Quiz 8
November 17, 2026

1. Let $Y_1, \ldots, Y_n$ be iid $N(\mu, \sigma^2)$, with the hypotheses $H_0 : \mu = \mu_0$ versus $H_A : \mu < \mu_0$, and β is evaluated at $\mu = \mu_A$ where $\mu_A < \mu_0$. Recall that $\beta = 1 - \text{power}(\mu_A)$, and let a power function be given by (with some constant c)

$$\text{power}(\mu) = P\left(Z < c + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}\right)$$

Now, we also have

$$\beta \;=\; P(\text{accept } H_0, \text{ when } H_A \text{ true}) = P(\overline{Y} \geq k, \;\; \mu = \mu_A)$$

Find the value of k (match the two β's above; the answer should contain c).

(Solution) First, with $\beta = 1 - \text{power}(\mu_A)$, we have

$$\beta \;=\; 1 - \text{power}(\mu_A) = 1 - P\left(Z < c + \frac{\mu_0 - \mu_A}{\sigma/\sqrt{n}}\right) = P\left(Z \geq c + \frac{\mu_0 - \mu_A}{\sigma/\sqrt{n}}\right)$$

but we also have that

$$\begin{aligned} \beta &= P(\text{accept } H_0, \text{ when } H_A \text{ true}) = P(\overline{Y} \geq k, \;\; \mu = \mu_A) \\ &= P\left(\frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} \geq \frac{k - \mu}{\sigma/\sqrt{n}}, \;\; \mu = \mu_A\right) = P\left(Z \geq \frac{k - \mu_A}{\sigma/\sqrt{n}}\right) \end{aligned}$$

from which we can obtain

$$\begin{aligned} \frac{k - \mu_A}{\sigma/\sqrt{n}} = c + \frac{\mu_0 - \mu_A}{\sigma/\sqrt{n}} \quad &\Longrightarrow \quad k - \mu_A = c(\sigma/\sqrt{n}) + \frac{\mu_0 - \mu_A}{\sigma/\sqrt{n}}(\sigma/\sqrt{n}) \\ &\Longrightarrow \quad k - \mu_A = c(\sigma/\sqrt{n}) + \mu_0 - \mu_A \\ &\Longrightarrow \quad k = \mu_0 + c\left(\frac{\sigma}{\sqrt{n}}\right) \end{aligned}$$

(This matches the k that was provided in the book/lecture/homework, with $c = -z_\alpha$. Also, it turns out that the power function given above is derived from LRT, so with the significance level set at α, the result should makes sense in this case.)