Quiz 7
November 10, 2026

1. Let $Y_1, \ldots, Y_n$ be iid Geometric(θ), so that each Y has pmf $P(Y = y) = (1-\theta)^{y-1}\theta$, where $y = 1, 2, \ldots$ and $0 \leq \theta \leq 1$. For testing $H_0 : \theta = 1/2$ versus $H_A : \theta \neq 1/2$, find the LRT statistic λ, and simplify/express in terms of $\overline{Y}$. You may use the results from Quiz 6.

   (Solution) We have seen (from Quiz 6) that the likelihood function is

$$L(\theta) \quad = \quad (1-\theta)^{(\sum_{i=1}^n y_i - n)}\theta^n$$

   and that

$$\hat{\theta}_{MLE} = \frac{n}{\sum_{i=1}^n Y_i} = \frac{1}{\overline{Y}}$$

   For testing $H_0 : \theta = 1/2$ versus $H_A : \theta \neq 1/2$, we have that

$$\begin{aligned}
\lambda &= \frac{L(\theta_0)}{L(\hat{\theta}_{MLE})} = \frac{L(1/2)}{L(1/\overline{Y})} = \frac{(1-1/2)^{(\sum_{i=1}^n Y_i - n)}(1/2)^n}{(1-1/\overline{Y})^{(\sum_{i=1}^n Y_i - n)}(1/\overline{Y})^n} = \frac{(1/2)^{n\overline{Y}}}{(1-1/\overline{Y})^{(n\overline{Y}-n)}(1/\overline{Y})^n} \\
&= \left[\frac{(1/2)^{\overline{Y}}}{(1-1/\overline{Y})^{(\overline{Y}-1)}(1/\overline{Y})}\right]^n = \left[\frac{(1/2)^{\overline{Y}}}{((\overline{Y}-1)/\overline{Y})^{(\overline{Y}-1)}(1/\overline{Y})}\right]^n = \left[\frac{(1/2)^{\overline{Y}}}{(\overline{Y}-1)^{(\overline{Y}-1)}(1/\overline{Y})^{\overline{Y}}}\right]^n \\
&= \left[\frac{(\overline{Y}/2)^{\overline{Y}}}{(\overline{Y}-1)^{(\overline{Y}-1)}}\right]^n
\end{aligned}$$