Quiz 1

September 8, 2026

1. If Y has a uniform distribution over $(0, 1)$, find the pdf of $U = -2\ln(Y)$, using any method.

(Solution)

If $Y \sim \text{Uniform}(0, 1)$, then the pdf of Y is given by $f_Y(y) = 1$, $0 < y < 1$.

(a) For the cdf method, we have that

$$\begin{aligned} F_U(u) &= P(U \le u) = P(-2\ln(Y) \le u) = P(Y \ge e^{-u/2}) \\ &= \int_{\{Y \ge e^{-u/2}\}} f_Y(y)\,dy = \int_{e^{-u/2}}^{1} 1\,dy = 1 - e^{-u/2} \end{aligned}$$

where u takes all the values from 0 and above, i.e., $0 < u < \infty$. To obtain the pdf of U,

$$f_U(u) = \frac{d}{du} F_U(u) = \frac{d}{du}(1 - e^{-u/2}) = \frac{1}{2} e^{-u/2}, \quad 0 < u < \infty.$$

(b) We see that the transformation $U = -2\ln(Y)$ is monotone, so we can use the pdf method, with $Y = e^{-U/2}$ and $\frac{dy}{du} = -(1/2)e^{-u/2}$. Hence,

$$f_U(u) = f_Y(y) \left| \frac{dy}{du} \right| = 1 \cdot \left| -\frac{1}{2} e^{-u/2} \right| = \frac{1}{2} e^{-u/2}, \quad 0 < u < \infty,$$

matching the result of part (a).

Note that the pdf of U is exponential density with $\beta = 2$, so that $U \sim \text{Exponential}(2)$.