Homework 3

Due Thursday, September 25

1. If Y has the pdf $f_Y(y) = cye^{-2y}$, $y > 0$, find the value c (recall that the pdf must integrate to one).

2. Suppose that a random variable Y has the pdf

$$f_Y(y) = ye^{-y}, \quad y \geq 0.$$

   Find the expected value of Y.

3. Suppose that a random variable Y has the pdf

$$f_Y(y) = \frac{3}{64}y^2(4-y), \quad 0 < y < 4$$

   Find the expected value and variance of Y.

4. If $Y \sim N(\mu, \sigma^2)$ so that it has the pdf

$$f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(y-\mu)^2}{2\sigma^2}}, \quad -\infty < y < \infty$$

   show that $E(Y) = \mu$ by starting with $E(Y) = \int y f_Y(y)\, dy$ and performing the integration. (HINT: Consider a substitution $z = (y-\mu)/\sigma$; also the fact that the pdf integrates to one will be helpful).

5. If Y is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute

   (a) $P(Y > 5)$

   (b) $P(4 < Y < 16)$

6. If Z is a standard normal random variable, find the value z_0 such that

   (a) $P(Z > z_0) = 0.5$

   (b) $P(-z_0 < Z < z_0) = 0.9$

7. If Y has an exponential distribution and $P(Y > 2) = 0.0821$, what is (a) $\beta = E(Y)$ and (b) $P(Y \leq 1.7)$? (Hint: Use the cdf of exponential).

8. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\beta = 2$. What is the probability that a repair time (a) exceeds 2 hours, (b) is at most 3.5 hours, (c) is between 2.3 and 5.2 hours? Solve this problem without using R.

9. Repeat the previous problem using R, providing the codes and numerical answers.

10. Let $Y \sim \text{Gamma}(2.1, 3.5)$. Find (a) $P(Y = 4)$, (b) $P(Y > 5)$, (c) $P(4 < Y < 16)$.

11. Consider a random variable Y with the pdf

$$f_Y(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 \leq y \leq 1$$

where $\alpha > 0$ and $\beta > 0$. If $\alpha = 4$ and $\beta = 2$, find $P(Y \leq 0.9)$.

12. Suppose that a random variable Y has a pdf given by $f_Y(y) = ky^3e^{-y/2}$, $y > 0$. Find the value of k (try to avoid integration) and conclude that Y follows a χ^2 distribution.