Math 629, Spring 2019 – Homework 4.
Due Monday, February 25.

Do Exercise **16** in Chapter 1.6[1] and Problems **1, 4** in Chapter 1.7.

Optional (not graded):

1. Let Σ denote the Borel σ-algebra in $\mathbb{R}^d$. Let $\widetilde{m} : \Sigma \to [0, \infty]$ be a map such that

(1) $\widetilde{m}([0,1]^d) = 1$,
(2) $\widetilde{m}(\cup_{j=1}^{\infty} E_j) = \sum_{j=1}^{\infty} \widetilde{m}(E_j)$ for pairwise disjoint sets $(E_j)_j$ in Σ,
(3) $\widetilde{m}(x + E) = \widetilde{m}(E)$ for every $x \in \mathbb{R}^d$ and every $E \in \Sigma$

Show that $\widetilde{m}$ coincides with the Lebesgue measure, i.e. that $m(E) = \widetilde{m}(E)$ holds for every $E \in \Sigma$.

2. (a) Read about the Brunn-Minkowski inequality in Chapter 1.5.
(b) For a measurable set $A \subset \mathbb{R}^d$ with $m(A) < \infty$ define A^* to be the open ball centered at the origin such that $m(A) = m(A^*)$. Prove that $A^* + B^* \subset (A + B)^*$.

[1] Numbering refers to the textbook *Real Analysis* by E. M. Stein and R. Shakarchi.