Homework 8 Solution

9.2.1 Since $\text{tr}(\mathbf{H}) = \sum h_{ii}$ and $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$, we see that

$$\sum h_{ii} = \text{tr}(\mathbf{H}) = \text{tr}(\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}') = \text{tr}(\mathbf{X}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}) = \text{tr}(\mathbf{I}) = (p+1) = p'$$

since the dimension of $\mathbf{I}$ is $(p+1) \times (p+1)$, proving (9.9).

Now, since $\mathbf{HX} = \mathbf{X}$, and the first column of $\mathbf{X}$ is $\mathbf{1}_n$, it follows immediately that $\mathbf{H1}_n = \mathbf{1}_n$, from which it follows that $\sum_{j=1}^n h_{ij} = 1$, for all $i = 1, \ldots, n$. Moreover, because $\mathbf{H}$ is symmetric, $\mathbf{H} = \mathbf{H}'$, so that $\mathbf{H}'\mathbf{X} = \mathbf{HX} = \mathbf{X}$, which implies that $\mathbf{H}'\mathbf{1}_n = \mathbf{1}_n$, from which it follows that $\sum_{i=1}^n h_{ij} = 1$, for all $j = 1, \ldots, n$. Therefore,

$$\sum_{i=1}^n h_{ij} = \sum_{j=1}^n h_{ij} = 1$$

proving (9.10).