Sequences

Part 1

Sequences

A **sequence** in mathematics is an unending succession of numbers.
e.g.

$$1, 2, 3, 4, \dots$$

$$2, 4, 6, 8, \dots$$

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

$$1, -1, 1, -1, \dots$$

The dots suggest that the sequence continues indefinitely, following the obvious pattern

Sequences

The numbers in a sequence are called the **terms** of the sequence, and are described by their position.

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

has: first term = 1

second term = $\frac{1}{2}$

third term = $\frac{1}{3}$

fourth term = $\frac{1}{4}$

n-th term = $\frac{1}{n}$

Sequences

We can express

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

as

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots$$

or more compactly in **bracket notation** as

$$\left\{\frac{1}{n}\right\}_{n=1}^{+\infty}$$

Example 1

List the first five terms of the sequence

$$\{2^n\}_{n=1}^{+\infty}.$$

<u>Solution</u>:

$$\{2^n\}_{n=1}^{+\infty} = 2^1, 2^2, 2^3, 2^4, 2^5, \ldots$$

$$= \boxed{2, 4, 8, 16, 32, \ldots}$$

Example 2

Express the following in bracket notation:

$a)\ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$b)\ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

c) $1, -1, 1, -1, \ldots$

$d)\ \frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

e) $1, 3, 5, 7, \ldots$

Example 2 (continued)

a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$

Term #	**1**	**2**	**3**	**4**
Term	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

$$\left\{\frac{n}{n+1}\right\}_{n=1}^{+\infty}$$

Example 2 (continued)

b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Term #	**1**	**2**	**3**	**4**
Term	$\frac{1}{2} = \frac{1}{2^1}$	$\frac{1}{4} = \frac{1}{2^2}$	$\frac{1}{8} = \frac{1}{2^3}$	$\frac{1}{16} = \frac{1}{2^4}$

$$\left\{\frac{1}{2^n}\right\}_{n=1}^{+\infty}$$

Example 2 (continued)

c) $1, -1, 1, -1, \ldots$

Term #	**1**	**2**	**3**	**4**
Term	$1 = (-1)^2$	$-1 = (-1)^3$	$1 = (-1)^4$	$-1 = (-1)^5$

$$\{(-1)^{n+1}\}_{n=1}^{+\infty}$$

Example 2 (continued)

d) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

Term #	**1**	**2**	**3**	**4**
Term	$\frac{1}{2} = (-1)^2 \frac{1}{2}$	$-\frac{2}{3} = (-1)^3 \frac{2}{3}$	$\frac{3}{4} = (-1)^4 \frac{3}{4}$	$-\frac{4}{5} = (-1)^5 \frac{4}{5}$

$$\left\{(-1)^{n+1} \cdot \frac{n}{n+1}\right\}_{n=1}^{+\infty}$$

Example 2 (continued)

e) $1, 3, 5, 7, \ldots$

Term #	**1**	**2**	**3**	**4**
Term	$1 = 2 \cdot 1 - 1$	$3 = 2 \cdot 2 - 1$	$5 = 2 \cdot 3 - 1$	$7 = 2 \cdot 4 - 1$

$$\{2n - 1\}_{n=1}^{+\infty}$$

It's hard getting dumped by a mathematician.

http://math-fail.com/2012/09/math-comic-time.html