

Homework 2

Due Tuesday, September 24

Show all your work. Data files available on class website. In all problems, assume the simple regression model $y_i = \beta_0 + \beta_1 x_i + e_i$, $i = 1, \dots, n$. For Problems 4 to 6, you are again encouraged to use R. All H_A are “ \neq ” alternative.

1. Show that $\sum \hat{e}_i = 0$ and that $\sum \hat{e}_i x_i = 0$.
2. Show that

$$SYY = RSS + SSreg = \sum (y_i - \hat{y}_i)^2 + \sum (\hat{y}_i - \bar{y})^2 = \sum \hat{e}_i^2 + \sum (\hat{y}_i - \bar{y})^2$$

(HINT: Subtract and add \hat{y}_i inside the parenthesis of $SYY = \sum (y_i - \bar{y})^2$ to start).

3. Show that

$$t = \frac{\hat{\beta}_1}{\text{se}(\hat{\beta}_1)} = \sqrt{n-2} \frac{r_{xy}}{\sqrt{1-r_{xy}^2}}$$

4. Refer to HW 1, Problem 4. Perform the t -tests for $H_0 : \beta_0 = 0$ and $H_0 : \beta_1 = 0$ (at the level $\alpha = 0.05$). Find the R^2 and the adjusted R^2 .
5. Refer to HW 1, Problem 5.
 - (a) Perform the t -tests for $H_0 : \beta_0 = 0$ and $H_0 : \beta_1 = 0$ (at the level $\alpha = 0.1$). Find the R^2 and the adjusted R^2 .
 - (b) Find the predicted value and the 95% prediction interval for `ht=170`.
6. Consider the dataset `heights.txt`, where *Dheight* and *Mheight* are daughter and mother heights, respectively, for each daughter-mother pair.
 - (a) Perform the regression of *Dheight* on *Mheight* (i.e., Y on X), and report the estimates for β_0 and β_1 , their standard errors, the value of the coefficient of determination (the R^2 and the adjusted R^2), and the residual standard error, $\hat{\sigma}$. Perform the t -tests for $H_0 : \beta_0 = 0$ and $H_0 : \beta_1 = 0$ (at the level $\alpha = 0.01$).
 - (b) Obtain a prediction and 99% prediction interval for a daughter whose mother is 64 inches tall.