Midterm Exam

October 29, 2024

1. Consider the model $y_i = \beta x_i + e_i$, $i = 1, \ldots, n$. Find $\hat{\beta}$ in terms of $\sum$ and x_i and y_i.

2. We are given the following codes in R (with line numbers on far left).

```
1> X<-matrix(c(rep(1,74),x1,x2,x3,x4,x5),ncol=?)
2> bhat<-solve(t(X)%*%X)%*%t(X)%*%y
3> yhat<-X%*%bhat
4> ehat<-y-yhat
5> rss<-t(ehat)%*%(ehat)
6> sigma2hat<-rss/(??)
7> varhatbhat<-sigma2hat[1,1]*solve(t(X)%*%X)
8> se<-sqrt(diag(varhatbhat))
9> tstat<-bhat/se
```

   (a) Find the missing values at lines 1 and 6 (need numerical values).

   (b) Write a command to compute the p-values of the t-statistics of line 9.

3. Suppose that we are given the model $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + e_i$ for $i = 1, 2, 3$, with the matrix form $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}$, and the design matrix $\mathbf{X}$ given by

$$\mathbf{X} = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$

   Suppose that $\hat{\sigma}^2$ is known and given by $\hat{\sigma}^2 = 12$. Show that $\mathrm{se}(\hat{\beta}_0) = 2$.