Quiz 5
October 20, 2026

1. Let $Y_1, \ldots, Y_n$ be iid random variables, each with pdf $f_Y(y) = \theta y^{-(\theta+1)}$, $y > 1$, where the parameter $\theta > 1$. Find the bias of $\overline{Y}$ (for the parameter θ).

   (Solution) First,

$$E(Y) = \int y f_Y(y)\, dy = \int_1^\infty y\theta y^{-(\theta+1)}\, dy = \theta \int_1^\infty y^{-\theta}\, dy = \theta \left[\frac{y^{-\theta+1}}{-\theta+1} \Big|_1^\infty \right] = \frac{\theta}{\theta - 1}$$

   and since $E(\overline{Y}) = E(Y)$, we conclude that

$$\text{Bias}(\overline{Y}) = E(\overline{Y}) - \theta = \frac{\theta}{\theta-1} - \theta = \frac{\theta - \theta(\theta-1)}{\theta-1} = \frac{\theta - \theta^2 + \theta}{\theta-1} = \frac{2\theta - \theta^2}{\theta-1} = \frac{\theta(2-\theta)}{\theta-1}$$