Trigonometric Integrals

Part 1: Some Prep Work

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$\int \sin^2 x\, dx$ and $\int \cos^2 x\, dx$

Recall:

$$\int \sin^2 x\, dx = \int \frac{1 - \cos 2x}{2}\, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

and

$$\int \cos^2 x\, dx = \int \frac{1 + \cos 2x}{2}\, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

Example 1

Evaluate

$$\int \cos^3 x \, dx$$

Solution:

Use $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x.$

Example 1 (continued)

$$\int \cos^3 x\, dx = \int \cos^2 x \cos x\, dx$$

$$= \int (1 - \sin^2 x) \cos x\, dx$$

$$u = \sin x$$
$$du = \cos x\, dx$$

$$= \int (1 - u^2)\, du$$

$$= u - \frac{1}{3}u^3 + C$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$

Example 2

Evaluate

$$\int \sin^4 x \, dx$$

Solution:

Use $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$.

Example 2 (continued)

$$\int \sin^4 x\, dx = \int (\sin^2 x)^2\, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \int \frac{1-2\cos 2x+\cos^2 2x}{4}\, dx$$

$$= \frac{1}{4}\int (1-2\cos 2x+\cos^2 2x)\, dx$$

Example 2 (continued)

$$\int \sin^4 x\, dx = \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1 + \cos 4x}{2}\right) dx$$

$$= \frac{1}{8}\int (2 - 4\cos 2x + 1 + \cos 4x)\, dx$$

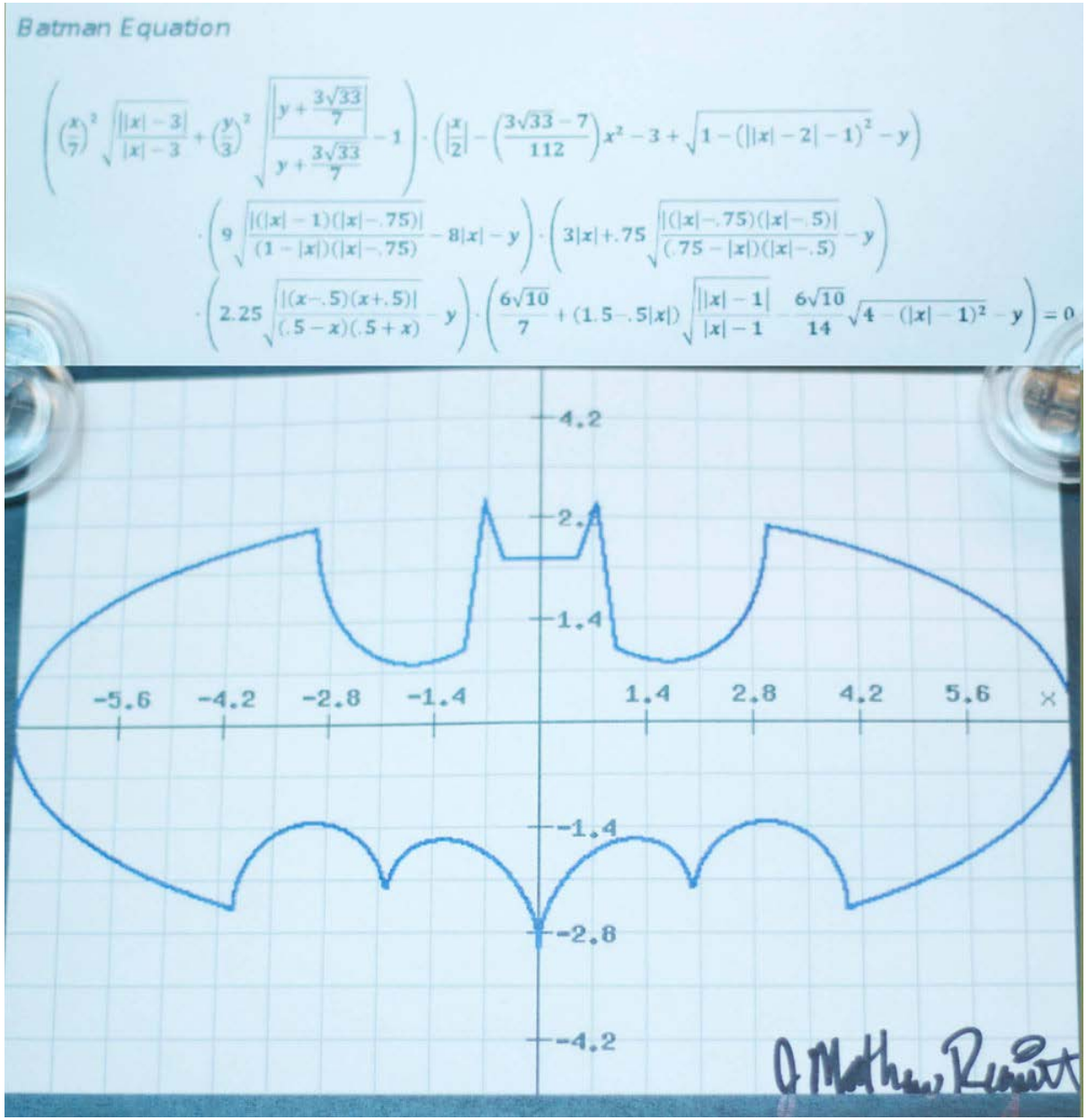
$$= \frac{1}{8}\int (3 - 4\cos 2x + \cos 4x)\, dx$$

Example 2 (continued)

$$\int \sin^4 x\, dx = \frac{1}{8}\int (3 - 4\cos 2x + \cos 4x)\, dx$$

$$= \frac{1}{8}\left(3x - 4 \cdot \frac{1}{2}\sin 2x + \frac{1}{4}\sin 4x\right) + C$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$



http://math.stackexchange.com/questions/54506/is-this-batman-equation-for-real