Arc Length

Part 2

Examples

Example 1

Find the length of the curve $x = \frac{y^{3/2}}{3} - y^{1/2}$ from $y = 1$ to $y = 9$.

<u>Solution</u>:

$$x = \frac{y^{3/2}}{3} - y^{1/2}$$

$$\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$$

Example 1 (continued)

$$L = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$$

$$= \int_1^9 \sqrt{1 + \left(\underbrace{\frac{1}{2} y^{1/2}}_{a} - \underbrace{\frac{1}{2} y^{-1/2}}_{b}\right)^2}\, dy$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$= \int_1^9 \sqrt{1 + \left(\frac{1}{2} y^{1/2}\right)^2 - 2\left(\frac{1}{2} y^{1/2}\right)\left(\frac{1}{2} y^{-1/2}\right) + \left(\frac{1}{2} y^{-1/2}\right)^2}\, dy$$

$$= \int_1^9 \sqrt{1 + \left(\frac{1}{2} y^{1/2}\right)^2 - \frac{1}{2} + \left(\frac{1}{2} y^{-1/2}\right)^2}\, dy$$

Example 1 (continued)

$$L = \int_1^9 \sqrt{1 + \left(\frac{1}{2}y^{1/2}\right)^2 - \frac{1}{2} + \left(\frac{1}{2}y^{-1/2}\right)^2}\, dy$$

$$= \int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2}\right)^2 + \frac{1}{2} + \left(\frac{1}{2}y^{-1/2}\right)^2}\, dy$$

$$= \int_1^9 \sqrt{\underbrace{\left(\frac{1}{2}y^{1/2}\right)}_{a}^2 + 2\underbrace{\left(\frac{1}{2}y^{1/2}\right)}_{a}\underbrace{\left(\frac{1}{2}y^{-1/2}\right)}_{b} + \underbrace{\left(\frac{1}{2}y^{-1/2}\right)}_{b}^2}\, dy$$

$$= \int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right)^2}\, dy$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

Example 1 (continued)

$$L = \int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right)^2}\,dy$$

$$= \int_1^9 \left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right) dy$$

$$= \cdots = \frac{32}{3}$$

Example 2

Find the length of the curve $y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x+4}$ on $0 \le x \le 2$.

<u>Solution</u>:

$$y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x + 4}$$

$$\frac{dy}{dx} = x^2 + 2x + 1 - \frac{4}{(4x + 4)^2}$$

$$= (x + 1)^2 - \frac{1}{4(x + 1)^2}$$

$$\frac{4}{(4x + 4)^2}$$
$$= \frac{4}{\left(4(x + 1)\right)^2}$$
$$= \frac{4}{4^2(x + 1)^2}$$
$$= \frac{1}{4(x + 1)^2}$$

Example 2 (continued)

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^2 \sqrt{1 + \left((x+1)^2 - \frac{1}{4(x+1)^2}\right)^2}\, dy$$

$$= \int_0^2 \sqrt{1 + \left((x+1)^2\right)^2 - 2\left((x+1)^2\right)\left(\frac{1}{4(x+1)^2}\right) + \left(\frac{1}{4(x+1)^2}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{1 + \left((x+1)^2\right)^2 - \frac{1}{2} + \left(\frac{1}{4(x+1)^2}\right)^2}\, dx$$

Example 2 (continued)

$$L = \int_0^2 \sqrt{1 + \left((x+1)^2\right)^2 - \frac{1}{2} + \left(\frac{1}{4(x+1)^2}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{\left((x+1)^2\right)^2 + \frac{1}{2} + \left(\frac{1}{4(x+1)^2}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{\left((x+1)^2\right)^2 + 2\left((x+1)^2\right)\left(\frac{1}{4(x+1)^2}\right) + \left(\frac{1}{4(x+1)^2}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{\left((x+1)^2 + \frac{1}{4(x+1)^2}\right)^2}\, dx$$

Example 2 (continued)

$$L = \int_0^2 \sqrt{\left((x+1)^2 + \frac{1}{4(x+1)^2}\right)^2}\, dx$$

$$= \int_0^2 \left((x+1)^2 + \frac{1}{4(x+1)^2}\right) dx$$

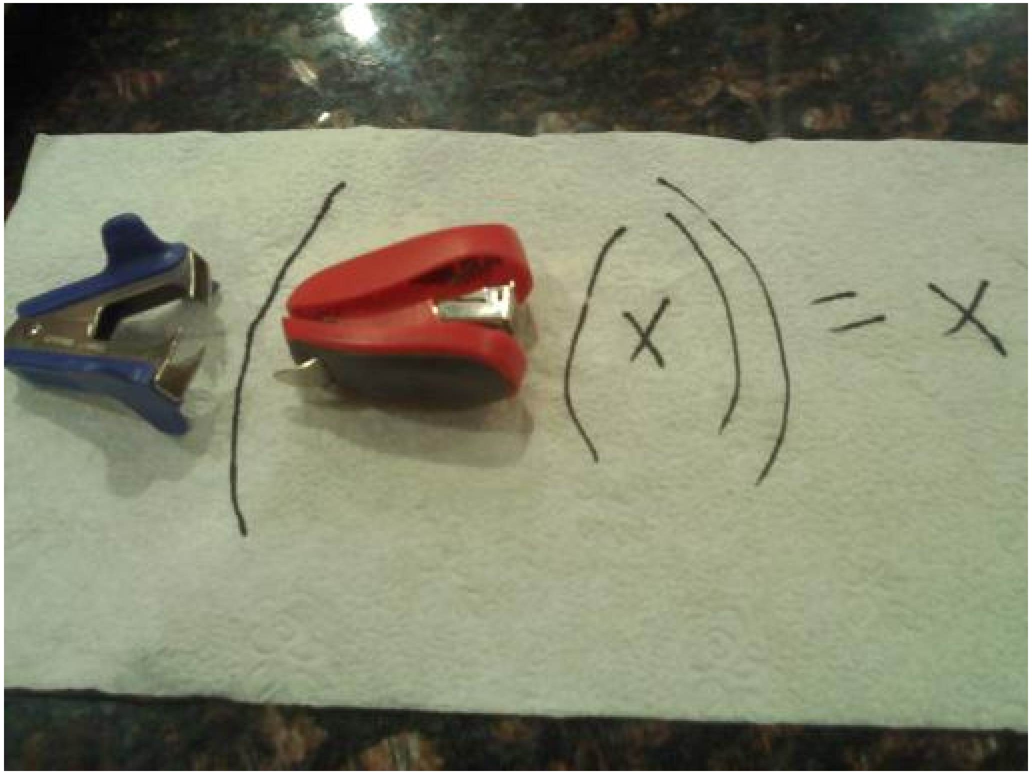
$$u = x + 1$$
$$du = dx$$
$$x = 2 \Rightarrow u = 3$$
$$x = 0 \Rightarrow u = 1$$

Example 2 (continued)

$$L = \int_0^2 \left((x+1)^2 + \frac{1}{4(x+1)^2} \right) dx$$

$$= \int_1^3 \left(u^2 + \frac{1}{4u^2} \right) du$$

$$= \cdots = \frac{53}{6}$$



http://mrhodotnet.blogspot.com/2011/11/function-and-its-inverse-office-supply.html