Section 9.2

After viewing the lecture videos and reading the textbook, you should be able to answer the following questions:

The $\boldsymbol{n}$**-th partial sum** of the series $u_1 + u_2 + u_3 + \cdots + u_k + \cdots = \sum_{k=1}^{\infty} u_k$ is $s_n = \sum_{k=1}^{n} u_k$.

A **geometric series** is a series which can be written in the form $\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots + ar^{k-1} + \cdots$. It converges if $|r| < 1$ and it diverges if $|r| \geq 1$. If the geometric series converges, then its sum is $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$.

The series $\sum_{k=1}^{\infty} \left(\frac{1}{k}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ is the **harmonic series** and it diverges.

The $\boldsymbol{n}$-th Term Test for Divergence

"If $\lim_{k\to\infty} u_k \neq 0$ then $\sum_{k=1}^{\infty} u_k$ diverges".

1. Find s_1, s_2, s_3, and s_4 for the series $1 + 1 + 1 + \cdots + 1 + \cdots = \sum_{k=1}^{\infty} 1$.
2. Which of the following are geometric series? If they are geometric series, what are a and r?
   a. $\sum_{k=3}^{\infty} (5 \cdot 2^{2k}) = 320 + 1280 + 5120 + 20480 + \cdots$
   b. $\sum_{k=1}^{\infty} \left(\frac{1}{3k^5}\right) = \frac{1}{3} + \frac{1}{96} + \frac{1}{729} + \cdots + \frac{1}{3k^5} + \cdots$
3. Do the following series converge or diverge?
   a. $\sum_{k=1}^{\infty} \left(\frac{1}{5k}\right) = \frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \cdots$
   b. $\sum_{k=15}^{\infty} \left(\frac{1}{k}\right) = \frac{1}{15} + \frac{1}{16} + \frac{1}{17} + \cdots$
4. If $\lim_{k\to\infty} u_k = 0$, then does $\sum_{k=1}^{\infty} u_k$ converge?