MATH 522 - FALL 2017 - TOPICS

1. Review and basic material

Sequences and series, uniform convergence, uniform continuity, power series

2. Approximation theory and compactness

2.1. Fourier series. Fourier coefficients, Fourier series, Dirichlet kernel, Fejér kernel, Fejér's theorem, Plancherel-Parseval theorem, approximation of unity, trigonometric polynomials

2.2. Weierstrass' theorem. Weierstrass' theorem, Bernstein polynomials

2.3. Compactness in metric spaces. Open cover, compact, Heine-Borel property, relatively compact, sequentially compact, Bolzano-Weierstrass property, bounded vs. totally bounded, characterization theorem of compactness in metric spaces

2.4. Compactness in C(K). Equicontinuity, uniform vs. pointwise boundedness, Arzelà-Ascoli theorem including the converse

2.5. Stone-Weierstrass theorem. Self-adjoint algebra, separating points, vanishing at no point

3. Linear operators and derivatives

3.1. Normed vector spaces. Normed vector space, Banach space, bounded/continuous linear maps, equivalent norms, norms on a finite dimensional space are equivalent, space of bounded linear operators, operator norm, dual spaces are Banach spaces, linear maps on finite dimensional spaces are bounded

3.2. Inequalities. L^p norms, Hölder's, Young's, Cauchy-Schwarz and Minkowski's inequalities in \mathbb{R}^n , duality of L^p norms, Hölder dual exponent

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3.3. **Differentiation.** Total/Fréchet derivative in normed vector spaces, chain rule, directional derivatives, partial derivatives in \mathbb{R}^n , Jacobian, gradient, C^1 functions, Lipschitz continuity, Lipschitz constant, partial differentiability vs. total differentiability

3.4. The contraction principle. Contractions, Banach's fixed point theorem including estimate for speed of convergence

3.5. Multivariable calculus. Inverse function theorem, implicit function theorem, higher order derivatives, Schwarz' theorem, multiindex notation, Taylor's theorem in \mathbb{R}^n , mean value theorem, Hessian matrix, local minima and maxima, critical point, second derivative test using Hessian matrix

3.6. Submanifolds of \mathbb{R}^n . Immersions, submanifold, regular curve, singular point, homeomorphism, local parametrization/coordinate system, alternative definitions for submanifold, tangent vectors, tangent space, normal vectors, normal space, Lagrange multipliers, [quadratic forms and the spectrum of real symmetric matrices¹]

3.7. Integrals depending on a parameter. Differentiating parameter integrals, differentiation under the integral, Fubini's theorem for double integrals

3.8. **Differential equations.** Initial value problem for first-order ordinary differential equations, geometric interpretation and examples, Picard-Lindelöf theorem on existence and uniqueness, Peano's existence theorem

4. Other topics

4.1. Baire category theorem and applications. nowhere dense, meager/first category, comeager/residual sets, Baire category theorem, applications to nowhere differentiable functions and sets of continuity, F_{σ} and G_{δ} sets

4.2. Elementary functional analysis. (This is not relevant for the final exam.)

uniform boundedness principle, application to Fourier series, open mapping theorem, closed graph theorem

4.3. Fractal dimensions. (*This is not relevant for the final exam.*) box-counting dimension, examples of fractals, Cantor set, sets of Lebesgue measure zero, Besicovitch-Kakeya sets and the Kakeya conjecture

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¹Not relevant for the final exam.