MATH 522 – SPRING 2018 – TOPICS

1. Review

Complex numbers, sequences and series, uniform convergence, uniform continuity

2. Approximation theory and compactness

2.1. **Power series.** power series, radius of convergence, uniform convergence of power series, derivatives of power series, Abel's theorem, Leibniz formula, fundamental theorem of algebra

2.2. **Fourier series.** Fourier coefficients, Fourier series, orthonormal systems, Dirichlet kernel, Fejér kernel, Fejér's theorem, Riemann-Lebesgue lemma, Bessel's inequality, Parseval's theorem, approximation of unity, trigonometric polynomials

2.3. **Weierstrass' theorem.** Weierstrass' theorem, Bernstein polynomials

2.4. **Compactness in metric spaces.** Open cover, compact, Heine-Borel property, relatively compact, sequentially compact, Bolzano-Weierstrass property, bounded vs. totally bounded, characterization theorem of compactness in metric spaces

2.5. **Compactness in $C(K)$.** Equicontinuity, uniform vs. pointwise boundedness, Arzelà-Ascoli theorem including the converse

2.6. **Stone-Weierstrass theorem.** Self-adjoint algebra, separating points, vanishing at no point

3. Linear operators and derivatives

3.1. **Normed vector spaces.** Normed vector space, Banach space, bounded/continuous linear maps, equivalent norms, norms on a finite dimensional space are equivalent, space of bounded linear operators, operator norm, dual spaces are Banach spaces, linear maps on finite dimensional spaces are bounded

3.2. **Inequalities.** L^p norms, Hölder's, Young's, Cauchy-Schwarz and Minkowski's inequalities in $\mathbb{R}^n$ and for sequential ℓ^p spaces

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3.3. **Differentiation.** Total/Fréchet derivative in normed vector spaces, chain rule, directional derivatives, partial derivatives in $\mathbb{R}^n$, Jacobian, gradient, C^1 functions, partial differentiability vs. total differentiability

3.4. **Contraction principle and ODEs.** Contractions, Banach's fixed point theorem, Picard-Lindelöf theorem on existence and uniqueness

3.5. **Multivariable calculus.** Inverse function theorem, implicit function theorem, higher order derivatives, Schwarz' theorem, multiindex notation, Taylor's theorem in $\mathbb{R}^n$, mean value theorem, Hessian matrix, local minima and maxima, critical point, second derivative test using Hessian matrix

3.6. **Convex optimization.** Linear regression, gradient descent, convex and strictly convex functions, characterization of convex functions using first- and second-order derivatives, critical points of convex functions, strong convexity, sublevel sets, existence of global minimum for strongly convex functions, convergence theorem for gradient descent