Exam 1 Solution

1. (a) (i) the same plot.
   (b) (ii) will give different numbers.
   (c) (ii) e_i is unknown and random.
   (d) (iii) must pass through both $\bar{x}$ and $\bar{y}$ (see page 28 of textbook).
   (e) (ii) $\bar{y}$ (can easily derive this, or note that $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$ but $\beta_1 = 0$ here).

2. (a) Since $T = \hat{\beta}/\text{se}(\hat{\beta})$, we just need `0.8/0.5846`
   (b) `pt(abs(4.312), df=3, lower.tail=F)*2`

3. (a) The R code that will fit the model $y = \beta_1 x + e$ is
   `lm(Late ~ Early - 1, data=ftcollinssnow)`
   (b) Following the ALR Primer, we can let `m1<-lm(Late ~ Early - 1, data=ftcollinssnow)` and take `sqrt(diag(vcov(m1)))`, or simply take `sqrt(vcov(m1))` in this case (why?), to obtain $\text{se}(\hat{\beta}_1)$.
   Or else, since $\text{se}(\hat{\beta}_1) = \sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}$, and since in this case $\text{Var}(\hat{\beta}_1) = \text{Var}(\hat{\beta}_1|x) = \sigma^2/\sum x_i^2$ (Problem 2.17 or Quiz 2), we must have for this problem that $\text{se}(\hat{\beta}_1) = \hat{\sigma}/\sqrt{\sum x_i^2}$.
   Also, $\hat{\sigma}^2 = RSS/(n-1)$, where $RSS = \sum \hat{e}_i^2 = \sum(y_i - \hat{y}_i)^2 = \sum(y_i - \hat{\beta}_1 x_i)^2$, with $\hat{\beta}_1 = \sum x_i y_i/\sum x_i^2$. Thus, the R codes to follow these are:

```
x<-ftcollinssnow$Early
y<-ftcollinssnow$Late
n<-dim(ftcollinssnow)[1] # can also do length(y) or nrow(ftcollinssnow)
beta1hat<-sum(x*y)/sum(x^2)
yhat<-beta1hat*x
ehat<-y-yhat
rss<-sum(ehat^2)
sigmahat2<-rss/(n-1)
sigmahat<-sqrt(sigmahat2)
se1<-sigmahat/sqrt(sum(x^2))
```

   The $\text{se}(\hat{\beta}_1)$ computed either way will match up with the output from the code `summary(lm(Late ~ Early - 1, data=ftcollinssnow))`.

4. Since $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{e}_i$,

$$\begin{aligned}
\sum \hat{e}_i x_i &= \sum(y_i - \hat{y}_i)x_i = \sum(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)x_i = \sum(y_i - \bar{y} + \hat{\beta}_1\bar{x} - \hat{\beta}_1 x_i)x_i \\
&= \sum[(y_i - \bar{y}) + \hat{\beta}_1(\bar{x} - x_i)]x_i = \sum[(y_i - \bar{y}) - \hat{\beta}_1(x_i - \bar{x})]x_i \\
&= \sum[(y_i - \bar{y})x_i - \hat{\beta}_1(x_i - \bar{x})x_i] = \sum(y_i - \bar{y})x_i - \hat{\beta}_1 \sum(x_i - \bar{x})x_i \\
&= SXY - \hat{\beta}_1 SXX = SXY - \frac{SXY}{SXX}SXX = SXY - SXY = 0
\end{aligned}$$