Quiz 6
November 3, 2026

1. Let $Y_1, \ldots, Y_n$ be iid Geometric(θ), so that each Y has pmf $P(Y = y) = (1-\theta)^{y-1}\theta$, where $y = 1, 2, \ldots$ and $0 \leq \theta \leq 1$. Find the MLE of θ.

   (Solution) First, we obtain the likelihood function

$$L(\theta) = \prod_{i=1}^{n} P(Y_i = y_i) = \prod_{i=1}^{n} [(1-\theta)^{y_i - 1}\theta] = (1-\theta)^{\sum_{i=1}^{n} y_i - n}\theta^n$$

   then

$$\ln L(\theta) = \left(\sum_{i=1}^{n} y_i - n \right) \ln(1-\theta) + n \ln \theta$$

   and setting

$$\frac{d}{d\theta} \ln L(\theta) = -\frac{(\sum_{i=1}^{n} y_i - n)}{(1-\theta)} + \frac{n}{\theta} = 0$$

   and solving for θ, we find that $\theta = n / \sum_{i=1}^{n} y_i$, or

$$\hat{\theta}_{MLE} = \frac{n}{\sum_{i=1}^{n} Y_i} = \frac{1}{\overline{Y}}$$

   (Note that in the homework 6, the sample mean $\overline{Y}$ was found to be a sufficient statistic, so the MLE is indeed a function of a sufficient statistic. Also makes sense since $E(\overline{Y}) = E(Y) = 1/\theta$, and by the invariance property of the MLE, we have that $\overline{Y}$ is the MLE of $1/\theta$.)