Homework 4

Due Thursday, October 2

1. If $E(Y) = 1$ and $\mathrm{Var}(Y) = 5$, find
   (a) $E((2+Y)^2)$
   (b) $\mathrm{Var}(4+3Y)$
2. Show that $\mathrm{Cov}(Y_1, Y_2) = \mathrm{Cov}(Y_2, Y_1)$.
3. Suppose that random variables Y_1 and Y_2 have the following: $E(Y_1) = 4$, $E(Y_2) = -1$, $\mathrm{Var}(Y_1) = 2$, and $\mathrm{Var}(Y_2) = 8$.
   (a) Given the above information, find the largest and smallest possible values of $\mathrm{Cov}(Y_1, Y_2)$ (HINT: consider what values the correlation ρ can take).
   (b) If $\mathrm{Cov}(Y_1, Y_2) = 3$, find $E(Y_1 Y_2)$.
4. Suppose that $Y_1 \sim \chi^2(\nu_1)$ and $Y_2 \sim \chi^2(\nu_2)$ are independent random variables. Find the following: (a) $E(Y_1 Y_2)$, (b) $E(Y_1 + Y_2)$, and (c) $\mathrm{Var}(Y_1 + Y_2)$.
5. For the previous problem, does the resulting distribution of $Y_1 + Y_2$ still follow a χ^2-distribution? Please explain your answer.
6. If $Y_1, Y_2, \ldots, Y_n$ are iid $N(\mu, \sigma^2)$ random variables, and $a_1 = a_2 = \cdots = a_n = 1/n$ are constants, find (a) $E(a_1Y_1 + a_2Y_2 + \cdots + a_nY_n)$ and (b) $\mathrm{Var}(a_1Y_1 + a_2Y_2 + \cdots + a_nY_n)$.
7. Suppose that $Z \sim N(0,1)$ and $W \sim \chi^2(\nu)$, with $\nu > 1$, are independent random variables, and let $T = Z/\sqrt{W}$. Find $\mathrm{Cov}(W, T)$.
8. (a) If Y has a t-distribution with ν d.f. (i.e., $Y \sim t(\nu)$), show that $Y^2 \sim F(1, \nu)$. (Just recall the composition of Y and see what Y^2 looks like).
   (b) If Y has an F distribution with ν_1 and ν_2 degrees of freedom (i.e., $Y \sim F(\nu_1, \nu_2)$), find the distribution of $1/Y$. (Again, recall the composition of Y to solve this problem).
9. If $Y \sim \chi^2(5)$, find (a) $f_Y(1.3)$, (b) $P(Y \le 1.3)$, (c) $P(Y > 9.2)$, (d) $P(1.5 \le Y \le 5.1)$, and (e) the value q if $P(Y \ge q) = 0.01$.
10. If $Y \sim t(4)$, find (a) $f_Y(0.7)$, (b) $P(Y \le 7)$, (c) $P(Y \ge 2.2)$, (d) $P(-1.5 \le Y < -0.2)$, and (e) the value q if $P(Y \ge q) = 0.1$.
11. If $Y \sim F(10, 15)$, find (a) $f_Y(0.5)$, (b) $P(Y \le 0.5)$, (c) $P(Y \ge 1.6)$, (d) $P(0.4 < Y < 1.1)$, and (e) the value q if $P(Y \ge q) = 0.05$.
12. For each of the random variables in Problems 9, 10, 11, find (a) the median of Y, (b) the first quartile (25-th percentile) of Y, (c) the third quartile (75-th percentile) of Y.