Homework 11

Due Wednesday, December 11

1. If Y_1, Y_2, Y_3, Y_4, Y_5 are iid exponential random variables with the parameter β, compute

$$P(\min(Y_1, Y_2, Y_3, Y_4, Y_5) > a)$$

2. Let $Y_1, \ldots, Y_n$ be iid random variables, each with pdf

$$f_Y(y) = e^{-(y-\theta)}, \quad y > \theta$$

   Find the pdf of $Y_{(1)} = \min\{Y_1, \ldots, Y_n\}$.

3. Let $Y_1, \ldots, Y_n$ be iid Beta$(2,2)$. Find (a) cdf and (b) pdf of $Y_{(n)} = \max\{Y_1, \ldots, Y_n\}$.

4. Consider a sample of size 5 from a uniform distribution over (0, 1). Compute the probability that the median is in the interval (1/4, 3/4).

5. Let $Y_1, \ldots, Y_n$ be iid Uniform$(0, \theta)$. Find the pdf of the k-th order statistic $Y_{(k)}$. In addition, if $\theta = 1$, show that $Y_{(k)}$ follows a beta distribution and determine the resulting beta random variable parameters α and β.

6. The fracture strength of tempered glass averages 14 and has standard deviation 2.
   (a) What is the probability that the average fracture strength of 100 randomly selected pieces of this glass exceeds 14.5?
   (b) Find an interval that includes, with probability 0.95, the average fracture strength of 100 randomly selected pieces of this glass.

7. An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.5 inches and if she randomly samples 100 men, find the probability that the difference between the sample mean and the true population mean will not exceed 0.5 inch.

8. Claims filed under auto insurance policies follow a distribution with mean 19,400 and standard deviation 6,000. Calculate the probability that the average of 36 randomly selected claims exceeds 20,000.

9. A charity receives 2025 contributions. Contributions are assumed to be mutually independent and identically distributed with mean 3125 and standard deviation 540. Calculate the approximate 95th percentile for the distribution of the average contributions received.

10. In a town, it was found that out of one out of every six people is insured. Consider a random sample of 612 people from the town. What is the probability that the number of insured people is strictly between 90 and 150, using a normal approximation with the continuity correction?

11. An airline finds that 5% of the persons who make reservations on a certain flight do not show up for the flight. If the airline sells 160 tickets for a flight, what is the probability that there are at most 5 no shows on a flight, using

    (a) Exact binomial probability?

    (b) Poisson approximation to binomial?

    (c) Normal approximation to binomial with the continuity correction?

12. **(5090*)** A student takes an actuarial exam with 30 questions. The probability that the student answers a given question correctly is 0.2, independent of all other questions. The probability that the student answers more than n questions correctly is greater than 0.1. The probability that the student answers more than $n+1$ questions correctly is less than 0.1. Calculate n using a normal approximation with the continuity correction.

13. **(5090*)** Let $X_1, \ldots, X_n \sim$ iid Uniform$(0, \theta)$. Let $Y_n = n(\theta - X_{(n)})$, where $X_{(n)} = \max\{X_1, \ldots, X_n\}$. Find the cdf of Y_n and investigate what happens to the cdf $P(Y_n \le y)$ when $n \to \infty$.

For the below problems, you will need to use the formula for the joint pdf of $(Y_{(j)}, Y_{(k)})$

$$\begin{aligned} & f_{Y_{(j)},Y_{(k)}}(y_j, y_k) \\ = \; & \frac{n!}{(j-1)!(k-1-j)!(n-k)!}[F_Y(y_j)]^{j-1}[F_Y(y_k) - F_Y(y_j)]^{k-1-j}[1 - F_Y(y_k)]^{n-k} f_Y(y_j) f_Y(y_k) \end{aligned}$$

where $y_j < y_k$ (compare with WMS formula on page 337).

14. **(5090*)** Suppose that $Y_1, \ldots, Y_n$ are iid Uniform$(0, \theta)$. Find the joint pdf of $(Y_{(1)}, Y_{(n)})$

15. **(5090*)** Let Y_1 and Y_2 be iid Uniform$(0, 1)$. Find $P(2Y_{(1)} < Y_{(2)})$.