Quiz 3
September 22, 2026

1. If Y_1 and Y_2 are independent with the mgfs

$$M_{Y_1}(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^5 \quad \text{and} \quad M_{Y_2}(t) = \exp(2e^t - 2)$$

find $E(Y_1Y_2)$.

(Solution) Since $M_{Y_1}(t) = \big(1/4 + (3/4)e^t\big)^5 = \big((1-3/4) + (3/4)e^t\big)^5 = \big((3/4)e^t + (1-3/4)\big)^5$ is the mgf of $Y_1 \sim \text{Binomial}(5, 3/4)$, and $M_{Y_2}(t) = \exp(2e^t - 2) = \exp(2(e^t - 1))$ is the mgf of $Y_2 \sim \text{Poisson}(2)$, and Y_1 and Y_2 are independent, it follows that

$$E(Y_1Y_2) = E(Y_1)E(Y_2) = (np)\lambda = (5 \cdot (3/4))2 = \frac{15}{2}$$

Even if do not recognize the distributions, you can still answer the question by computing

$$\begin{aligned} E(Y_1) &= \frac{d}{dt}M_{Y_1}(t)\Big|_{t=0} = \frac{d}{dt}\big(1/4 + (3/4)e^t\big)^5\Big|_{t=0} = 5\big(1/4 + (3/4)e^t\big)^4(3/4)e^t\Big|_{t=0} \\ &= 5 \cdot (3/4) = 15/4 \end{aligned}$$

and

$$E(Y_2) = \frac{d}{dt}M_{Y_2}(t)\Big|_{t=0} = \frac{d}{dt}\exp(2e^t - 2)\Big|_{t=0} = \exp(2e^t - 2)2e^t\Big|_{t=0} = 2$$

so that

$$E(Y_1Y_2) = E(Y_1)E(Y_2) = (15/4)2 = \frac{15}{2}$$