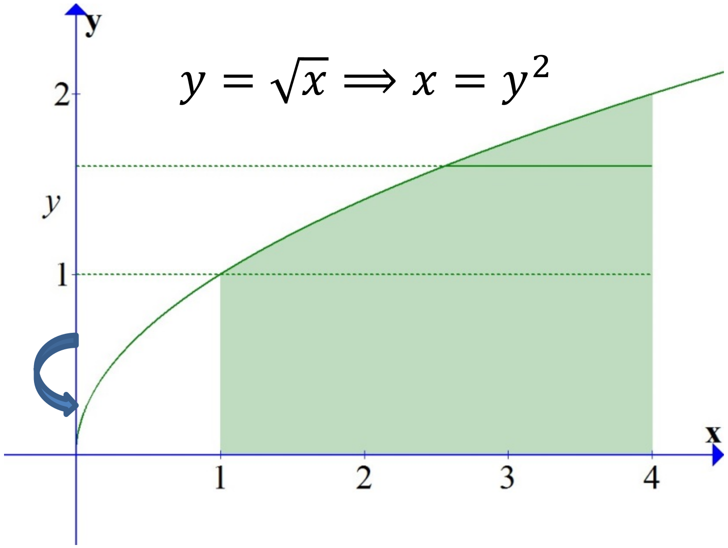
(Find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.)



Solution:

Sketch the curve.

Example 2 (continued)



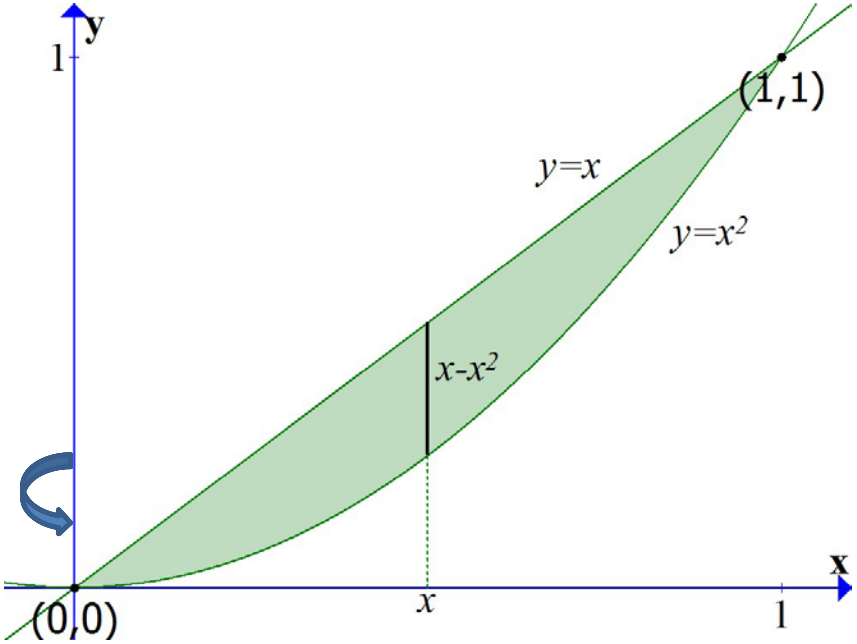
$$V = \int_0^1 \pi(4^2 - 1^2)\, dy$$

$$+ \int_1^2 \pi(4^2 - (y^2)^2)\, dy$$

$$= \cdots = \frac{124\pi}{5}$$

Example 3

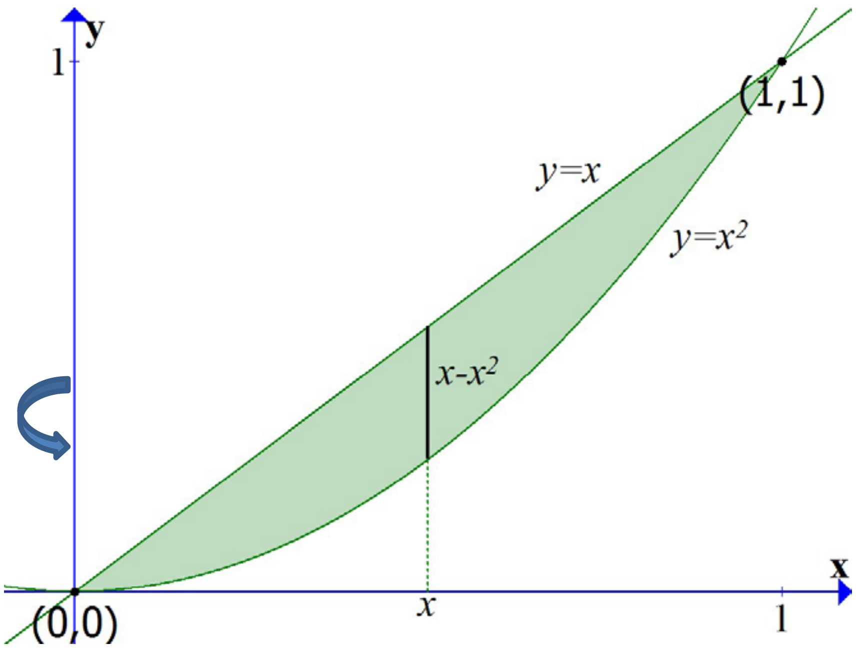
Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis.



Solution:

Sketch region to be revolved.

Example 3 (continued)



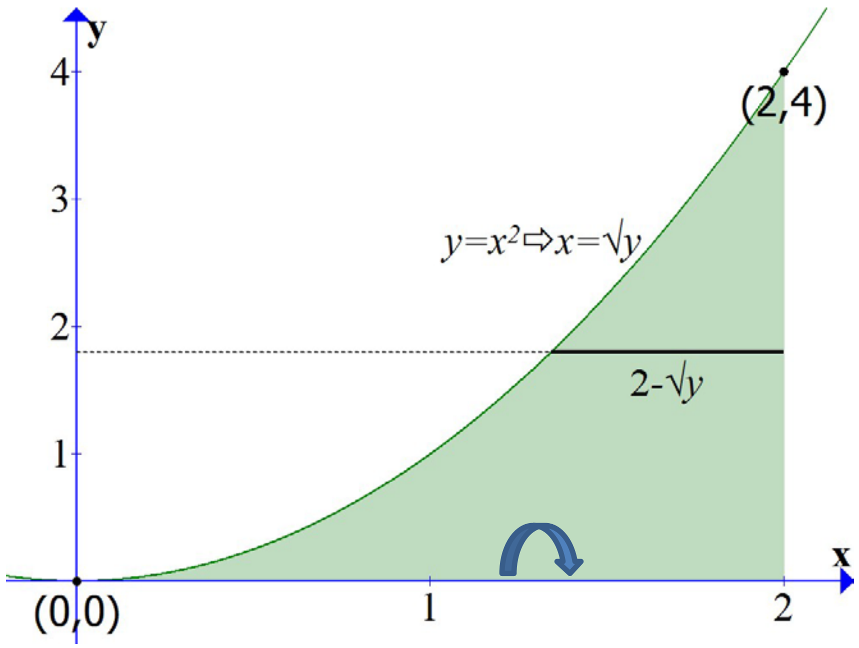
$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, dx$$

$$= \int_0^1 2\pi(x)(x - x^2)\, dx$$

$$= \cdots = \frac{\pi}{6}$$

Example 4
(Revolve $x = u(y)$ about the x-axis)

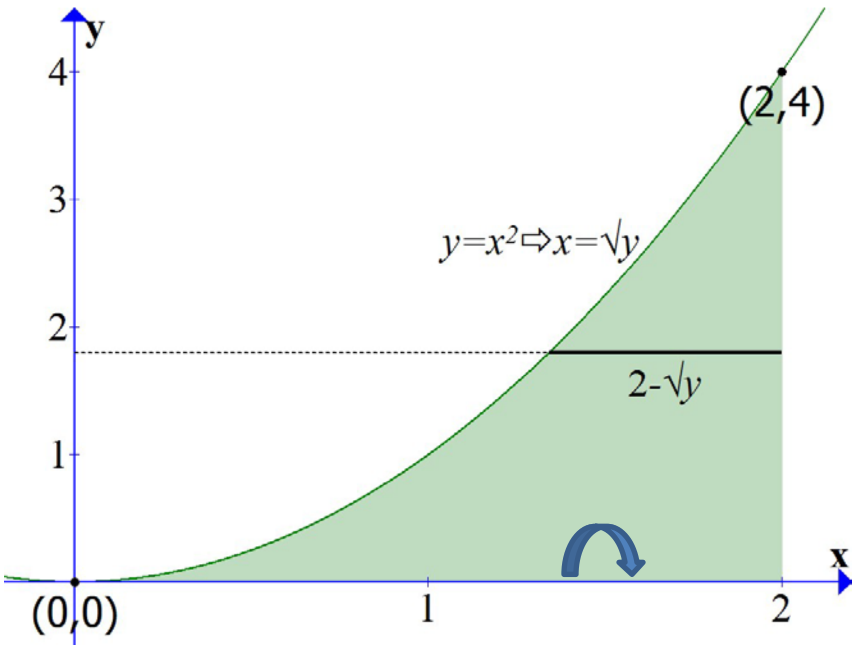
Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0,2]$ is revolved about the x-axis.



Solution:

Sketch region to be revolved.

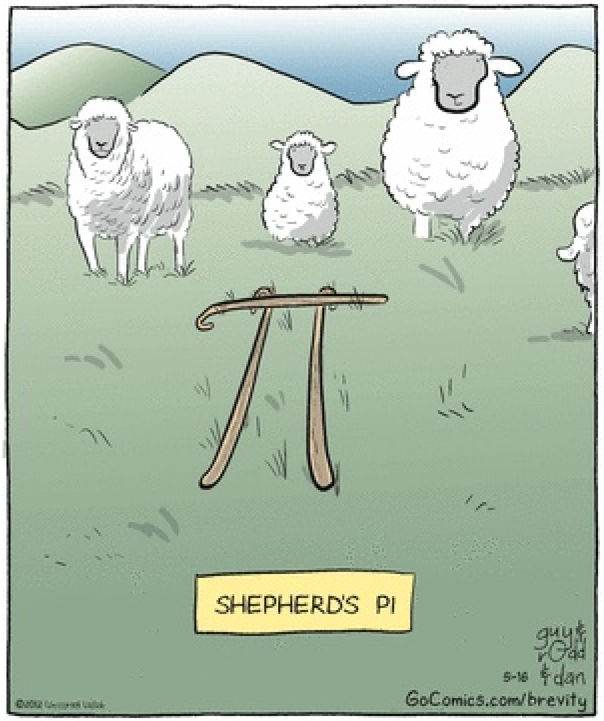
Example 4 (continued)



$$V = \int_a^b 2\pi(\text{radius})(\text{height})\, dy$$

$$= \int_0^4 2\pi y(2 - \sqrt{y})\, dy$$

$$= \cdots = \frac{32\pi}{5}$$



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